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More on testing exact rational expectations in cointegrated vector autoregressive models: Restricted drift terms

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Abstract

In this note we consider testing of a type of linear restrictions implied by rational expectations hypotheses in a cointegrated vector autoregressive model for $I(1)$ variables when there in addition is a restriction on the deterministic drift terms.

Keywords: VAR models, cointegration, restricted drift terms, rational expectations.

1 Introduction

The present paper is a continuation of the paper Johansen and Swensen (1999), from now J&S, which treats the situation where there are no restrictions in the vector autoregressive (VAR) model except that the impact matrix has reduced rank. In this note we will consider the situation where the basic VAR model in addition satisfies a restriction on the deterministic drift term. This is a natural restriction since without this constraint the data generating VAR will contain a quadratic trend.

The approach in this paper is, as also described in J&S, well known in certain rational expectation models, and have been used in several contexts. The starting point is a VAR model. The *exact* linear rational expectations we consider will entail certain restrictions on the coefficients of the VAR model, and these restrictions can be tested by conventional statistical methods, such as Wald or likelihood ratio tests.

When the variables described in the VAR are stationary, this approach was proposed by Hansen and Sargent (1981 and 1991). Baillie (1989) advocated treating I(1) variables similarly by first transforming them to stationarity. A well known application in this vein is the treatment of present value models in Campbell and Shiller (1987). The transformations usually depend on unknown parameters which must be estimated before the restrictions are tested. This is a drawback, and the contribution of J&S is to show how one can in some situations perform likelihood ratio tests without such a preliminary transformation.

An important feature of the models we deal with, and which is worth emphasizing, is that the restriction is exact in the sense that the expectations are formed on the basis of the variables of the model. There are no expectations involving stochastic processes unobservable to the econometrician. It is precisely this assumption that allows us to derive testable restrictions on the coefficients of the VAR model based on the economic restrictions. This distinction is stressed by Hansen and Sargent (1981 and 1991).

Another point worth emphasizing is that the coefficients specifying the linear restrictions must be completely known. This is not always the case, but the results of the present paper is also of interest in the more general case where the restrictions involve some unknown parameters. Since we find the numerical value of the likelihood for fixed values of such parameters, it can be the basis for determining the maximum likelihood estimators by using an optimization algorithm or simply a grid search.

The paper is organized as follows. In the next section we state the type of relationships among expectations we shall consider, and derive the implications for the VAR model when expectations are considered to be rational in the sense described earlier. In section 3 we show how a likelihood ratio test can be developed when the restrictions have this form. The last section contains an illustration of how the results developed in section 3 can be applied.

2 The form of the restrictions.

This section defines the statistical model which is assumed to generate the data and formulates the rational expectation hypothesis. As mentioned in the introduction it is a common modification of the vector autoregressive (VAR) model considered in J&S.

We assume that the p-dimensional vectors of observations are generated according to the VAR model

$$\Delta X_t = \Pi X_{t-1} + \Pi_2 \Delta X_{t-1} + \cdots + \Pi_k \Delta X_{t-k+1} + \mu_0 + \mu_1 t + \Phi D_t + \epsilon_t, \ t = 1, \ldots, T \quad (1)$$

where $X_{-k+1}, \ldots, X_0$ are assumed to be fixed and $\epsilon_1, \ldots, \epsilon_T$ are independent, identically distributed Gaussian vectors, with mean zero and covariance matrix Σ. The matrices $D_t, t = 1, \ldots, T$ consist of other deterministic series. We assume that $\{X_t\}_{t=1,2,\ldots}$ is $I(1)$ so that the matrix Π has reduced rank $0 < r < p$ and thus may be written

$$\Pi = \alpha\beta', \quad (2)$$

where α and β are $p \times r$ matrices of full column rank. In addition we consider the situation where the coefficient of the linear term satisfies $\alpha_\perp' \mu_1 = 0$, where $\alpha_\perp$ is a $p \times (p-r)$ matrix of full column rank consisting of columns that are orthogonal to the columns of the matrix α. This is a common restriction which means that the process $\{X_t\}$ does not contain a quadratic drift.

The restriction on the linear term is incorporated in the usual way. Since μ_1 belongs to the linear space spanned by the columns of α, it can be written $\mu_1 = \alpha\kappa_1$ where κ_1 is an r-dimensional vector. Letting β^* denote the $(p+1) \times r$ matrix $(\beta', \kappa_1)'$, $\Pi^* = \alpha\beta^{*\prime}$ and $X_{t-1}^* = (X_{t-1}', t)'$, the model may be reformulated as

$$\Delta X_t = \Pi^* X_{t-1}^* + \Pi_2 \Delta X_{t-1} + \cdots + \Pi_k \Delta X_{t-k+1} + \mu_0 + \Phi D_t + \epsilon_t, \ t = 1, \ldots, T. \quad (3)$$

Since the model (3) contains a deterministic trend, it is reasonable to allow for the possibility that the hypotheses may involve restrictions on the coefficients of these terms. That rational expectations model can have such implications was pointed out by West (1989).

The set of restrictions we consider is of the form

$$E[c_1' X_{t+1} | \mathcal{O}_t] + c_0' X_t + c_{-1}' X_{t-1} + \cdots + c_{-k+1}' X_{t-k+1} + c_c + c_\tau (t+1) + c_\phi D_{t+1} = 0. \quad (4)$$

Here $E[\cdot|\mathcal{O}_t]$ denotes the conditional expectation in the probabilistic sense taken in model (1), given the variables $X_1, \ldots, X_t$. Thus, $\mathcal{O}_t$ denotes the information contained in the observed variables of the VAR-model up to time t. The $p \times q$ matrices $c_i, i = -k+1, \ldots, 1$ are *known* matrices, possibly equal to zero and so is the q-dimensional vector c_τ and the matrix c_ϕ. The q-dimensional vector c_c can contain unknown parameters and is of the form $c_c = H_c \omega_c$ where the $q \times s_c$ dimensional matrix H_c is *known*, and ω_c is an s_c dimensional vector consisting of unknown parameters, $0 \leq s_c \leq q$. In addition we will assume that the two matrices c_1 and $c_{-k+1} + \cdots + c_0 + c_1$ are of full column rank. Note that we allow lagged values of X_t to be included in the restrictions.

On letting $d_{-i+1} = -\sum_{j=i-1}^{k-1} c_{-j}, i = 0, \ldots, k$ the restrictions (4) may be reformulated as

$$E[c_1' \Delta X_{t+1} | \mathcal{O}_t] - d_1' X_t + d_{-1}' \Delta X_t + \cdots + d_{-k+1}' \Delta X_{t-k+2} + c_c + c_\tau (t+1) + c_\phi D_{t+1} = 0. \tag{5}$$

Thus theoretical restrictions may be expressed in the form (4) or (5) according to what is convenient in the particular application.

We conclude this section by reformulating restrictions (4) and (5) as restrictions on the coefficients of the statistical model (1). Taking the conditional expectation of ΔX_{t+1} given $X_1, \ldots, X_t$, we get, by using (3) and multiplying by c_1'

$$\begin{aligned} c_1' E[\Delta X_{t+1} | \mathcal{O}_t] = c_1' \Pi^* X_t^* + c_1' \Pi_2 \Delta X_t + \cdots + c_1' \Pi_k \Delta X_{t-k+2} \\ + c_1' \mu_0 + c_1' \Phi D_{t+1}. \end{aligned} \tag{6}$$

Inserting this expression into (5) implies that the following conditions must be satisfied

$$c_1' \Pi^* = d_1^{*\prime}, c_1' \Pi_i = -d_{-i+1}', i = 2, \ldots, k, c_1' \mu_0 = -c_c = -H_c \omega_c \text{ and } c_1' \Phi = -c_\phi.$$

where d_1^* is the $(p+1) \times q$ matrix $d_1^* = (d_1', -c_\tau)'$.

This can be summarized as:

Proposition 1 *Restrictions (4) or (5) take the following form in terms of model (3):*

$$\begin{aligned} (i) \quad & \beta \alpha' c_1 = -\textstyle\sum_{j=-1}^{k-1} c_{-j} = d_1, c_1' \alpha \kappa_1 = -c_\tau, \\ (ii) \quad & c_1' \Pi_i = \textstyle\sum_{j=i-1}^{k-1} c_{-j}' = -d_{-i+1}', i = 2, \ldots, k, \\ & c_1' \mu_0 = -H_c \omega_c \text{ and } c_1' \Phi = -c_\phi. \end{aligned} \tag{7}$$

Remark 1. It may be worthwhile to comment on how to deal with the simpler model without linear trend but with restriction on the constant term, i.e $\mu_1 = 0$ and $\alpha_\perp' \mu_0 = 0$ in (1). Then $\mu_0 = \alpha \kappa_0$, and restrictions of the form (4) can be formulated as described provided $c_\tau = 0$ and c_c is a given constant. The restriction in (i) in Proposition 1 is then $c_1'\alpha\kappa_0 = -c_c$ and there are no restriction on μ_0 in (ii). This is the reason why we assume that c_c is known in this case. Otherwise there would be complicated restrictions between the parameters in α, κ_0 and those describing c_c.

3 Derivation of the maximum likelihood estimators and the likelihood ratio test.

We now provide an analogous result to Proposition 2 in J&S, which allows the constraints following from the restrictions in part (i) in Proposition 1 to be expressed in terms of freely varying parameters. We will use the usual notation that $\overline{a} = a(a'a)^{-1}$, such that $a'\bar{a} = I$.

Proposition 2 *Let b and d be matrices of full column rank with dimensions $p \times q$ and $(p+1) \times q$ respectively. Then the $p \times (p+1)$ matrix Π^* has reduced rank r and satisfies*

$$\Pi^{*\prime} b = d^* \tag{8}$$

if and only if there exist a $(p-q)\times(r-q)$ matrix η and a $(p+1-q)\times(r-q)$ matrix ξ^* of full rank and a $(p-q) \times q$ matrix Θ such that Π^* has the representation

$$\overline{b} d^{*\prime} + \overline{b_\perp} \Theta \overline{d}^{*\prime} + \overline{b_\perp} \eta \xi^{*\prime} \overline{d_\perp}^{*\prime}. \tag{9}$$

Proof. The argument is the same as in J&S by pre-multiplying $\Pi^{*\prime}$ by the $(p+1) \times (p+1)$ matrix $(d^*, d^*_\perp)'$ and post-multiplying it with the $p \times p$ matrix $(b, b_\perp)$.

We can now proceed in the same way as in J&S. First we use the reformulation (9) of the restriction (i) in Proposition 1 taking $b = c_1$ and $d^* = d_1^*$. Then we multiply with $b'_\perp$ and b' and take the other restrictions into account, and end up with simplified equations for $b'_\perp \Delta X_t$ and $b'\Delta X_t$.

Then the model equation (3) is decomposed into two sets of regression equations. The equation for $b'_\perp \Delta X_t$ is a non-linear regression because of the reduced rank matrix $\eta\xi^{*\prime}$, and the equation for $b'\Delta X_t$ is a simple linear regression to determine ω_c given by

$$\begin{aligned} b'\Delta X_t &= d^{*\prime} X^*_{t-1} - d'_{-1}\Delta X_{t-1} - \cdots - d'_{-k+1}\Delta X_{t-k+1} \\ &- H_c\omega_c - c_\phi D_t + b'\epsilon_t. \end{aligned} \tag{10}$$

The error terms $b'_\perp \epsilon_t$ and $b'\epsilon_t$ are correlated, hence we derive the conditional model for $b'_\perp \Delta X_t$ given $b'\Delta X_t$ and the past using $\rho = b'_\perp \Sigma b (b'\Sigma b)^{-1}$. This gives

$$\begin{aligned} b'_\perp \Delta X_t &= \eta \xi^{*\prime} \overline{d^*_\perp}' X^*_{t-1} + \rho b' \Delta X_t + (\Theta (d^{*\prime} d^*)^{-1} - \rho) d^{*\prime} X^*_{t-1} \\ &+ (b'_\perp \Pi_2 + \rho d'_{-1}) \Delta X_{t-1} + \cdots + (b'_\perp \Pi_k + \rho d'_{-k+1}) \Delta X_{t-k+1} \\ &+ (b'_\perp \mu_0 + \rho H_c \omega_c) + (b'_\perp \Phi + \rho c_\phi) D_t + u_t, \end{aligned} \tag{11}$$

where the errors are $u_t = (b'_\perp - \rho b')\epsilon_t$. Note that they are independent of the errors $b'\epsilon_t$ in the equation for $b'\Delta X_{t-1}$.

That there are no restrictions between the parameters in the two equations can be argued as in J&S. We can therefore find the maximum likelihood estimators and the maximal value of the likelihood by considering separately the marginal model given by (10), and the conditional model (11) described above. This can be done since the likelihood can be written as a product of the likelihoods of the conditional and the marginal models. Remark that all the parameters, except ω_c and $b'\Sigma b$, are estimated using the conditional model.

Consider the estimation of the conditional system. There are two cases. If $q = r$ all cointegrating relations are known, $sp(\beta) = sp(d)$, and the matrices η and ξ^* are zero. In this case inference can proceed with regression of the variable $b'_\perp \Delta X_t$ on $b'\Delta X_t, d^{*\prime}X^*_{t-1}, \Delta X_{t-1}, \ldots, \Delta X_{t-k+1}, D_t$ and 1, $t = 1, \ldots, T$. Let R^*_{1t} be the residuals and compute $S^*_{11} = \sum_{t=1}^T R^*_{1t} R^*_{1t}/T$.

For the case where $q = 1, \ldots, r-1$, we have to use the techniques for reduced rank regression. In addition the restrictions on the linear term must be taken into account. Thus we modify the approach used in J&S by incorporating the methods for models for deterministic drift terms, see e.g. Johansen (1996, section 6.2). We recall the main points.

Regress the variables $b'_\perp \Delta X_t$ and $\overline{d^*_\perp}' X^*_{t-1}$ on $b'\Delta X_t, d^{*\prime} X_{t-1}, \Delta X_{t-1}, \ldots, \Delta X_{t-k+1}, D_t$ and 1, $t = 1, \ldots, T$ with the residuals as R^*_{1t} and R^*_{2t}. Define the moment matrices $S^*_{ij}, i, j = 1, 2$ by

$$S^*_{ij} = \frac{1}{T} \sum_{t=1}^{T} R^*_{it} R^{*\prime}_{jt}, i, j = 1, 2. \qquad (12)$$

The dimension of the matrices S^*_{11}, S^*_{22} and S^*_{12} are $(p-q)\times(p-q)$, $(p+1-q)\times(p+1-q)$ and $(p-q)\times(p+1-q)$ respectively. The maximum likelihood estimators of ξ^* are given by $\tilde{\xi}^* = (v^*_1, \ldots, v^*_{r-q})$ where $v^*_1, \ldots, v^*_{p+1-q}$ are eigenvectors in the eigenvalue problem

$$|\lambda S^*_{22} - S^*_{21} S^{*-1}_{11} S^*_{12}| = 0, \qquad (13)$$

which has solutions $\tilde{\lambda}^*_1 > \ldots > \tilde{\lambda}^*_{p+1-q}$. Since the matrix $S^*_{21} S^{*-1}_{11} S^*_{12}$ has rank $p-q$, the eigenvalue $\tilde{\lambda}^*_{p+1-q} = 0$. Here the normalization $\tilde{\xi}^{*\prime} S^*_{22} \tilde{\xi}^* = I_{r-q}$ is used. The estimator of η is given by

$$\tilde{\eta} = S^*_{12} \tilde{\xi}^*.$$

Furthermore, the part of the maximized likelihood function stemming from the conditional model is

$$L^{-2/T}_{1\cdot 2, max} = \begin{cases} |S^*_{11}| \prod_{i=1}^{r-q} (1 - \tilde{\lambda}^*_i)/|b'_\perp b_\perp| & when \quad q = 1, \ldots, r-1 \\ |S^*_{11}|/|b'_\perp b_\perp| & when \quad q = r. \end{cases}$$

The part stemming from the marginal model (10) follows from results for standard multivariate Gaussian models, and equals

$$L_{2,max}^{-2/T} = |\tilde{\Sigma}_{22}^*|/|b'b|,$$

where

$$\begin{aligned}\tilde{\Sigma}_{22}^* &= \frac{1}{T}\sum_{t=1}^{T}(b'\Delta X_t - d^{*\prime}X_{t-1}^* + d'_{-1}\Delta X_{t-1} + \cdots + d'_{-k+1}\Delta X_{t-k+1} + H_c\tilde{\omega}_c + c_\phi D_t) \\ &\quad (b'\Delta X_t - d^{*\prime}X_{t-1}^* + d'_{-1}\Delta X_{t-1} + \cdots + d'_{-k+1}\Delta X_{t-k+1} + H_c\tilde{\omega}_c + c_\phi D_t)', \quad (14)\end{aligned}$$

and $\tilde{\omega}$ is the maximum likelihood estimator for ω. Hence the maximum value of the likelihood function under the hypothesis is given by

$$L_{H,max}^{-2/T} = |\tilde{\Sigma}_{22}^*||S_{11}^*|\prod_{i=1}^{r-q}(1-\tilde{\lambda}_i^*)/|b'b||b'_\perp b_\perp|, \quad (15)$$

where the product is to be taken as 1 when $q = r$.

In Johansen (1996, section 6.2) it is shown the maximum value of the likelihood in the reduced rank model defined by (3) and satisfying the restrictions (2) and $\alpha'_\perp\mu_1 = 0$ is given by $L_{max}^{-2/T} = |S_{00}^*|\prod_{i=1}^{r}(1-\hat{\lambda}_i^*)$, where $S_{00}^*, \hat{\lambda}_i^*, i = 1,\ldots,r$ arise from maximizing the likelihood in a manner similar to the one described above. In this case only restrictions (2) and $\alpha'_\perp\mu_1 = 0$ are taken into account.

Collecting the results above we therefore have the following result for the likelihood ratio test of restrictions (4) in the VAR model (1) with the reduced rank condition (2) and the restriction $\alpha'_\perp\mu_1 = 0$ imposed.

Proposition 3 *Consider the rational expectation restrictions of form (4) with $c_{-k+1},\ldots,c_0,,c_1$ known. Assume that $b = c_1, b_\perp = c_{1\perp}$ and $d = -(c_1 + c_0 + \cdots + c_{-k+1})$ have full column rank. The likelihood ratio statistic of a test for the restrictions (4) in the reduced rank VAR model satisfying (2) and $\alpha'_\perp\mu_1 = 0$ against a VAR model satisfying only the reduced rank condition (2) and $\alpha'_\perp\mu_1 = 0$, is*

$$\begin{aligned}-2lnQ &= Tln|S_{11}^*| - T\sum_{i=1}^{r}ln(1-\hat{\lambda}_i^*) + Tln|\tilde{\Sigma}_{22}^*| \\ &- Tln|S_{00}^*| + T\sum_{i=1}^{r-q}ln(1-\tilde{\lambda}_i^*) - Tln(|b'b||b'_\perp b_\perp|),\end{aligned}$$

where S_{11}^*, $\tilde{\lambda}_i^*, i = 1,\ldots,r-q$ and $\tilde{\Sigma}_{22}^*$ are given by (12), (13) and (14), and $S_{00}^*, \hat{\lambda}_i^*, i = 1,\ldots,r$ are estimates from the VAR model (3) satisfying (2) and $\alpha'_\perp\mu_1 = 0,$.

Remark 2. Although it is an essential part of the restrictions (4) that the matrices $c_{-k+1}, \ldots, c_0, c_1$ are completely known we can, as pointed out in the introduction, combine the results above with a grid search or a numerical optimization procedure to obtain the maximum likelihood estimators in the cases where the matrices are specified as functions of some unknown parameters. In case models of this kind are non-linear, however, this has to be taken into account in the statistical inference. We refer to J&S for a closer explanation of this point.

Remark 3. By inspecting the proof of Proposition 3 we see that the hypothesis described in Remark 1 can be tested by using a constant instead of a linear term in the reduced rank regression, and not including any constant terms in the conditional and marginal part of the model.

4 An illustration.

To illustrate the methods of the previous section we consider a present value model similar to the one treated in J&S, and take as point of departure the VAR model reported in Engsted (2002). The annual data covers the period 1922-1996 and consists of a time series P_t of real stock prices at the end of year t and a time series D_t of real dividends paid during year t. As reported by Engsted (2002), it is possible to use a VAR-model of order 1 for $X_t^{*\prime} = (X_t', 1)' = (P_t, D_t, 1)'$, although there is an indication of a structural break in the early 1940's. The reduced rank VAR-model with restricted constant term, has cointegrating vector equal to $(1, -16.203, -28.368)$, which agrees with the result reported in Engsted (2002). The maximal value of -2 log-likelihood is equal to -240.4. The situation is thus the one described in remark 1. Consider now a present value model of the form

$$\delta E[P_{t+1} + D_{t+1} | \mathcal{O}_t] = P_t. \tag{16}$$

It is not difficult to see that when δ and is considered as known, and equal to δ_0 and say, (16) has the form, (4) or (5) with

$$c_1 = -\delta_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } d_1 = \begin{pmatrix} \delta_0 - 1 \\ \delta_0 \end{pmatrix}.$$

Hence,

$$c_{1\perp} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ and } d_1^* = \begin{pmatrix} \delta_0 - 1 \\ \delta_0 \\ 0 \end{pmatrix},$$

and present value models of this form can be tested using the results from the previous section.

Since $q = r = 1$ no reduced rank regression is necessary. The conditional model is estimated by regressing $c_{1\perp}' \Delta X_t = \Delta(P_t - D_t)$ on $c_1' \Delta X_t = -\delta_0 \Delta(P_t +$

D_t) and $d^{*\prime}X^*_{t-1} = (\delta_0 - 1)P_{t-1} + \delta_0 D_{t-1}$. The part of the likelihood from the conditional model is the mean residual sum of squares divided by $2 = c'_{1\perp}c_{1\perp}$. The part of the likelihood from the marginal model is just the mean sum of squares of $c'_1\Delta X_t - d^{*\prime}_1 X^*_{t-1} = P_{t-1} - \delta_0(P_t + D_t)$ divided by $2\delta_0^2 = c'_1 c_1$.

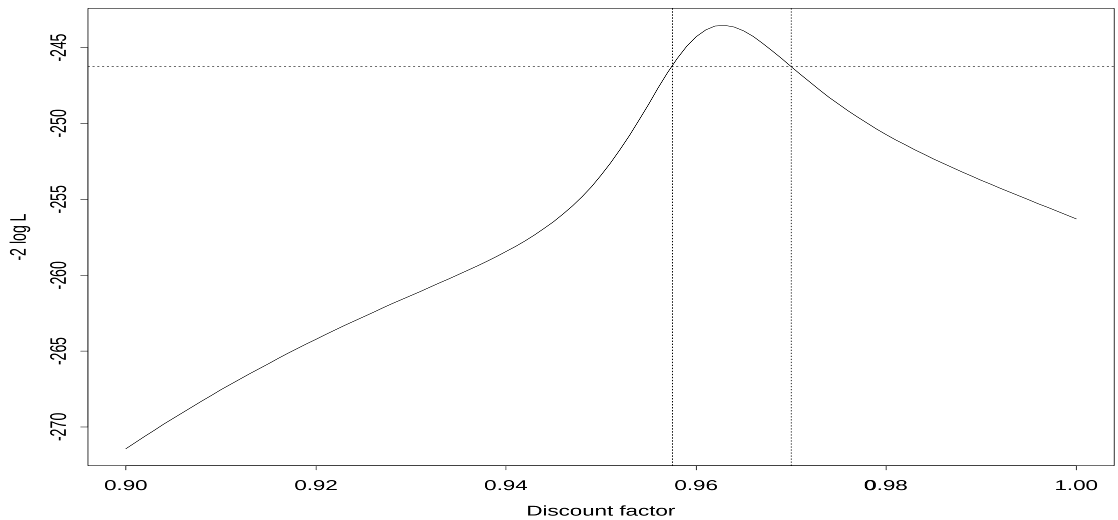

Figure 1: Plot of -2 log-likelihood for discount factor δ varying in $[0.9, 1.0]$.

A likelihood ratio test of the implications of (16) can now be carried out by comparing the values of -2 log-likelihood from the two fitted models. There are three degrees of freedom in this case. The first model contains seven parameters, i.e two in the cointegration vector, two adjustment parameters and three parameters in the covariance matrix. The present value model contains four parameters, i.e two coefficients of the regressors and a variance parameter in each of the conditional and marginal model.

Often it is not reasonable to start out with known values for δ. Then the likelihood for the present value hypothesis described above can be evaluated for various choices of this parameter. The value corresponding to the maximum is the maximum likelihood estimate.

In Figure 1 the values of -2 log-likelihood is plotted for values of δ in the interval $[0.9, 1.0]$. The maximal value is -243.5 corresponding to the discount factor 0.963. Comparing this to the value for the VAR-model with restricted constant yields a -2 log-likelihood ratio of 3.1. Since one degree of freedom is lost when estimating δ, the corresponding P-value is 0.21.

It is also easy to construct confidence intervals for the discount factor from Figure 1. If $\chi^2_1(\alpha)$ is the α quantile of the χ^2-distribution with 1 degree of freedom, those values of δ for which the difference of -2 log-likelihood is less than $\chi_1^2(1-\alpha)$ from -243.5 define an asymptotic $1-\alpha$ confidence interval. With $\alpha = 0.1$ and $-243.5 - \chi_1^2(0.9) = -246.2$ one can see from igure 1 that the confidence interval in this case is $(0.958, 0.970)$.

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