ON THE COMPUTATIONS OF THE COMPLETE WAVE DRIFT DAMPING PROBLEM FOR A SHIP MODEL

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Abstract

Computations of wave drift damping and related quantities like the induced wave responses of a 230m long ship are performed for wave periods in the range \(7s \leq T \leq 14.8s\) and a large number of wave angles using the computer code WAMITUIO. The (moored) ship model slowly moves in the three horizontal modes of motion while responding to incoming waves in six degrees of freedom. The numerical model consistently accounts for the coupled fluid and ship motions. We find that the individual components of the wave drift damping matrix are particularly sensitive to a few wave directions, being different for the various components, however. Most important effect occurs for wave periods in the range \(9s \leq T \leq 13s\). The linear responses of the ship are significant for all the computed parameters.

Key words: Wave drift damping, wave and current forces, ship motion

1 Introduction

An important aspect of marine hydrodynamics concerns the effects of the ocean wave environment on stationary floating bodies like a moored ship. Of particular interest from both a theoretical and practical point of view are how to compute the induced wave forces and the resulting responses of the ship. In an irregular sea, the linear wave forces and responses oscillate with the frequencies of the incoming waves. Non-linear effects, however, lead to wave forces and responses oscillating with the sum and difference frequencies of the individual wave components. We shall here be concerned with the damping of the induced difference frequency motions of the ship, more specifically the wave drift damping problem.

We consider a model of a (moored) floating body moving slowly in the three horizontal modes of motion (surge, sway and yaw) while responding to incoming waves in six degrees of freedom. Consider first the time-averaged horizontal force \((F_x, F_y)\) and the moment \(M_z\) about the vertical axis acting on the body. These
quantities are proportional to the wave amplitude squared, and may be obtained by

\[
\begin{pmatrix}
F_x \\
F_y \\
M_z
\end{pmatrix} = \begin{pmatrix}
F_{x0} \\
F_{y0} \\
M_{z0}
\end{pmatrix} - \begin{pmatrix}
B_{11} & B_{12} & B_{16} \\
B_{21} & B_{22} & B_{26} \\
B_{31} & B_{32} & B_{36}
\end{pmatrix} \begin{pmatrix}
U \omega/g \\
V \omega/g \\
\Omega/\omega
\end{pmatrix}
\]

(1)

where \(B_{ij}\) denotes the wave drift damping matrix and \(U\) and \(V\) are the slow horizontal velocities along the \(x\) and \(y\) directions, respectively, while \(\Omega\) is the slow angular velocity of the floating body. \((F_{x0}, F_{y0}, M_{z0}) = (F_x, F_y, M_z)\) for \(U = V = \Omega = 0\). \(\omega\) denotes the frequency of the incident waves upon the floating body while \(g\) is the acceleration due to gravity. Furthermore, \(x, y\) denote horizontal coordinates and \(z\) vertical coordinate, with \(z = 0\) in the mean free surface of the sea.

During recent years a complete mathematical and numerical model for solving the wave drift damping problem (obtaining \(B_{ij}\)) has been developed at the University of Oslo. The theoretical model is based on the application of potential theory. The formal solution of the problem is expressed in terms of integral equations, while numerical solution is obtained by means of a low-order panel method. The code is developed and adapted to the WAMIT program (developed at MIT) and is called WAMITUJO. We have now come that far that the code may be applied to extensive wave analysis of realistic marine bodies. This is the purpose of the present paper. The geometry under consideration is a model of a turret production ship (TPS) being 230m long with 41m beam and 15m draught (see figure 5).

The transitory part of the wave drift damping problem includes the mathematical and numerical problem of a ship moving (drifting) with a small (constant) speed in waves. The latter problem is mathematically equivalent to the problem of interaction between waves, a current and a floating body. Thus, results for the latter problems are included in the results presented.

Our procedure has the main goal of obtaining the wave drift damping matrix, which determines an important contribution to the equation governing the slow motions of the (moored) ship. A complete method for obtaining the wave drift damping matrix for marine bodies of general shape has until now been lacking. Along the track of solving the wave drift problem, we also treat the linear part of the coupled problem of a ship responding to incoming waves while moving horizontally with a slow motion at a significantly larger time scale than the periods of the waves. Thus, we shall also present results for the linear wave-induced motions of the ship, accounting for the effects introduced by a slow drift of the body.

We assume that the sea may be characterized by a spectrum due to longcrested waves. The latter means that the sea is unidirectional. We further assume that the individual components of the wave spectrum may be analyzed separately and that the effect due to the various wave components may be combined subsequently. The theory and results described below, under the assumption of incoming monochromatic unidirectional waves, is applicable to irregular seas in this context.

2 Brief review of the theory

The following description is summarized from [2-5,7]. The mathematical problem is formulated in a relative frame of reference \(O-xyz\) following the slow motion of the body. Unit vectors \(i, j, k\) are introduced in the \(x, y, z\) directions, respectively. The water depth is constant and equal to \(h\). We assume that potential theory may be applied to determine the fluid motion.
Incoming waves in the relative frame of reference are described by the potential

$$\phi^I = \text{Re}[(Aig/\omega)\varphi^I e^{i\omega t}]$$

(2)

where

$$\varphi^I = \frac{\cosh k(z + h)}{\cosh kh}e^{-ikR \cos(\beta - \phi)},$$

(3)

$$\sigma = \omega - Uk \cos \beta - Vk \sin \beta,$$

(4)

and \(A, k, \beta\) are the wave amplitude, the wavenumber and the (time-dependent) wave angle, respectively. \(\omega\) is the wave frequency obeying the dispersion relation

$$\omega^2 = gk \tanh kh.$$ 

(5)

Furthermore, polar coordinates are introduced such that \(x = R \cos \theta\) and \(y = R \sin \theta\).

Let \(\mathbf{v}\) denote the fluid velocity in the relative frame of reference. \(\mathbf{v}\) may be decomposed by \(\mathbf{v} = \mathbf{v}' - U \mathbf{i} - V \mathbf{j} - \Omega k \times \mathbf{x}\), where the three latter components are introduced to an observer which change his position from the fixed to the relative frame of reference. \(\mathbf{v}'\) may be described by a velocity potential \(\Phi'\) satisfying the Laplace equation \(\nabla^2 \Phi' = 0\). The velocity potential may be decomposed by

$$\Phi' = U \chi^U + V \chi^V + \Omega \chi^\Omega + \phi + \varphi^{(2)},$$

(6)

where \(U \chi^U + V \chi^V + \Omega \chi^\Omega\) denotes the potential due the slow motion of the body in the absence of incoming waves, \(\phi\) is the linear potential due to the incoming, scattered, and radiated waves, which all are proportional to the wave amplitude \(A\), and \(\varphi^{(2)}\) is a time-averaged potential which is proportional \(A^2\). The latter potential contributes to the time-averaged force and moment acting on the body, and to the equation governing the slow motions of the body [3,5].

All potentials satisfy boundary conditions at the free surface, at the wetted body surface, in far field and at the sea floor. The free surface boundary condition is obtained using that the individual derivative of the pressure at the free surface is zero. In particular, the free surface boundary condition for \(\phi\) becomes, upon linearizing with respect to the wave amplitude,

$$\phi_{tt} + 2w \cdot \nabla_h \phi_t + \phi_h \nabla_h \cdot \mathbf{w} + g\phi_z = 0 \quad \text{at} \quad z = 0,$$

(7)

where

$$\mathbf{w} = U(-\mathbf{i} + \nabla \chi^U) + V(-\mathbf{j} + \nabla \chi^V) + \Omega(-k \times \mathbf{x} + \nabla \chi^\Omega)$$

(8)

and \(\nabla_h\) is the horizontal gradient. We next introduce the following decomposition of the velocity potential

$$\phi = \text{Re}[(iAg/\omega)\varphi_D e^{i\omega t} + \sum_{j=1}^{6} \frac{d}{dl}(\xi_j e^{i\omega t} \varphi_j)],$$

(9)

where the first part represents the incoming and scattered waves, and the second part the radiation potential. \(\xi_j\) denotes the amplitude of oscillation of mode \(j\) \((j = 1,...,6)\). The motion may be expanded after the slow velocities by

$$\xi_j = \xi_j^0 + (U \omega/g)\xi_j^U + (V \omega/g)\xi_j^V + (\Omega/\omega)\xi_j^\Omega.$$

(10)
Corresponding expansions are introduced for the potentials \( \varphi_D \) and \( \varphi_j, \ j = 1, \ldots, 6 \). Further description of the mathematical and numerical analysis may be found in the references [2-5,7]. Other relevant work on the wave drift damping problem may be found in the reference list.

Having formulated the perturbed boundary value problems, the formulae for the wave drift damping matrix are obtained following the main steps:

- Compute the potentials to governing the flow field for the geometry being considered. The potentials are obtained from the sets of the formulated boundary value problems.
- Evaluate the added mass and damping coefficients, the exciting forces, and the linear motion of the body.
- The formulae for the wave drift damping are derived, starting from the equations of conservation of linear and angular momentum. All terms proportional to the wave amplitude squared times the slow velocities of the body are accounted for.

When the wave drift damping matrix and the drift forces in (1) are obtained, we may solve the equation governing the slow drift motion of a (moored) ship, see [1, eq. 1].

3 Results and discussion

Computations of the wave-drift damping matrix \( \{B_{ij}\}, \ i, j = 1, 2, 6 \) and the linear body responses \( \xi_j \) are performed. We have also evaluated the drift-force and moment \( (F_{x0}, F_{y0}, M_{z0}) \) for zero speed of the body, but results are not shown here for these quantities. The range of the period of the incident waves is \( 7 \leq T \leq 14.8 \). With an infinite water depth in the computations, the corresponding range of the wave length is \( 77m \leq \lambda \leq 342m \). The wave angles \( \beta \) have been taken for every increment of \( 15^\circ \), namely \( 0^\circ, 15^\circ, 30^\circ, \ldots, 360^\circ \).

In figure 1, we show the relation between the components of the wave-drift damping matrix and the wave period \( T \) and angle \( \beta \). The intersections between the solid lines in the plots correspond to the values which are computed. The figure is presented just like the matrix in equation (1). In figure 2, we present some samples of \( B_{ij} \) as functions of the wave period \( T \) for several values of \( \beta \).

The ship is symmetric with respect to the length direction \( (y = 0) \), giving

- \( B_{11}, B_{22}, B_{33}, B_{66} \) are symmetric about \( \beta = 180^\circ \) (head waves) and \( \beta = 0^\circ \) (following waves).
- \( B_{12}, B_{16}, B_{21}, B_{31} \) are anti-symmetric about \( \beta = 180^\circ \) and \( \beta = 0^\circ \). (These coefficients become zero at \( \beta = 0^\circ \) and \( 180^\circ \).)

The ship may, roughly speaking, be regarded as being symmetric with respect to the mid-beam \( (x = 0) \), giving

- \( B_{22} \) and \( B_{66} \) are almost symmetric about \( \beta = 90^\circ \) and \( \beta = 270^\circ \) (beam seas)
- \( B_{12}, B_{21}, B_{36}, \) and \( B_{63} \) are close to being anti-symmetric about an angle close to \( \beta = 90^\circ \) and \( \beta = 270^\circ \).
The diagonal members $B_{ii}$ of the matrix are all positive. This means that the wave drift damping provides a positive damping of induced slow motions of the ship. We note, however, that negative wave drift damping may occur for marine bodies with a complex geometry, like the one of an oil-platform, see e.g. [7].

The results show that the individual wave drift damping coefficients are particularly sensitive to a few wave directions. For the diagonal terms we have: $B_{11}$ is most pronounced for headings 45°, 135° (and 315°, 225°). $B_{11}$ is also pronounced for head seas and following seas. The terms $B_{22}$ and $B_{66}$ are most pronounced in beam seas, however.

For the off-diagonal terms we find that $B_{21}$ and $B_{12}$ are most pronounced for headings 60°, 120° (and 300°, 240°), while $B_{62}$ and $B_{26}$ are most pronounced for headings 75°, 105° (and 285°, 255°). The terms $B_{61}$ and $B_{16}$ are rather large for several wave headings.

For all components $B_{ij}$, most important effect occurs for wave periods in the range $9s < T < 13s$, roughly speaking.

We note that the magnitude of $B_{66}$ is significantly larger than any other components of the wave drift damping matrix. Comparing the values of $B_{66}$ with the yaw-moment $M_{6}$ at zero speed (results not shown), the former may be about 100 times larger than the latter, which means that a slow rotation of the ship gives a significant contribution to the moment $M_{z}$.

The linear responses of the ship are determined by

$$\xi_j = \xi_j^0 + (U\omega/g)\xi_j^U + (V\omega/g)\xi_j^V + (\Omega/\omega)\xi_j^\Omega.$$ 

Examples of the responses at zero speed ($\xi_j^0$) and the perturbations due to a forward speed $U$ of the ship ($\xi_j^U$) are presented in figures 3-4. Most of the components of $\xi_j^0$ and $\xi_j^U$, for the same mode of motion, are of comparable magnitude. A larger effect of the perturbed pitch motion ($j = 5$) is observed for wave period about 10s, however. A forward speed of $U = 1m/s$ means that $U\omega/g \approx 0.06$, giving as example, for $\beta = 45°, 135°$,

$$\xi_5 = \xi_5^0 + (U\omega/g)\xi_5^U \approx (0.68 + 0.06 \cdot 3.3)(A/B_0) \approx 0.88 \cdot (A/B_0)$$

which means a rather pronounced effect due to the forward speed of the body. ($B_0$ denotes the beam of the ship.) Similar results for the pitch responses $\xi_6^U$ and $\xi_3^U$ are true. More results will be shown at the conference.

Finally we note that the simulations may in the next step be applied to find the forces induced in moorings of the ship, and to estimate the motions of and the induced tensions in eventual risers connected to the ship.

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Figure 1. Wave drift damping matrix $B_{ij}$ organized as in eq. (1) vs. wave period $T$ and wave angle $\beta$, nondimensionalized by $\rho g A^2 B_0$ ($i = 1, 2$, $j = 1, 2, 6$), $\rho g A^2 B_0^2$ ($i = 6$, $j = 1, 2, 6$). $B_0$ beam of the ship. Solid line corresponds to computed values.
Figure 2. Samples of $B_{ij}$ as functions of wave period $T$ for several wave angles $\beta$ arranged like the matrix in Figure 1.
Figure 3. Zero speed responses \( \xi_0/(A/B_0^n) \). \( n = 0 \) \((j = 1, 2, 3)\), \( n = 1 \) \((j = 4, 5, 6)\). First column: \( j = 1, 3, 5 \). Second column: \( j = 2, 4, 6 \).
Figure 4. Perturbed responses $\zeta_j^U/(A/B_0^n)$. $n = 0 \ (j = 1, 2, 3), \ n = 1 \ (j = 4, 5, 6)$. First column: $j = 1, 3, 5$. Second column: $j = 2, 4, 6$. 
Figure 5. The ship model. Wetted surface (total ship) discretized by 760 panels. Free surface (total) discretized by 1800 panels, with an outer radius of 2l, approximately. Computations of the symmetric model are performed with half geometry.

References


