On the difference in the speed of gravity waves in a physical and numerical wave tank

by

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Precise measurements of gravity waves with very small wave slope in a physical wave tank are compared with an explicit linear inviscid wave maker theory. The main purpose is to measure the speed of the physical waves relative to those computed. We find that the wave speed in the physical wave tank is slightly less than in the computations. The small difference in the wave speed leads to a relative phase difference between the real waves and the inviscid computations of about 0.01±0.006 radians per wave length (0.16%±0.1%), which is comparable to an estimated phase delay due the boundary layer at the tank walls. In this result, the estimated effects of weak nonlinearity and surface tension in the experiments are subtracted. This relative phase difference is significantly smaller than what previous investigations in wave tanks of similar size have indicated.

1 Introduction

Several recent papers concern the development of a numerical wave tank which is the theoretical-numerical counterpart of a physical wave tank. The purpose may be a detailed study of e.g. strongly nonlinear gravity waves. Another objective may be to simulate the forces acting on a body exposed to a wave field. Such a numerical wave tank was developed by e.g. Dommermuth *et al.* (1988) several years ago who implemented an improved version of the algorithm due to Vinje and Brevig (1981). They succeeded in modeling a breaking wave up to re-entry. Moreover, they compared the numerical simulations with laboratory experiments by Melville and co-authors, see e.g. Melville and Rapp (1985). The comparison between the numerical and physical waves was rather good, apart from a small phase difference between the real waves and the numerical simulations. It was tentatively speculated in Dommermuth *et al.* (1988) that the cause was dissipation in the physical wave tank. Later, Skyner (1996) performed a similar investigation comparing PIV measurements of a breaking wave with the theoretical-numerical model of Dold & Peregrine (1986). Skyner noted a rather significant phase difference between the experiments and the theoretical model. Such a phase difference was also noted between some previous laboratory experiments carried out at the University of Oslo and a numerical code for analyzing nonlinear wave loads on large volume marine structures, described by Cai and Mehlum (1996). It turns out that the phase difference noted in these examples is an order of magnitude larger than can be explained by physical effects like dissipation and surface tension, where the former effect is expected to reduce the wave speed while the latter may increase the wave speed.

From the above examples one may question: should a rather significant phase difference, which is increasing in time, between waves in a physical wave tank and precise theoretical inviscid simulations be expected? This question has motivated the present study. We shall perform precise measurements of waves with very small wave slope in a physical wave tank and compare with an explicit linear inviscid wave maker theory. Input to the latter is the motion of the physical wave maker. The ultimate goal is precise measurements of the phase of the physical waves relative to the theoretical model which produce numerical results to desired accuracy.

2 Experiments and theoretical model

The experiments are carried out in a wave tank in the Hydrodynamical Laboratory at the University of Oslo. The wave tank is 24.6m long, 0.5m wide and the water depth is 0.6m. In one end of the tank there is a hydraulic piston wave maker. At the other end there is an absorbing beach to damp the waves. Measurements of the wave elevation are performed at several positions of the tank. The measurements are terminated before any (small) reflected wave has returned to the measurement position. The wave tank and the motion of the wave maker are very precise. Documentation of the accuracy of the wave tank complementary to this description may be found in Huseby and Grue (1999).

For controlling and monitoring the wave maker and to record the surface elevation we use a computer with the data acquisition cards AT-MIO-16E-1 and AT-MIO-64E-3 from National Instruments. The update rate of the wave maker and the sampling rate of the data acquisition is 1000Hz. (Some experiments using a sampling rate of 100Hz were also performed, but results are not shown here. The results of these experiments were not sufficiently accurate. We have found that a sampling rate of 1000Hz was needed to get conclusive results of the investigation.) The motion of the wave maker is measured with a magnetic digital scale system with accuracy about $\pm$ 0.1mm. These measurements are used for input to the linear theoretical model (see below) that we compare with experiments. Thus it is important that the position of the wave maker is determined with high precision.

Relevant to the previous experiments we compare with, a group of transient waves focusing at a certain distance from the wave maker is investigated. The input signal to the wave maker is given by

$$\bar{\eta}(t) = \frac{V_0\gamma}{\frac{27}{256}{t_g}^4} t^3(t_g - t)\sin[\varphi(t)], \quad \varphi(t) = \omega_0 t(1 - \frac{\alpha t}{t_g}), \quad 0 \leqslant t < t_g, \tag{1}$$

where γ is a constant for the calculation from volt to cm, t_g is the length of the time series of the wave maker, α is a constant, ω_0 is frequency and V_0 is the amplitude in volt. We perform a total of sixteen experiments with $\alpha = 0.3$, $\omega_0 = 2\pi \cdot 2.0\text{s}^{-1}$ giving a focusing of the waves at 11.5m. The amplitude V_0 is either 0.04V or 0.06V. Four wave gauges are used to measure the surface elevation in each run. The resolution of these gauges is 0.1mm. They are static calibrated.

The experimental results are compared with the linear transient wave maker solution presented in Dommermuth *et al.* (1988). The free surface elevation η is in non-dimensional form determined by

$$\eta = \frac{4}{L}\sum_{n=1}^{\infty}[\sum_{m=1}^{\infty}(k_n^2+\kappa_m^2)^{-1}]\cos k_n x\int_0^t d\tau U(\tau)\cos\omega_n(t-\tau)+\frac{1}{L}\int_0^t d\tau U(\tau), \qquad (2)$$

where $\kappa_m = (m+1/2)$, $k_n = n\pi/L$, $\omega_n^2 = k_n \tanh k_n$ and L is the non-dimensional length of the tank (with unit depth). $U(\tau)$ is the input velocity from the time history of the wave maker. Equation (2) is the version due to a tank of finite length, derived from the infinite length case of Kennard (1949).

3 Results

In figure 1 we present the free-surface elevation at three different positions from the wave maker at rest: 2.0m, 6.0m and 12.5m. Both theory and experiments due to four different runs are shown. The experimental results compare excellent with the linear theory, apart from a small reduction in the experimental wave elevation.

With the purpose of a closer investigation of the results we apply Fourier transform of the measured and computed elevation $\eta(t)$ in a time interval between T_1 and T_2, i.e.

$$\mathcal{F}(\omega) = \int_{T_1}^{T_2}\eta(t)e^{-i\omega t}dt, \qquad (3)$$

where ω denotes frequency. The magnitude of $\mathcal{F}(\omega)$ is determined by $|\mathcal{F}(\omega)|$ and the phase of $\mathcal{F}(\omega)$ by

$$arg\mathcal{F}(\omega) = Im(ln\mathcal{F}(\omega)). \qquad (4)$$

We first consider $|\mathcal{F}(\omega)|$. Results due to four separate runs are displayed in each of the figures 2a-b. The experiments are very repeatable. The comparisons between theory and experiment show good agreement apart from a small continuous reduction of the experimental $|\mathcal{F}(\omega)|$ as the wave group propagates along the tank. The damping rate due to dissipation in the the boundary layer at the walls and the bottom of the wave tank was studied by Mei and Liu (1973). From their eqs. (2.11) and (4.19) the damping of the amplitude of the waves, per wave length, is

$$\exp(-2\pi\delta/b), \qquad (5)$$

where $\delta = \sqrt{\nu/2\omega}$ denotes the boundary layer thickness, b the half width of the wave tank and ν the kinematic viscosity. The damping rate given by (5) fits with the damping observed in the experiments.

Before we consider the results for $arg\mathcal{F}(\omega)$ we estimate the effects on the wave speed due to the boundary layer at the tank walls, the weak nonlinearity and surface tension in the experiments. The former effect may be obtained from Mei and Liu (1973) eqs. (2.11) and (4.19), giving a wave attenuation after a distance Δx of

$$-(\delta/b)\cdot(k\Delta x), \tag{6}$$

where δ and b are defined after (5), Δx denotes the travel distance of the wave and k the wavenumber. With $\nu = 10^{-6}$ m^{2}s^{-1} and ω =7s^{-1}, (6) predicts a phase delay of -0.007 radians per wave length λ ($\Delta x = \lambda$), and -0.06 radians for the distance between the recording positions at 2m and 12.5m. These estimates apply to the peak frequency of the wave group.

The nonlinear dispersion relation for deep water gravity waves, $\omega^2 = gk(1 + A^2k^2)$, where A denotes wave amplitude and g acceleration due to gravity, introduces an excess speed of waves with a finite Ak as compared to waves with $Ak = 0$. This excess speed is determined by $c/c_{lin} - 1 = A^2k^2$, where $c_{lin} = c(Ak = 0)$. A corresponding phase difference between nonlinear and linear waves traveling a distance Δx then becomes

$$A^2k^2\cdot(k\Delta x). \tag{7}$$

We perform the experiments with two different small but finite amplitudes. For the smallest waves with $Ak \simeq 0.025$ at the peak frequency (figures 1, 2a, 3a), (7) predicts a positive phase difference of 0.004 radians per wave length, and 0.03 radians when the waves travel from the position at 2m to that at 12.5m. For the larger waves with $Ak \simeq 0.038$ at the peak frequency (figures 2b, 3b), the corresponding positive phase differences are 0.009 radians and 0.08 radians.

A phase difference due to surface tension is

$$\frac{\tau}{\rho g}k^2\cdot(k\Delta x), \tag{8}$$

where $(\tau/\rho g)^{1/2} \simeq$ 3mm. (8) gives approximately a phase difference of 0.001 radians per wave length and 0.01 radians when the waves travel from the position at 2m to that at 12.5m. Thus, the estimated effects of a boundary layer, nonlinearity and surface tension in the physical experiments nearly cancel.

We now consider the difference between the theoretical and the experimental $arg\mathcal{F}(\omega)$. When $arg\mathcal{F}_{theory}(\omega) - arg\mathcal{F}_{exp}(\omega) < 0$ the computed waves are ahead of the the waves in the physical wave tank. On the other hand, when $arg\mathcal{F}_{theory}(\omega) - arg\mathcal{F}_{exp}(\omega) > 0$, the physical waves are ahead of the computations. The results in figure 3 show that this difference is close to zero. More precisely,

$$arg\mathcal{F}_{theory}(\omega) - arg\mathcal{F}_{exp}(\omega) \simeq -0.05 \pm 0.05 \tag{9}$$

radians for most of the frequencies, for the smallest waves ($Ak = 0.025$), when the waves have traveled the distance between the positions at 2m and 12.5m. This corresponds to a phase delay of -0.006 ± 0.006 radians per wave length. We note in particular that $arg\mathcal{F}_{theory}(\omega) - arg\mathcal{F}_{exp}(\omega) \simeq 0$ at the recording position at 6m. Furthermore,

$arg\mathcal{F}_{theory}(\omega) - arg\mathcal{F}_{exp}(\omega)$ is the same at the positions at 2m and 12.5m, for frequencies in the vicinity of the peak frequency. This means that the dominant part of the wave group travels with the same (average) speed in the computations and in the physical experiments between the positions at 2m and 12.5m.

The results in figure 3 fit with the discussion above on the effects of boundary layer, nonlinearity and surface tension, which, as noted, nearly cancel in the experiments. If we subtract the effects of nonlinearity and surface tension in the experiments, we expect a phase delay of -0.01 ± 0.006 radians per wave length between experiments with linear waves without surface tension, and the computations.

We may now compare our results with those of earlier investigations, Dommermuth *et al.* (1988) and Skyner (1996), both carried out in wave tanks of similar size as ours. In Dommermuth *et al.* (1988) the wave group had a peak frequency of $\omega_{peak} = 5.53\text{s}^{-1}$ and a corresponding wave length of $\lambda_{peak} = 1.93\text{m}$. It was noted that the numerical simulations were ahead of the physical waves with 0.06s after a travel distance of about six wave lengths. This gives a phase difference (after six waves) of $\omega_{peak}\Delta t \simeq 0.34$ radians, or 0.06 radians per wave length. In Skyner (1996), with a peak frequency of the wave group of $\omega_{peak} \simeq 5.14\text{s}^{-1}$ and a corresponding $\lambda_{peak} = 2.26\text{m}$, it appears that the numerical simulations were behind the physical waves with 0.125s, or $\omega_{peak}\Delta t \simeq 0.63$ radians, after a travel distance of about 2.2 wave lengths. This gives a phase difference of 0.29 radians per wave length. The results of our investigation, with $\omega_{peak} = 7\text{s}^{-1}$ and $\lambda_{peak} = 1.25\text{m}$, are compared with the previous investigations in table 1. We obtain here a relative phase difference between the experiments and the theoretical model which is significantly smaller than indicated by the other investigations.

Our experimental results (both phases and amplitudes) were found not to be sensitive to the presence of a surface film produced using a wetting agent (results not shown). This effect can thus effectively be ruled out as an explanation for the large phase shifts reported in Dommermuth *et al.* (1988) and Skyner (1996).

4 Conclusion

The purpose of the investigation has been to quantify the relative phase difference between waves propagating in a physical wave tank and precise inviscid linear computations. We have found that:

- The wave speed in the physical wave tank is slightly less than in the computations. The small difference in the wave speed leads to a relative phase difference between the real waves and the inviscid linear computations of about 0.01 ± 0.006 radians per wave length ($0.16\% \pm 0.1\%$). In this result, the estimated effects of weak nonlinearity and surface tension in the experiments are subtracted.

- This relative phase difference is significantly smaller than what previous investigations in wave tanks of similar size have indicated.

- The (small) relative phase difference is of the same size as the estimated effect of the boundary layer at the tank walls (Mei and Liu, 1973).

- A data acquisition system with a high sampling rate (1000Hz) was crucial to obtain precise results in the experiments.

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	A	B	C
Dommermuth *et al.* (1988)	−0.06s	−0.34	−0.06 (-1%)
Skyner (1996)	0.125s	0.63	0.29 (4.6%)
Present			-0.01 ± 0.006 ($-0.16\% \pm 0.1\%$)
Boundary layer (Mei and Liu, 1973)			−0.007 (−0.1%)

Table 1: Column A: Difference in time Δt at a recording position, between arrival of waves in a physical and a numerical wave tank. Column B: $\omega_{peak}\Delta t$ ($=arg\mathcal{F}_{theory}(\omega) - arg\mathcal{F}_{exp}(\omega)$). Column C: $\omega_{peak}\Delta t$ per wave length.

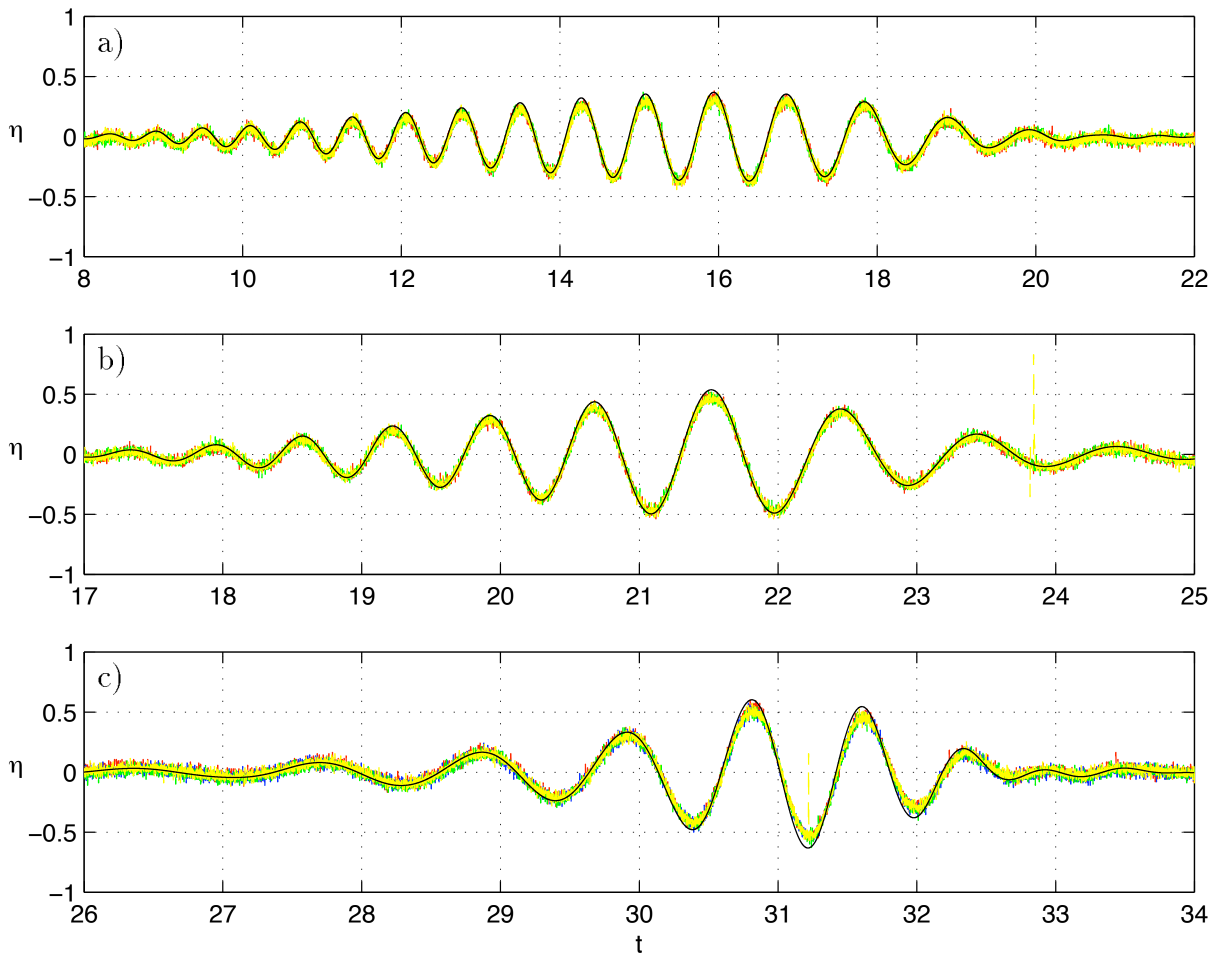

Figure 1: Surface elevation (cm) vs. time (sec) at three different positions, from top: x=2.0m, 6.0m and 12.5m. Four subsequent experiments (dashed lines) and linearized theory (solid lines). Peak frequency $\omega_{peak} = 7s^{-1}$, $V_0 = 0.04V$ and $\eta_{max}k_{peak} \simeq 0.025$. Note the difference in horizontal scale between figures a and b & c.

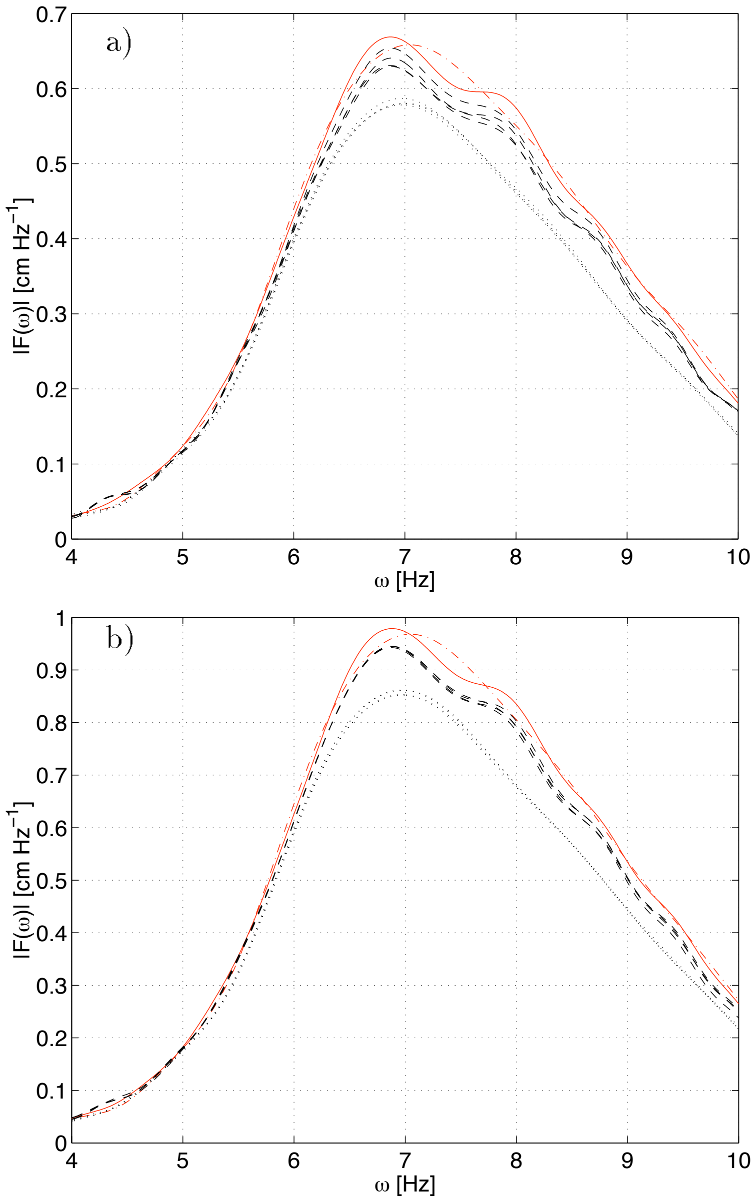

Figure 2: $|\mathcal{F}(\omega)|$ for four experiments at two positions and theory. Dashed lines are experiments at $x = 2m$ (solid is theory) and dotted lines at $x = 12.5m$ (dash dot is theory). a): $V_0 = 0.04V$, $\eta_{max}k_{peak} \simeq 0.025$. b): $V_0 = 0.08V$, $\eta_{max}k_{peak} \simeq 0.038$.

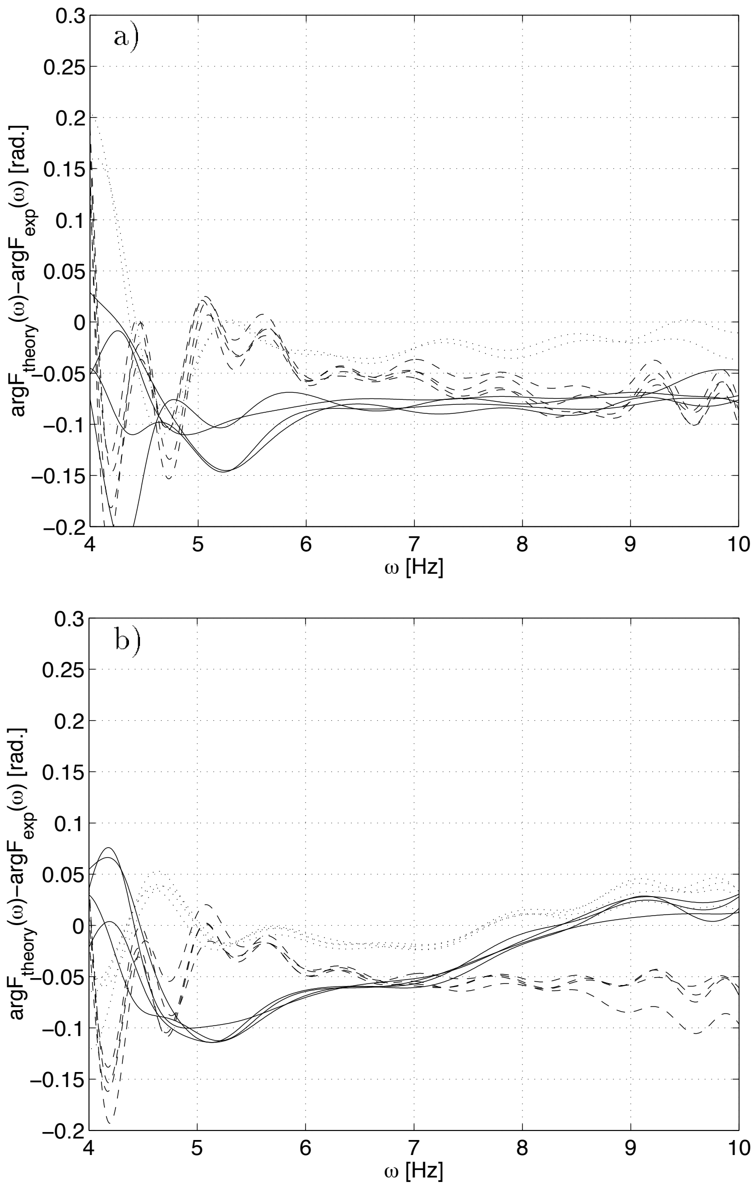

Figure 3: $arg\mathcal{F}_{theory}(\omega) - arg\mathcal{F}_{exp}(\omega)$. Dashed lines are at $x = 2m$, dotted lines at $x = 6m$ and solid lines are at $x = 12.5m$. a): $V_0 = 0.04V$, $\eta_{max}k_{peak} \simeq 0.025$. b): $V_0 = 0.08V$, $\eta_{max}k_{peak} \simeq 0.038$.

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