

# ANALYTICAL APPROXIMATION FOR THE PRICE DYNAMICS OF SPARK SPREAD OPTIONS

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**ABSTRACT.** This paper presents an analytic approximation for the pricing dynamics of spark spread options in terms of Fourier transforms. We propose to model the spark spread, that is, the price difference of electricity and gas, directly using a mean-reverting model with diffusion and jumps. The model is analyzed empirically, and shown to fit observed data in the UK reasonably well. The main advantage with the model is that the spark spread of electricity and gas forwards, being forwards with delivery over periods, can be priced analytically. The price dynamics for different spark spread options with electricity and gas forwards as underlyings, is analytically derived through Fourier transforms. These pricing expressions allow for efficient numerical valuations via the fast Fourier transform (FFT) technique.

## 1. INTRODUCTION

Spark spread options are option contracts written on the price difference of electricity and gas, being e.g., the spot or forward price. Such contracts can provide a producer of electricity, say, protection against too high gas prices, or be used as a tool to value a power plant (as a real option). The spark spread also appears in exotic derivatives contracts like tolling agreements, where the holder is allowed to produce and sell electricity at advantageous prices, or simply hold back if the price differential is less than a given threshold. Tolling agreements are typical examples of swing options, where the holder has additional optionality in choosing the level and timing for striking the contract.

It is well-known, at least in the case when the two underlying processes are geometric Brownian motions, that there are no analytical pricing formula like the Black & Scholes formula for spark spread options, except in the case where the strike is equal to zero. When the option is a call or put on the price differential directly, then one may use the Margrabe Formula (see Margrabe [14]) to value the option. The most common models for spot energies are of an exponential Ornstein-Uhlenbeck type (known as the Schwartz model, see Schwartz [16]), and include features like mean-reversion and jumps. These processes

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generalize the geometric Brownian motion dynamics, and just add up the complexity of the pricing problem for spread options.

Most commonly, the spark spread options are written on the difference of the forward prices of the two underlying commodities. Forwards in gas and electricity are traded with delivery over a *period* rather than at a fixed future point in time. If we use a Schwartz model for both energies, it is possible to derive explicit dynamics for fixed-delivery forwards (see e.g. Benth et al. [3]), but for forward contracts delivering over a time period there exist no explicit dynamics. Thus, to price spark spread options when the spot price processes are modelled marginally by exponential processes, we are forced to apply numerical valuation techniques for price path simulation and numerical integration. The standard approaches are Monte Carlo simulation or numerical solution of multi-dimensional partial differential equations.

Insisting on modeling the price processes of the two underlyings using geometric Brownian motion, and considering spark spread options on fixed-delivery forward contracts, lead us to calculating the expected value of a payoff function depending on the difference between to lognormal random variables. In the paper by Carmona and Durrleman [8] it is suggested to approximate the difference of two lognormal random variables with a normal random variable. Lima [12] observed that the difference between two exponential Ornstein-Uhlenbeck processes could not be approximated very well with a normal distribution, but instead showed features like skewness and heavy tails. This led Lima [12] to propose to model the difference using the Normal inverse Gaussian distribution (NIG) introduced in finance by Barndoff-Nielsen [1].

Taking the ideas above one step further, one may ask whether the spark spread *data* can be modelled *directly* using a mean-reverting process with jumps, having additive noise rather than geometric. This paper proposes a seasonally varying Ornstein-Uhlenbeck process to model the spark spread data, where the random innovations are given by a Wiener process and compound Poisson processes. We let the small price difference variations be modelled by a diffusion, while the bigger jumps that we observe in the market is captured by a compound Poisson process. Analysing gas and electricity data from the UK demonstrates that this additive model fits the observed spark spreads reasonably well.

The main advantage with this model is the pricing tractability. We are able to derive an explicit dynamics of the spark spread forwards for arbitrary delivery periods, and the price dynamics of general option contracts written on the spark spread of electricity and gas forwards can be represented by explicit inverse Fourier transforms. The pricing formula when we include jumps in the modeling is not as explicit as the Margrabe formula, say, but can easily be calculated using numerical methods like the fast Fourier transform. To base the pricing on a two-dimensional stochastic process, one must use advanced numerical tools which will lead to more complicated and less efficient methods for finding the price. Moreover, if one wants to analyse the price dynamics, one is forced to numerically simulate conditional expectations in the case of a Monte Carlo approach, which needs a large number of simulations before converging. Our simple formula is more efficient in finding the price dynamics, and we claim that it is better if one wants to perform risk evaluations

on a portfolio, say, consisting of several spark spread option contracts and other nonlinear products.

The disadvantage with our way to analyse the problem is that we loose the connection with the marginal processes in the pricing formula (except indirectly via the estimated parameters). To find a two-dimensional stochastic process that marginally model gas and electricity, and also includes the correct dependency structure may be a challenging task. It is known that energies must be modelled by jump processes, however, it is not straightforward to build reasonable models where the dependency comes into play in an empirically correct way. Copulas is a modeling tool that seems to be promising, however, we shall not pursue this idea further in the present paper, but leave it for future research (see Benth and Kettler [5]). We remark that such a modeling does not lead to any simple pricing formula for spark spread options, on the contrary, they need to be calculated by numerical techniques as discussed above.

The paper is organized as follows: In Section 2 we define the market and introduce the spark spread options that we are going to analyze, together with some discussions about our proposed model. In Section 3 we continue with defining the stochastic dynamics of the spark spread, along with the characteristic functions required for calculating the forward price dynamics. Using Fourier methods, we present formulas for spark spread options price dynamics in Section 4, being in a suitable form for applying fast Fourier transform techniques. Finally, in Section 5 we perform an empirical study of electricity and gas data collected in the UK, and demonstrate the reasonability of our model together with a discussion of option pricing.

## 2. SPARK SPEAD OPTIONS

In this section we introduce at a formal level the spark spread options that we are going to analyze, and establish some connections related to our proposed modeling view. Let  $E(t)$  be the electricity spot price and  $G(t)$  the gas spot price at time  $t$ , modelled as two stochastic processes on the complete probability space  $(\Omega, \mathcal{F}, P)$  equipped with a filtration  $\{\mathcal{F}_t\}_{t \in [0, T]}$ .  $T$  is the time horizon of the market. If  $c$  is the heat rate, that is, the factor converting gas prices into the units of electricity prices, the spark spread difference between electricity and gas is

$$(2.1) \quad S(t) = E(t) - cG(t).$$

We call  $S(t)$  the *spark spread*. A forward contract on electricity and gas deliver over a period rather than at a fixed point in time. We consider a forward contract written on a spot with price  $X(t)$  having delivery over the period  $[\tau_1, \tau_2]$ . The forward price at time  $t \leq \tau_1$  is defined to be the risk-neutral expectation of the delivery, that is;

$$(2.2) \quad F(t, \tau_1, \tau_2) = \mathbb{E}_Q \left[ \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} X(u) du \mid \mathcal{F}_t \right].$$

This forward price can be derived using the theory of no-arbitrage together with the fact that it is costless to enter a forward contract. In the market, such contracts are settled

either physically or financially. Let  $F_{\text{el}}$  and  $F_{\text{gas}}$  denote the electricity and gas forward, respectively.

Consider a spark spread option written on the difference of electricity and gas. The price of an option with exercise at time  $T$  with strike  $K$  is defined as

$$(2.3) \quad C(t) = e^{-r(T-t)} \mathbb{E}_Q [\max(F_{\text{el}}(T, \tau_1, \tau_2) - cF_{\text{gas}}(T, \tau_1, \tau_2) - K, 0) | \mathcal{F}_t],$$

where  $r$  is the risk-free interest rate. The risk-neutral probability  $Q$  is the same for both forwards and the option. It is a probability measure that will change the underlying characteristics of the processes. We suppose that there is a measure  $Q$  describing the risk preferences in both the gas and electricity markets. This can be done without loss of generality since we model both markets jointly, and the  $Q$ -probability refer to the stochastic processes.

We now elaborate a bit on the underlying forward price difference in the spark spread option price:

$$(2.4) \quad \begin{aligned} F_{\text{spread}}(t, \tau_1, \tau_2) &= F_{\text{el}}(t, \tau_1, \tau_2) - cF_{\text{gas}}(t, \tau_1, \tau_2) \\ &= \frac{1}{\tau_2 - \tau_1} \mathbb{E}_Q \left[ \int_{\tau_1}^{\tau_2} E(u) - cG(u) du | \mathcal{F}_t \right] \\ &= \mathbb{E}_Q \left[ \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} S(u) du | \mathcal{F}_t \right]. \end{aligned}$$

Thus, we see that the spark spread option can be considered as an option written on a forward contract with delivery of the spread difference over the period  $[\tau_1, \tau_2]$ . Motivated from this, we can price the spread option based on a model for the spread difference directly, rather than modeling the electricity and gas separately.

Note that this view is not far from the idea introduced in Carmona and Durrleman [8]. There the authors model each commodity as a geometric model, linked by a correlated two-dimensional Brownian motion. Only for  $K = 0$  it is possible to price analytically in terms of a Black & Scholes type formula. To deal with a strike which is different than zero, they argue that the difference between two lognormal variables can be approximated by a normal distribution, for which prices can be explicitly found. In Lima [12] it is demonstrated that the difference for electricity and gas is not very well captured by the normal distribution when basing the marginal dynamics on a mean-reversion model. The joint geometric model gives heavy tails in the difference, something that is important for spread options which are far out of the money. We argue further that when applying geometric models marginally, we loose in general the analytical expression for the forward price dynamics. The approach of modeling the spark spread directly by an additive model, provides us with an explicit dynamics for the spark spread forward and thus tractable expressions in terms of Fourier transforms for the option prices. The reader is referred to Benth, Kallsen and Meyer-Brandis [4] for additive models applied to electricity markets. We now move on with a more detailed analysis.

## 3. MODELING AND ANALYSIS OF THE SPARK SPREAD

In this Section we propose a model for the spark spread. Moreover, we derive some key formulas involving the characteristic function of different expressions of the spark spread, that becomes useful when pricing forward contracts and options on these.

Our proposal is to model the spark spread directly by a non-Gaussian Ornstein-Uhlenbeck process. Suppose the dynamics of  $S(t)$  is decomposed into a seasonal part  $\Lambda(t)$  and a stochastic part  $\tilde{S}(t)$  as follows,

$$(3.1) \quad S(t) = \Lambda(t) + \tilde{S}(t).$$

The dynamics of  $\tilde{S}$  is given by a non-Gaussian Ornstein-Uhlenbeck process,

$$(3.2) \quad d\tilde{S}(t) = -\alpha\tilde{S}(t) dt + \sigma dB(t) + dL(t),$$

where  $B$  is a Brownian motion and  $L$  is a pure-jump Lévy process. We suppose that  $B$  and  $L$  are independent processes. The volatility  $\sigma$  is a non-negative constant. We denote by  $\psi(\theta)$  the cumulant function of  $L$ , defined as ( $\theta$  being a real number)

$$(3.3) \quad \psi(\theta) = \ln \mathbb{E} [\exp (i\theta L(1))] .$$

The Lévy-Kintchine Formula gives an explicit expression for  $\psi$  in terms of the Lévy measure of  $L$ ,

$$(3.4) \quad \psi(\theta) = \int_{\mathbb{R}} \{e^{i\theta z} - 1 - i\theta z 1_{|z|<1}\} \ell(dz).$$

Here, the Lévy measure  $\ell(dz)$  is a  $\sigma$ -finite Borel measure on the real line with  $\ell(\{0\}) = 0$  and

$$\int_{\mathbb{R}} \min(1, z^2) \ell(dz) < \infty.$$

We suppose that  $L$  has finite variance, which is equivalent to assuming that

$$(3.5) \quad \int_{\mathbb{R}} z^2 \ell(dz) < \infty.$$

We note that for a measurable and bounded function  $g$ , the characteristic function of  $\int_a^b g(u) dL(u)$ ,  $0 \leq a < b$  is

$$(3.6) \quad \mathbb{E} \left[ \exp \left( i\theta \int_a^b g(u) dL(u) \right) \right] = \exp \left( \int_a^b \psi(\theta g(u)) du \right).$$

A useful observation for later is the relation

$$(3.7) \quad \mathbb{E} \left[ \int_a^b g(u) dL(u) \right] = (-i\psi'(0)) \int_a^b g(u) du.$$

For more theory on Lévy processes and their applications in finance, see the book by e.g. Shiryayev [16].

We need to specify the dynamics of  $\tilde{S}$  under a risk-neutral probability  $Q$ , and we suppose from now on that  $Q$  is defined via the Girsanov transform for which

$$(3.8) \quad dW(t) = dB(t) - \frac{\mu_{\text{mpr}}}{\sigma} dt$$

is a Brownian motion. Here,  $\mu_{\text{mpr}}$  is a parameter that we will interpret as the market price of risk, measuring the reward that market traders charge for the risk of not being able to hedge perfectly any derivative using gas and/or electricity. The market price of risk is an additional parameter in the model specification that needs to be calibrated from historical derivatives price data. Note that under this change of probability, the characteristics of  $L$  remains unaltered, which in other words mean that we suppose that there is no market price of risk connected to the jumps. This is a frequently used assumption (see e.g. Cartea and Figueroa [11]), simplifying the further analysis considerably. The  $Q$ -dynamics of  $\tilde{S}$  becomes

$$(3.9) \quad d\tilde{S}(t) = \left( \mu_{\text{mpr}} - \alpha \tilde{S}(t) \right) dt + \sigma dW(t) + dL(t).$$

We have the following explicit results for  $\tilde{S}(t)$  and its time integral:

**Lemma 3.1.** *Under the risk-neutral measure  $Q$ , we have*

$$(3.10) \quad \tilde{S}(t) = \tilde{S}(0)e^{-\alpha t} + \frac{\mu_{\text{mpr}}}{\alpha}(1 - e^{-\alpha t}) + \sigma e^{-\alpha t} \int_0^t e^{\alpha u} dW(u) + e^{-\alpha t} \int_0^t e^{\alpha u} dL(u),$$

and

$$(3.11) \quad \int_{\tau_1}^{\tau_2} \tilde{S}(u) du = -\frac{1}{\alpha} \tilde{S}(\tau_2) + \frac{1}{\alpha} \tilde{S}(\tau_1) + \frac{\mu_{\text{mpr}}}{\alpha}(\tau_2 - \tau_1) + \frac{\sigma}{\alpha} (W(\tau_2) - W(\tau_1)) \\ + \frac{1}{\alpha} (L(\tau_2) - L(\tau_1)).$$

*Proof.* Equation (3.11) follows by a direct integration of the stochastic differential equation (3.2). Appealing to the Itô Formula of semimartingales we find (3.10).  $\square$

We can from these two explicit representations derive the characteristic functions of  $\tilde{S}(t)$  and  $\int_{\tau_1}^{\tau_2} \tilde{S}(t) dt$ , being key formulas in calculating forward prices and options on these. The characteristic functions are stated in next two Lemmas:

**Lemma 3.2.** *For  $\theta \in \mathbb{R}$  and  $0 \leq t \leq \tau$ , it holds that*

$$(3.12) \quad \mathbb{E}_Q \left[ \exp \left( i\theta \tilde{S}(\tau) \right) \mid \mathcal{F}_t \right] = \exp \left( i\theta (\mu(t, \tau) + e^{-\alpha(\tau-t)} \tilde{S}(t)) \right. \\ \left. - \frac{\theta^2 \sigma^2}{4\alpha} (1 - e^{-2\alpha(\tau-t)}) + \Psi(t, \tau; \theta) \right),$$

where

$$\mu(t, \tau) = \frac{\mu_{\text{mpr}}}{\alpha} \{1 - e^{-\alpha(\tau-t)}\} \\ \Psi(t, \tau; \theta) = \int_t^\tau \psi(\theta e^{-\alpha(\tau-u)}) du.$$

*Proof.* For notational simplicity, define  $\tilde{L} = \sigma W + L$ , which is a Lévy process. Integrating (3.2) from  $t$  to  $\tau$  yields

$$(3.13) \quad \tilde{S}(\tau) = \tilde{S}(t)e^{-\alpha(\tau-t)} + \frac{\mu_{\text{mpr}}}{\alpha}(1 - e^{-\alpha(\tau-t)}) + e^{-\alpha\tau} \int_t^\tau e^{\alpha u} d\tilde{L}(u).$$

From the  $\mathcal{F}_t$ -measurability of  $\tilde{S}(t)$  and independence property of the increments of  $\tilde{L}$ , we find

$$\begin{aligned} \mathbb{E}_Q \left[ \exp \left( i\theta \tilde{S}(\tau) \right) \mid \mathcal{F}_t \right] &= \exp \left( i\theta \mu(t, \tau) + i\theta e^{-\alpha(\tau-t)} \tilde{S}(t) \right) \\ &\quad \times \mathbb{E}_Q \left[ e^{i\theta e^{-\alpha\tau} \int_t^\tau e^{\alpha u} d\tilde{L}(u)} \right]. \end{aligned}$$

Hence, the Lemma follows after appealing to the characteristic function of the integral of the Lévy process  $\tilde{L}$ , as given in (3.6).  $\square$

The next Lemma states the similar result for  $\int_{\tau_1}^{\tau_2} \tilde{S}(u) du$ :

**Lemma 3.3.** *For  $\theta \in \mathbb{R}$  and  $0 \leq t \leq \tau_1 < \tau_2$ , it holds that*

$$(3.14) \quad \mathbb{E}_Q \left[ \exp \left( i\theta \int_{\tau_1}^{\tau_2} \tilde{S}(u) du \right) \mid \mathcal{F}_t \right] = \exp \left( i\theta \lambda_1(t, \tau_1, \tau_2) + \Psi(t, \tau_1, \tau_2; \theta) + i\theta \lambda_2(t, \tau_1, \tau_2) \tilde{S}(t) \right),$$

where

$$\begin{aligned} \lambda_1(t, \tau_1, \tau_2) &= \frac{\mu_{\text{mpr}}}{\alpha} \left\{ (\tau_2 - \tau_1) - \frac{1}{\alpha} (e^{-\alpha(\tau_1-t)} - e^{-\alpha(\tau_2-t)}) \right\} \\ \lambda_2(t, \tau_1, \tau_2) &= \frac{1}{\alpha} \left\{ e^{-\alpha(\tau_1-t)} - e^{-\alpha(\tau_2-t)} \right\} \\ \Psi(t, \tau_1, \tau_2; \theta) &= \int_t^{\tau_1} \psi \left( \frac{\theta}{\alpha} \left\{ e^{-\alpha(\tau_1-u)} - e^{-\alpha(\tau_2-u)} \right\} \right) - \frac{\theta^2 \sigma^2}{2\alpha^2} \left\{ e^{-\alpha(\tau_1-u)} - e^{-\alpha(\tau_2-u)} \right\}^2 du \\ &\quad + \int_{\tau_1}^{\tau_2} \psi \left( \frac{\theta}{\alpha} \left\{ 1 - e^{-\alpha(\tau_2-u)} \right\} \right) - \frac{\theta^2 \sigma^2}{2\alpha^2} \left\{ 1 - e^{-\alpha(\tau_2-u)} \right\}^2 du. \end{aligned}$$

*Proof.* The proof follows the same lines as in Lemma 3.2: Use  $\tilde{L} = \sigma W + L$ , and observe from (3.13) that

$$\begin{aligned} \tilde{S}(\tau_1) - \tilde{S}(\tau_2) &= \tilde{S}(t) \left\{ e^{-\alpha(\tau_1-t)} - e^{-\alpha(\tau_2-t)} \right\} - \frac{\mu_{\text{mpr}}}{\alpha} \left\{ e^{-\alpha(\tau_1-t)} - e^{-\alpha(\tau_2-t)} \right\} \\ &\quad + e^{-\alpha\tau_1} \int_t^{\tau_1} e^{\alpha u} d\tilde{L}(u) - e^{-\alpha\tau_2} \int_t^{\tau_2} e^{\alpha u} d\tilde{L}(u). \end{aligned}$$

Hence, from (3.11), the  $\mathcal{F}_t$ -measurability of  $\tilde{S}(t)$  and the independent increment property of  $\tilde{L}$ , we get that

$$\mathbb{E}_Q \left[ \exp \left( i\theta \int_{\tau_1}^{\tau_2} \tilde{S}(u) du \right) \mid \mathcal{F}_t \right] = \exp \left( i\theta \lambda_1(t, \tau_1, \tau_2) + i\theta \lambda_2(t, \tau_1, \tau_2) \tilde{S}(t) \right)$$

$$\begin{aligned} & \times \mathbb{E}_Q \left[ \exp \left( i \frac{\theta}{\alpha} \int_{\tau_1}^{\tau_2} 1 - e^{-\alpha(\tau_2-t-u)} d\tilde{L}(u) \right) \right] \\ & \times \mathbb{E}_Q \left[ \exp \left( i \frac{\theta}{\alpha} (e^{-\alpha(\tau_1-t)} - e^{-\alpha(\tau_2-t)}) \int_t^{\tau_1} e^{\alpha u} d\tilde{L}(u) \right) \right]. \end{aligned}$$

The Lemma follows from (3.6).  $\square$

We are now in the position to derive the dynamics for the forward price difference underlying the spark spread option:

**Proposition 3.4.** *For  $0 \leq t \leq \tau_1 < \tau_2$ , we have that*

$$(3.15) \quad F_{\text{spread}}(t, \tau_1, \tau_2) = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \Lambda(u) du + \lambda(t, \tau_1, \tau_2) + \delta(t, \tau_1, \tau_2) (S(t) - \Lambda(t)),$$

where

$$(3.16) \quad \lambda(t, \tau_1, \tau_2) = \frac{\mu_{\text{mpr}} - i\psi'(0)}{\alpha} (1 - \delta(t, \tau_1, \tau_2))$$

$$(3.17) \quad \delta(t, \tau_1, \tau_2) = \frac{1}{\alpha(\tau_2 - \tau_1)} (e^{-\alpha(\tau_1-t)} - e^{-\alpha(\tau_2-t)}).$$

*Proof.* Differentiating the right-hand side of the key formula in Lemma 3.3 and letting  $\theta = 0$  lead to the desired result after observing that  $\Psi(t, \tau_1, \tau_2; 0) = 0$ .  $\square$

We remark that applying exponential Ornstein-Uhlenbeck models for the spot prices will not lead to any explicit dynamics for the forward, but has to be represented as an integral. This makes it impossible to derive any analytical option prices for the spark spread forwards. Further, note that both functions  $\mu$  and  $\lambda$  defined in Lemma 3.2 and Prop. 3.4, respectively, are functions involving the market price of risk  $\mu_{\text{mpr}}$ , and note that by definition the spread forward  $F_{\text{spread}}$  is a martingale with respect to  $Q$ .

In Benth and Koekebakker [6] it is shown that there exists no analytical expression for the forward dynamics with delivery over a period, when the underlying spot price process follows a Schwartz model. The forward is not even a Markov process, but depends on all other forwards with shorter delivery periods in a quite complicated way.

If  $\tau_1, \tau_2 \rightarrow \infty$  with  $\tau_2 - \tau_1$  being bounded, we have that  $\delta(t, \tau_1, \tau_2) \rightarrow 0$ . Thus, it follows

$$F_{\text{spread}}(t; \tau_1, \tau_2) \rightarrow \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \Lambda(u) du + \frac{\mu_{\text{mpr}} - i\psi'(0)}{\alpha}.$$

Hence, the forward prices will converge to the average seasonal price modified by the market price of risk and the drift introduced by the Lévy jump process for deliveries in the far future. We also note that the price becomes independent on the current spark spread and will become non-stochastic in the limit. This provides us with a simple tool to estimate the market price of risk from forward price data as long as the market trades in forwards where the delivery period is far into the future.



## 4. PRICING OF SPARK SPREAD OPTIONS USING FOURIER ANALYSIS

In this subsection we analyze the option pricing dynamics written on the spark spread using Fourier analysis. For this purpose, consider an option written on the spark spread with payoff  $f(F_{\text{spread}}(T, \tau_1, \tau_2))$  at the exercise time  $T \leq \tau_1$  for some real-valued Borel measurable function  $f$  such that  $f(F_{\text{spread}}(T, \tau_1, \tau_2)) \in L^1(Q)$ . Denote the price at time  $t \leq T$  of this option by

$$C_{\text{spread}}(t, T, \tau_1, \tau_2) = e^{-r(T-t)} \mathbb{E}_Q [f(F_{\text{spread}}(T, \tau_1, \tau_2)) | \mathcal{F}_t] .$$

Since we shall be mostly interested in call options with payoff function  $f(x) = \max(x - K, 0)$ , we note that for such options the payoff function is at most linearly growing. The following Lemma tells us that options with at most linearly growing payoff functions are indeed  $Q$ -integrable under the hypothesis of square-integrability of the Lévy process  $L$ :

**Lemma 4.1.** *Suppose that  $|f(x)| \leq c(1 + |x|)$  for some positive constant  $c$ . Then we have that  $f(F_{\text{spread}}(T, \tau_1, \tau_2)) \in L^1(Q)$ .*

*Proof.* If  $F_{\text{spread}}(T, \tau_1, \tau_2) \in L^1(Q)$ , then by the linear growth of  $f$  we have that the Lemma holds. We prove this.

From the dynamics in Prop. 3.4, we see that  $F_{\text{spread}}(T, \tau_1, \tau_2) \in L^1(Q)$  whenever

$$\int_0^T e^{\alpha u} dL(u) \in L^1(Q) .$$

Recall that the characteristics of  $L$  is the same under  $P$  and  $Q$ , and appealing to Cauchy-Schwarz' inequality and Itô's Formula for semimartingales, we find that the Lévy integral is integrable under  $Q$  whenever  $L$  is square integrable, which we have assumed in (3.5).  $\square$

Let us first restrict our attention to payoff functions being integrable, i.e., assuming that  $f \in L^1(\mathbb{R})$ . Note that neither call nor put options are included in this case, but we can consider different types of knock-out options. However, call and put options can be analyzed after an appropriate exponential damping, which we will consider in a moment.

Let  $\widehat{f}$  denote the Fourier transform of  $f$ , and  $\star$  be the convolution product. Recall from the inverse Fourier transform that

$$f(x) = \frac{1}{2\pi} \int \widehat{f}(y) e^{-ixy} dy .$$

We derive the following price dynamics for a spark spread option expressed in terms of an inverse Fourier transform:

**Proposition 4.2.** *Let  $f \in L^1(\mathbb{R})$  and  $0 \leq t \leq T \leq \tau_1$ . If  $f(F_{\text{spread}}(T, \tau_1, \tau_2)) \in L^1(Q)$ , then the price dynamics*

$$C_{\text{spread}}(t, T, \tau_1, \tau_2) = C_{\text{spread}}(F_{\text{spread}}(t, \tau_1, \tau_2); t, T, \tau_1, \tau_2)$$

is given by

$$(4.1) \quad C_{\text{spread}}(x; t, T, \tau_1, \tau_2) = e^{-r(T-t)} \frac{1}{2\pi} \int \widehat{f}(y) \widehat{p}_{t, T, \tau_1, \tau_2}(y) e^{\Psi(t, T; -y\delta(T, \tau_1, \tau_2))} e^{-iyx} dy .$$

Here,  $p_{t,T,\tau_1,\tau_2}$  is the probability density of a normal random variable with expectation

$$\frac{-i\psi'(0)}{\alpha}\delta(0, \tau_1, \tau_2) (e^{\alpha T} - e^{\alpha t}) .$$

and variance

$$\frac{\sigma^2}{2\alpha}\delta^2(T, \tau_1, \tau_2) (1 - e^{-2\alpha(T-t)}) .$$

Furthermore, the function  $\Psi$  is defined in Lemma 3.2 and  $\delta$  in Prop. 3.4.

*Proof.* From Prop. 3.4 we find,

$$\begin{aligned} & \mathbb{E}_Q [f(F_{\text{spread}}(T, \tau_1, \tau_2)) | \mathcal{F}_t] \\ &= \frac{1}{2\pi} \int \widehat{f}(y) \mathbb{E}_Q \left[ e^{-iy \left( \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \Lambda(u) du + \lambda(T, \tau_1, \tau_2) + \delta(T, \tau_1, \tau_2) \widetilde{S}(T) \right)} | \mathcal{F}_t \right] dy \\ &= \frac{1}{2\pi} \int \widehat{f}(y) \mathbb{E}_Q \left[ e^{-iy \delta(T, \tau_1, \tau_2) \widetilde{S}(T)} | \mathcal{F}_t \right] e^{-iy \left( \lambda(T, \tau_1, \tau_2) + \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \Lambda(u) du \right)} dy . \end{aligned}$$

The Proposition follows from the key formula in Lemma 3.2.  $\square$

The pricing dynamics of the spread option becomes independent of the market price of risk  $\mu_{\text{mpr}}$ . It comes from a Girsanov change in the Wiener part of the spark spread dynamics only, a risk that can be hedged away. The jump risk, on the other hand, is not priced in our model and therefore there should be no dependency on any market price of risk in the option price dynamics. We also see that in the case of no jumps in the spark spread, i.e.  $L = 0$ , the expectation of the normal random variable becomes zero, in addition to  $\Psi = 0$ . The price can then be easily read off as an expectation of a function of a normal random variable with expectation equal to the forward spread price at time  $t$  and variance as in the Proposition.

We proceed to analyze call and put options from the Fourier perspective. Since the payoff function  $f(x) = \max(x - K, 0)$  for a call option is not in  $L^1(\mathbb{R})$ , it is not covered by the above Proposition. However, following the idea in Carr and Madan [10], we can treat the call option by similar tools as in Prop. 3.4 after an appropriate damping of the linear growth of the payoff. Introduce the following exponentially dampened payoff function

$$(4.2) \quad f_\gamma(x) = e^{-\gamma x} \max(x - K, 0)$$

for  $\gamma > 0$ , and note that

$$f(x) = \frac{e^{\gamma x}}{2\pi} \int \widehat{f}_\gamma(y) e^{-ixy} dy = \frac{1}{2\pi} \int \widehat{f}_\gamma(y) e^{(\gamma - iy)x} dy .$$

The function  $f_\gamma$  is obviously integrable for every positive constant  $\gamma$ , and the price dynamics of a call option is given by the following Proposition:

**Proposition 4.3.** *Suppose that the Lévy measure  $\ell(dz)$  of  $L$  satisfies the exponential integrability condition*

$$\int_1^\infty e^{\gamma z} \ell(dz) < \infty$$

for some  $\gamma > 0$ . Then, the price of a call option with strike  $K$  at exercise time  $T$  can be represented as

$$(4.3) \quad C_{\text{spread call}}(x; t, T, \tau_1, \tau_2) = e^{-r(T-t)} c(t, T, \tau_1, \tau_2) \times \frac{1}{2\pi} \int \widehat{f}_\gamma(y) \widehat{p}_{t,T,\tau_1,\tau_2}^\gamma(y) \exp(\Psi(t, T; (-y - i\gamma)\delta(T, \tau_1, \tau_2))) e^{(-iy+\gamma)x} dy,$$

where  $f_\gamma$  is defined in (4.2),

$$c(t, T, \tau_1, \tau_2) = \exp\left(\frac{i\psi'(0)}{\alpha} \delta(0, \tau_1, \tau_2)(e^{\alpha T} - e^{\alpha t}) - \frac{\gamma^2 \sigma^2}{4\alpha} (1 - e^{-2\alpha(T-t)})\right)$$

and  $p_{t,T,\tau_1,\tau_2}^\gamma$  is the probability density function of a normal random variable with expectation

$$\frac{-i\psi'(0)}{\alpha} \delta(0, \tau_1, \tau_2)(e^{\alpha T} - e^{\alpha t}) - \frac{\gamma \sigma^2}{4\alpha} (1 - e^{-2\alpha(T-t)})$$

and variance

$$\frac{\sigma^2}{2\alpha} (1 - e^{-2\alpha(T-t)}).$$

The other functions are as in Prop. 4.2 above.

*Proof.* Denote by  $f(x) = \max(x - K, 0)$ . We have from the considerations above that

$$\begin{aligned} & \mathbb{E}_Q [f(F_{\text{spread}}(T, \tau_1, \tau_2)) | \mathcal{F}_t] \\ &= \frac{1}{2\pi} \int \widehat{f}_\gamma(y) \mathbb{E}_Q \left[ e^{(-iy+\gamma)\left(\frac{1}{\tau_2-\tau_1} \int_{\tau_1}^{\tau_2} \Lambda(u) du + \lambda(T, \tau_1, \tau_2) + \delta(T, \tau_1, \tau_2) \widetilde{S}(T)\right)} | \mathcal{F}_t \right] dy \\ &= \frac{1}{2\pi} \int \widehat{f}_\gamma(y) \mathbb{E}_Q \left[ e^{(-iy+\gamma)\delta(T, \tau_1, \tau_2) \widetilde{S}(T)} | \mathcal{F}_t \right] e^{(-iy+\gamma)(\lambda(T, \tau_1, \tau_2) + \frac{1}{\tau_2-\tau_1} \int_{\tau_1}^{\tau_2} \Lambda(u) du)} dy. \end{aligned}$$

In order for the expectation to be well-defined, the random variable  $\exp(\gamma\delta(T, \tau_1, \tau_2)\widetilde{S}(t))$  must be integrable. However, it holds that

$$\mathbb{E}_Q \left[ e^{\gamma \int_0^t e^{-\alpha(t-u)} dL(u)} \right] = e^{\int_0^t \psi(-i\gamma e^{-\alpha(t-u)}) du}$$

as long as  $\psi(-i\gamma e^{-\alpha(t-u)})$  is integrable over  $[0, t]$ , which by definition of  $\psi$  and the exponential integrability assumption in the Proposition, holds true.

By extending the key formula in Lemma 3.2 to complex constants  $\theta$ , and using  $\theta = -iy + \gamma$ , we derive the expression in the Proposition.  $\square$

To price put options on the spread, we apply the relation

$$\max(x - K, 0) - \max(K - x, 0) = x - K,$$

to derive that

$$(4.4) \quad C_{\text{spread put}}(x; t, T, \tau_1, \tau_2) = C_{\text{spread call}}(x; t, T, \tau_1, \tau_2) + Ke^{-r(T-t)} - x,$$

with  $x = F_{\text{spread}}(t, \tau_1, \tau_2)$ . This is the put-call parity, which must hold in order to prevent arbitrage opportunities.

The Fourier transforms in Props. 4.2 and 4.3 are tailor-made for applying the fast Fourier transform (FFT) numerical technique. For given models of the jump processes (see the last Section for more details) we will frequently have an analytic expression of the integrand  $\psi$  of  $\Psi$  available, thus, after a numerical or analytical integration the inverse Fourier transform can be calculated by FFT directly, as long as we know the Fourier transform of the payoff function  $f$ . In the next Lemma we state the Fourier transform of the payoff from a call option, which is derived by a straightforward integration:

**Lemma 4.4.** *The Fourier transform of the function  $f_\gamma$  for  $\gamma > 0$  defined in (4.2) is given as*

$$(4.5) \quad \widehat{f}_\gamma(y) = \frac{e^{(iy-\gamma)K}}{\gamma^2 - 2iy\gamma - y^2}.$$

We refer to Carr and Madan [10] for further discussions and numerical examples on the application of the FFT for valuation of option prices.

If we would have modelled gas and electricity spot prices separately as two exponential Ornstein-Uhlenbeck processes, we would have faced two problems. Firstly, the dynamics of the two forward price processes underlying the spark spread option would not have been analytically available, which means that we will have to evaluate numerically the forward price. Secondly, even if we would have the forward prices with delivery over a period available, we face the Fourier transform of a two dimensional payoff function, which is hard to derive for a call option with general strike price  $K$ . Thus, our modeling approach has a big advantage over marginal modeling of the two commodities underlying the spark spread option.

Let us briefly mention how we can represent the pricing formula in terms of convolutions products involving the option payoff function, the normal distribution and a jump distribution. For simplicity, we restrict our attention to integrable pay-off functions  $f(x) \in L^1(\mathbb{R})$ .

Using the definitions of  $\psi$  and  $\Psi$ , we have that

$$\begin{aligned} \exp(\Psi(t, T; -y\delta(t, \tau_1, \tau_2))) &= \exp\left(\int_t^T \psi(-y\delta(t, \tau_1, \tau_2)e^{-\alpha(T-u)})\right) \\ &= \mathbb{E}\left[\exp\left(-iy\delta(t, \tau_1, \tau_2) \int_t^T e^{-\alpha(T-u)} dL(u)\right)\right]. \end{aligned}$$

Hence, if we denote the probability distribution function of

$$-\delta(t, \tau_1, \tau_2) \int_t^T e^{-\alpha(T-u)} dL(u),$$

by  $F_L(x)$ , then the above calculation reveals that the Fourier transform of this measure is equal to

$$\exp(\Psi(t, T; -y\delta(t, \tau_1, \tau_2))).$$

Using this information, we can write the spread option price by the following convolution formula:

$$(4.6) \quad C_{\text{spread}}(x; t, T, \tau_1, \tau_2) = \int_{\mathbb{R}} \{f \star p_{t,T,\tau_1,\tau_2}\}(y-x) F_L(dy)$$

with  $p_{t,T,\tau_1,\tau_2}$  defined in Prop. 4.2. Thus, the price dynamics of a spark spread option can be interpreted as a weighted average of spark spread options written on a forward dynamics without jumps. The weighting is over a modification of the distribution of the jumps.

## 5. MODELING THE SPARK SPREAD FOR UK ELECTRICITY AND GAS

We test our model on spot price data for gas and electricity observed in UK, and discuss different issues concerning the modeling of price spikes by jump processes.

**5.1. Empirical analysis and model fitting.** We had available electricity and gas spot prices recorded in the period 06 02 2001 to 27 04 2004 in United Kingdom. Observations are daily market prices, quoted at each working day. This resulted in 806 data over the period of observation. Electricity spot prices are recorded in £/MWh, while gas is in pence/therm. Data are transformed to spark spread prices according to (2.1) by taking the difference of the electricity spot prices and the gas spot prices multiplied by heat rate  $c$ . The heat rate converts the gas prices into the units of electricity, and was chosen to be  $c = 0.85$ , which seems to be a reasonable number from practice. Spark spread differences are presented in Fig. 1. From this time series we see that in the first two years the spark spreads are less volatile than in the more recent part. This may be a sign that the market conditions changed, however, we shall not perform a more detailed analysis of this.

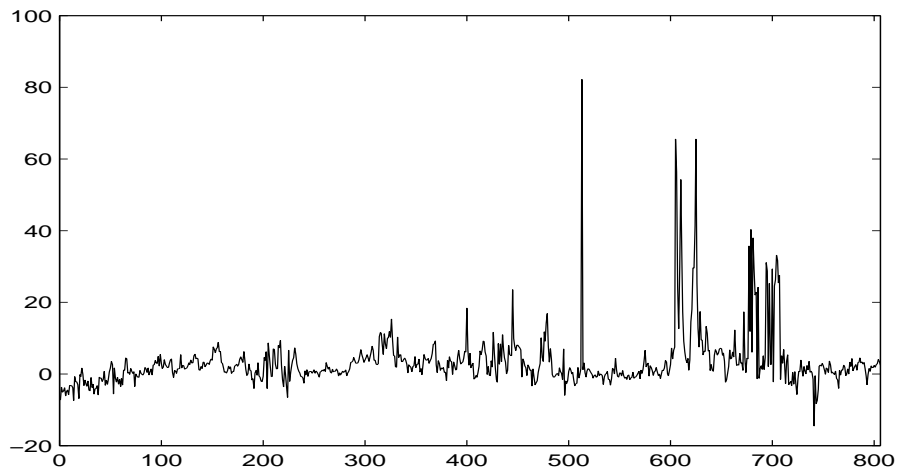


FIGURE 1. Spark spread differences.

To understand better the statistical properties of the spark spread differences, we build a histogram of empirical values and present it together with a fitted normal distribution

density function in Fig. 2. The histogram shows obvious non-normality of data: it has an extremely high peak (kurtosis is equal to 30.22) and very heavy tails. Moreover, it has clear right skewness (skewness is equal to 4.55). Thus, it is not surprising that the normality hypothesis is rejected at the 1% level of significance.

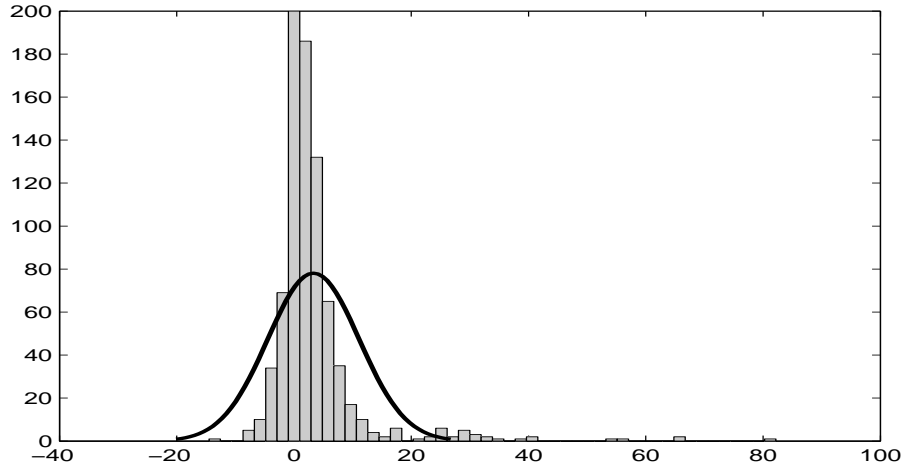


FIGURE 2. Histogram of the spark spread differences with normal density curve.

We assume the seasonality function  $\Lambda(t)$  to have the form  $\Lambda(t) = \Lambda_1(t) + \Lambda_2(t)$ , where

$$(5.1) \quad \Lambda_1(t) = a_0 + a_1 t$$

is the linear trend component, and

$$(5.2) \quad \Lambda_2(t) = b_0 + b_1 \cos(2\pi(t - b_2)/252)$$

describes seasonal effects. By simple linear regression, the slope and intercept were estimated to be  $a_0 = 0.008$  and  $a_1 = 0.273$ , respectively. The value of intercept is statistically insignificant, however the slope is significant at the 1% level. Hence, during the period of observation there has been significant increase in values of the spark spread differences. The seasonality part  $\Lambda_2(t)$  of  $\Lambda(t)$  is fitted to the de-trended data using nonlinear least squares, implemented in Matlab as the function `nlinfit`. The estimated values are presented in Table 1. From the Fig. 3 it is clear that there are quite weak seasonal effects in data. We checked for weekly, monthly and quarterly effects in spark spread differences as well, however they were basically not present and thus ignored. In the further analysis we remove the seasonality function  $\Lambda(t)$  from the spark spread data, and consider the remaining time series.

To detect the mean-reversion in model (3.2), we regress today's data on the data of the previous day. The estimated mean-reverting parameter is  $\alpha = 0.508$ , being significant at the 1% level. Not unexpectedly, the fitted intercept appeared to be statistically insignificant, and is therefore assumed to be equal to zero as in the model (3.2). The residuals from the regression analysis are presented as a time series in Fig. 4.

TABLE 1. Fitted parameters of seasonal function

$b_0$	$b_1$	$b_2$
0.180	-3.307	11.526

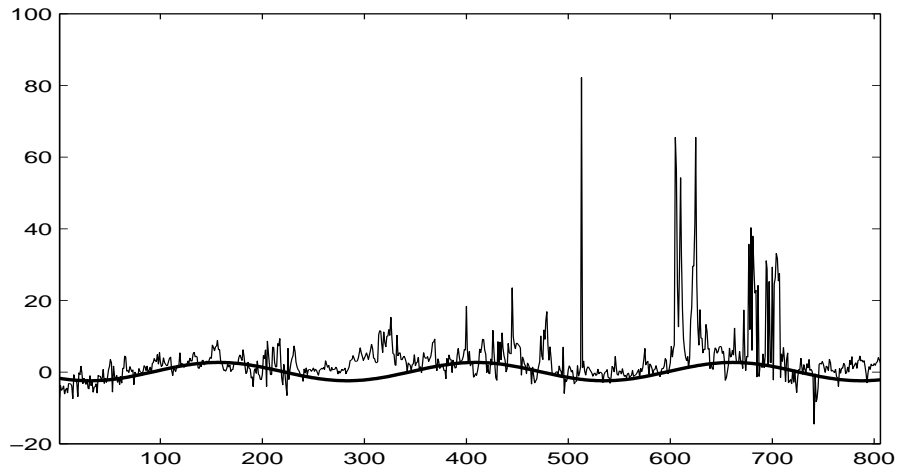


FIGURE 3. De-trended spark spread differences with the seasonality function.

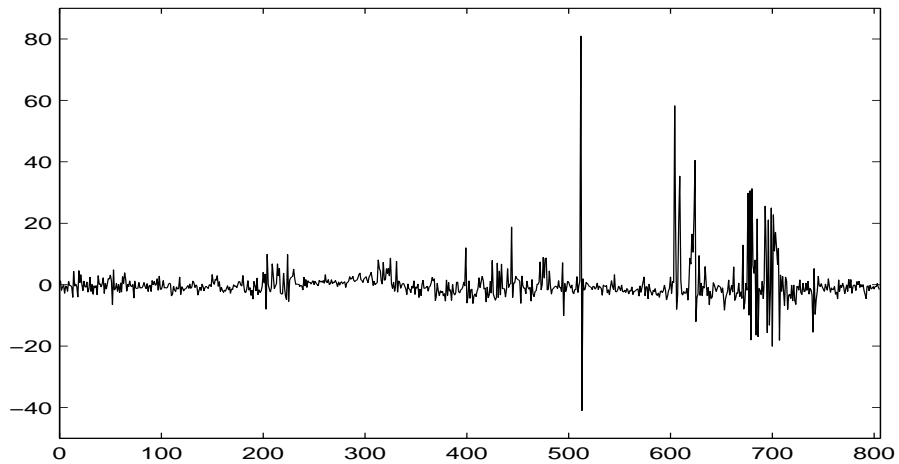


FIGURE 4. Spark spread differences residuals.

The histogram of the residuals of the spark spread differences with the fitted normal density curve is presented in Fig. 5, where we see that the residuals are far from being normally distributed. The residuals has a mean close to zero, and standard deviation

equal to 6.23. They have an extremely high peak (kurtosis is equal to 53.14) and very heavy tails. Moreover, the skewness is equal to 4.80, indicating quite distinct-right hand skewness in the residuals. We continue our analysis by investigating different specifications of the Lévy process  $L(t)$  in the spark spread dynamics (3.2) to fit these residuals.

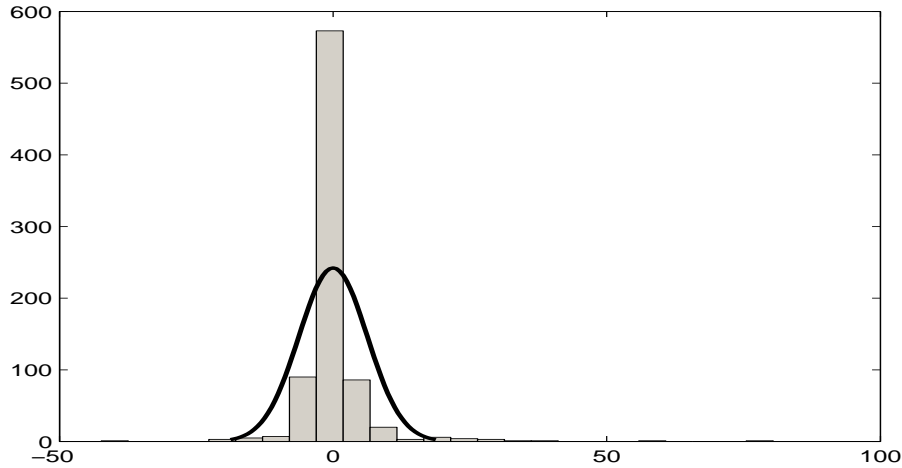


FIGURE 5. The histogram of the spark spread differences residuals.

An analytically tractable and flexible class of models for residuals having skewness and heavy tails is the normal inverse Gaussian (NIG) Lévy process. The NIG Lévy process  $L(t)$  is defined by letting  $L(1)$  being NIG distributed (and in addition putting  $\sigma = 0$ ). This class of Lévy processes was applied to oil and gas data in Benth and Šaltytė-Benth [7], and is a candidate for modeling the spark spread residuals as well (see Barndorff-Nielsen [1] and Barndorff-Nielsen and Shephard [2] for applications to finance). However, we shall in this paper follow a different, but more traditional way of modeling jumps in a commodity time series. Observing that the residuals are mostly having small fluctuations around zero, but from time to time showing rather extreme jumps, it may be natural to consider the Lévy process  $L(t)$  to be a pure jump process. A model of this form is conveniently fitted using the method of recursive filtering (see e.g. Clewlow and Strickland [9] for a description of this algorithm), which we now apply to our data set.

We begin by calculating the empirical standard deviation of the residuals, being equal to 6.23. Then we identify all residuals larger in absolute value than  $3 \times 6.23 = 18.68$  as jumps. In the first step, we find that there are 16 positive and 2 negative values that exceed this limit. We calculate the daily jump frequency by dividing the number of jumps by the total number of data, which results in the value 0.0223. After removing these jumps, we recalculate the standard deviation (becoming 3.36), and repeat the procedure of identifying jumps. We iterate this algorithm until the standard deviation of the residuals with jumps removed converges and no new jumps are identified. The results of the recursive filtering procedure are presented in Table 2.



TABLE 2. Summary of the recursive filtering procedure

Iteration	Std.dev.	Cumul. # jumps	Daily jump frequency
1	3.36	18	0.0223
2	2.68	34	0.0422
3	2.35	50	0.0620
4	2.22	58	0.0720
5	2.13	64	0.0794
6	2.08	68	0.0844
7	2.07	69	0.0856
8	2.07	69	0.0856

Let us first discuss the “normal variations” given by the filtered residuals. From Table 2 we can read off the standard deviation (or volatility) of the filtered residuals to be  $\sigma = 2.07$ , and a histogram of normalized residuals is given in Fig. 6. It has a clear bell shape, with kurtosis and skewness being 0.32 and 0.22, respectively. The Kolmogorov-Smirnov statistics is significant at the level of 8%, meaning that the distribution is very close to normal. These facts indicate that the choice of Brownian motion as an error process is reasonable.

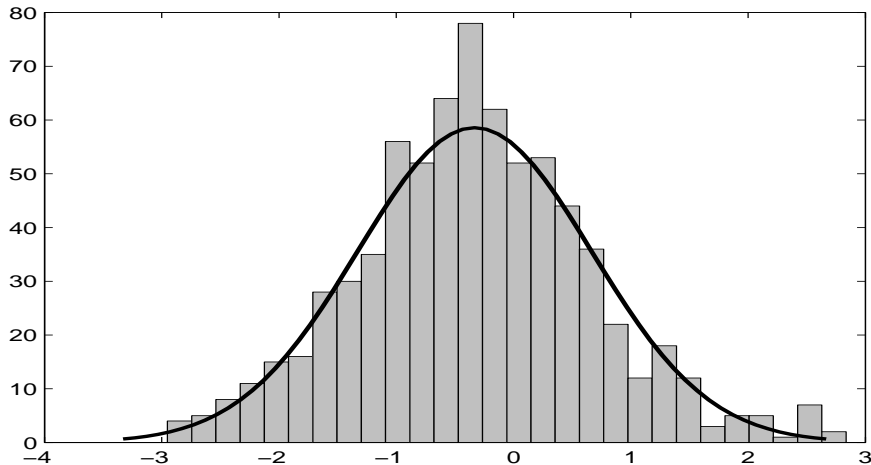


FIGURE 6. Histograms of the filtered residuals.

We next turn our attention to the filtered jumps. A suitable model for  $L(t)$  is a compound Poisson process defined as

$$(5.3) \quad L(t) = \sum_{i=1}^{P(t)} X_i$$

for an independent sequence of *i.i.d* random variables  $X_i$  and  $P(t)$  being a Poisson process with jump intensity  $\lambda$ . The jump intensity is read off from Table 2 as  $\lambda = 0.0856$ . We did not detect any seasonal pattern of the jump intensity, which supports the choice of a constant  $\lambda$ . In Fig. 7 we have plotted the histogram of the filtered jumps. To have an

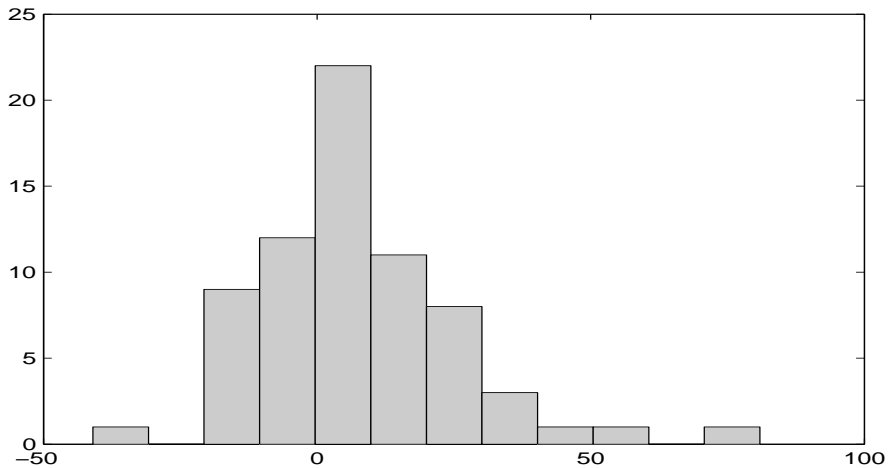


FIGURE 7. Histograms of the values of jumps

analytically tractable model (see Subsection 5.2 below for a discussion), we assume that the jumps are normally distributed, with estimated mean equal to 7.33 and standard deviation 18.78. Hence, we model  $X_i$  as a normal random variable with this as specified mean and standard deviation. We remark that the choice of  $L(t)$  in (5.3) gives an additive version of the jump dynamics model for asset prices suggested by Merton [15].

One may not be convinced by applying the normal distribution to fit the jump size histogram. Indeed, from Fig. 8, we see that the histograms for the negative and positive jumps are different. Moreover, the jump intensities are estimated to be 0.0273 for the negative jumps, while the positive have an intensity of 0.0583, which means that they occur more than twice as often. Such a skewness in jump behaviour occurs for other data series as well (see e.g. Clewlow and Strickland [9] for more on this), and a natural model may be

$$(5.4) \quad L(t) = N^+(t) - N^-(t)$$

where  $N^\pm(t)$  are two independent compounded Poisson processes, defined as

$$N^\pm(t) = \sum_{i=1}^{P^\pm(t)} X_i^\pm$$

for two independent sequences of *i.i.d* lognormal variables  $X_i^\pm$  with parameters  $s^\pm$  (shape) and  $m^\pm$  (scale). The Poisson processes  $P^\pm(t)$  are assumed to have jump intensities  $\lambda^\pm$ , which we have estimated to be  $\lambda^+ = 0.0583$  and  $\lambda^- = 0.0273$ . The estimated shape

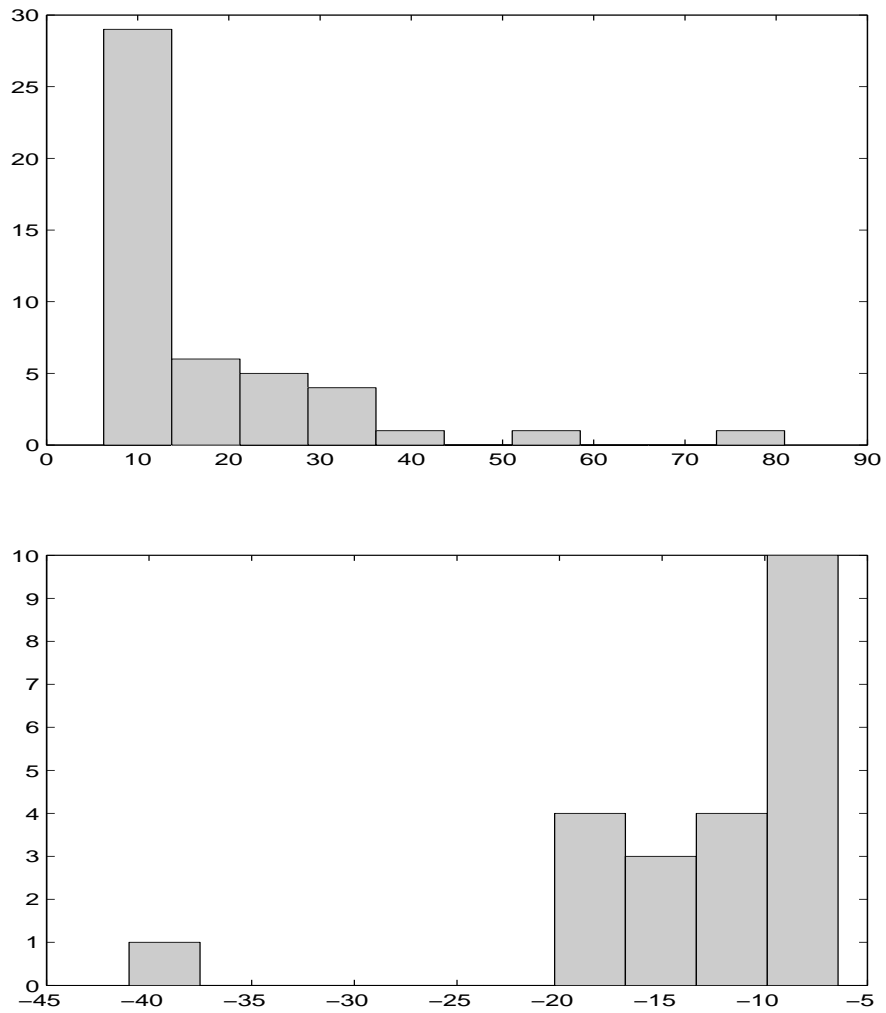


FIGURE 8. Histograms of the values of positive (top) and negative (bottom) jumps.

TABLE 3. Estimated parameters of jumps distribution

Parameter	Positive jumps	Negative jumps
$m$	13.25	21.27
$s$	0.64	1.52

and scale parameters of the positive and negative jump size distributions are presented in Table 3. The unfortunate effect of choosing lognormal distributions for the jump sizes is that the integrability condition for call options does not hold.

**5.2. Discussion of option pricing.** Let us discuss the relation to option pricing of the proposed jump processes. Consider first the jump model stated in (5.3). If the cumulant function of  $X$ , the jumps size, is denoted by  $\phi$ , we have that

$$\psi(\theta) = \ln \mathbb{E} [\exp(i\theta L(1))] = \lambda (e^{\phi(\theta)} - 1) .$$

When  $X$  is supposed to be a normally distributed random variable with expectation  $m_j$  and variance  $\sigma_j^2$ , its cumulant becomes

$$\phi(\theta) = i\theta m_j - \frac{1}{2}\theta^2 \sigma_j^2 .$$

Noting that the exponential integrability condition in Prop. 4.3 is equivalent with  $L(1)$  having exponential moments, that is, the expectation of  $\exp(\gamma L(1))$  being finite, we see that for the above jump model we can price call options on the spark spread using the Fourier approach.

Turning our attention to the model (5.4), we find that

$$\psi(\theta) = \lambda(e^{\phi^+(\theta)} + e^{\phi^-(\theta)} - 2) ,$$

where  $\phi^\pm(\theta)$  are the cumulant functions for  $X^\pm$ , respectively. We can calculate these for the lognormal model, ending up with an infinite series representation. However, we observe that exponential moments do not exist for the Lévy process  $L$  when  $X$  is lognormally distributed, and thus we can not use this model to price call options by FFT.

In order to apply the fast Fourier technique, we need the function  $\Psi$ , which is given as an integral of the cumulant  $\psi$  (see Lemma 3.3). In practical applications of the models proposed above, we need to approximate  $\Psi$  by a numerical integration procedure since the integral can not be analytically calculated.

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