PARTICLE TRACKING IN IDEALIZED BAROTROPIC FLOW ON THE CATALAN SHELF

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Abstract

Particles\(^1\) are traced on the Catalan shelf to investigate the cross and along shelf transport. The trajectories are found by using a Lagrangian particle tracking technique incorporated into the nonlinear idealized barotropic model of the flow. The rate of the dispersion, in cross and along shelf direction, is evaluated from statistics of the particles released. Horizontal diffusion coefficients are calculated.

Key words: Particle Motion, Shelf Dynamics, Diffusion, Topographic Steering, Mixing

1 Introduction

A nonlinear idealized barotropic model of the shelf edge flow on the Catalan shelf is developed in Gjevik et al. (1999). The topography of the shelf domain is characterized by a transition zone from a narrow to a broader shelf which is expected to have a profound effect on the shelf flow. In order to study the structure of this flow, with particular interest in cross and along shelf dispersion in the transition zone, a particle tracking algorithm is implemented into the nonlinear barotropic model.

The numerical model, which simulates barotropic flow on the Catalan Shelf, solves the nonlinear depth integrated equation of motion and the equation of conservation of mass by a finite difference method. Figure 1 shows the model domain and the shelf edge jet imposed at the upstream boundary. The jet is centered 10 km outside the shelf break and has a cosine profile

\[
\bar{v} = v_0 s(t) \cos \frac{\pi(x - L_B)}{B} \quad L_B - \frac{B}{2} < x < L_B + \frac{B}{2}
\]

\(^1\) Water parcels, zooplankton, eggs, pollution etc. treated as passive participants drifting with flow, assuming swimming and migration of minor importance.

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where $v_0 = 0.2 \text{ m/s}$, $B = 20 \text{ km}$ and $L_B = 130 \text{ km}$. The function $s(t)$ increases the strength of the jet from rest at $t = 0$. A throughout description of the barotropic model, the shelf slope profile and simulations with different forcing on the upstream boundary is found in Gjevik et al. (1999). The current field used in this report is forced by the jet described in (1), which correspond to the simulation depicted in figure 9 of Gjevik et al. (1999).

![Diagram](image)

Fig. 1. Model domain and jet profile at upstream boundary. Isobaths for 100, 500, 1000 and 1500 m. Shelf break at 100 m isobath. Impenetrable boundaries at $x = 0$ and $x = 150$.

The long shelf variation in shelf width is modelled by

$$L(y) = L_0 + \frac{L_D - L_0}{2} \left[ 1 + \tanh \frac{2(y - y_0)}{L_T} \right]$$

(2)

where $L_0 = 139 \text{ km}$ and $L_D = 101 \text{ km}$ scale the upstream and downstream width of the shelf respectively and $L_T = 30 \text{ km}$ is the width of the transition zone. The parameter $y_0 = 150 \text{ km}$ gives the center of the transition zone at the long shelf coordinate with $|\frac{2(y - y_0)}{L_T}| \gg 1$ both at the upstream and downstream boundaries of the model domain.

2 Particle tracking

Lagrangian motion of particles released in the current field is found by integrating the equations

$$\frac{dx_k}{dt} = \overline{u}(x_k, y_k, t) \quad \frac{dy_k}{dt} = \overline{v}(x_k, y_k, t)$$

(3)

Here $\overline{u}$ and $\overline{v}$ are the depth mean current velocity, subscript $k$ denotes the particle number ($k = 1 \ldots M$) and $(x_k, y_k)$ its position. The particle tracking algorithm is
implemented into the barotropic nonlinear ocean model. For simplicity a standard Euler method is used as initiator to an Adam-Moulton method for the numerical integration of the equations (3).

The necessary continuity in the velocity field, required by an Lagrangian particle tracking technique (Hunter (1987)), is obtained by bilinear interpolation from the 4 closest spatial current nodes and numerical integration of (3) performed with the same time step as used by the barotropic model. The use of unmodified bilinear interpolation is source to unwanted effects near land. Nearest neighbour interpolation, which simply returns the value of the closest grid point, is therefore used if the nearest neighbour is a dry grid point. This technique deadlocks particles which encounter a region defined as solid land.

A validation of the tracking algorithm was performed through simulations from a slightly modified version of the model in Ommundsen (1999) with a much courser resolution. The computational cost, if a moderate amount of particles are released, is low. However, the nonlinear flow model may have to be reran several times, due to the fact that one may want to change the predefined amount and initial position of the particles released.

3 Simulation

The simulations performed by Gjevik et al. (1999) show strong tendency for topographic steering of the flow along the shelf edge. Plots of the current and the dispersion of particles released, reveal that the jet is unstable with eddy structures inside and along the shelf edge. In figure 2 and 3, taken from Gjevik et al. (1999), the dispersion of two clusters of particles released in the current field is shown. In figure 2 the cluster of particles is initially shaped as a rectangle and placed inside the 500 isobath. The figure shows the steering of the jet and the location of smaller scale eddies in the transition zone from the narrow to the broader part of the shelf. The shape of the cluster is deformed into an elongate feature in the deeper regions, while on the shelf the cluster is somewhat strained. In figure 3 the particles are released at the transition zone from a narrow to a broad shelf. There are evidence of mixing of the water masses near the 100 isobath. Inside the cluster, there are structures of the prevailing circulation in the area. The maximum strength of the mean current is located at the 500 isobath.

For a further study of the cross and along shelf dispersion a particle-source is placed in coordinate (138.3, 120) on the 100 meter isobath. From this source 50 particles are uniformly released within a time interval of 100 hours. Each particle is traced over a time interval of 360 hours.

Figure 4 shows the trajectories of particles released from the described source. A combination of eddy structures along the 100 isobath and a strong topographic steering of the jet are visible. The numerical barotropic model ran 100 hours before the first particle was released, this to ensure that the complex flow pattern set by the jet at the upstream boundary had been developed.

To quantify the cross shelf dispersion a displacement-function which measure the
Fig. 2. Particle location from initial release, $t = 0$, with 72 hours consecutive intervals, ordered left-right top-bottom. Total number of released particles 6767. Dark shading represent dense concentration of particles.
Fig. 3. Particle location from initial release, $t = 0$, with 72 hours consecutive intervals, ordered left-right top-bottom. Total number of released particles 7175. Dark shading represent dense concentration of particles.
distance from the shelf edge is defined by

$$F_{k,n}(y) = \sqrt{(x_{k,n} - L(y))^2 + (y_{k,n} - y)^2}$$  \(\text{(4)}\)

Here \(x_{k,n}\) and \(y_{k,n}\) are the particle position at time index \(n\) and \(L(y)\) is the tanh function of the 100 isobath (shelf break), equation 2. The time index \(n\) is individual connected to each particle, starting when the particle is released from the source \(n = 0, 0.25, 0.5 \ldots 360\) hours. The minimum of \(F_{k,n}\) is then the shortest distance between particle number \(k\) and the 100 isobath \(n\) hours after the particle has been released from the source. This minimum is found using the conjugate gradient algorithm of Shanno and Phua (1980). \(F_{k,n}\) is defined positive if the particle is located in shallow waters (depth < 100 meter). To quantify the along shelf dispersion a distance-function \(B_{k,n}\) is calculated by projecting the particles in \(y\)-direction onto the 100 isobath and calculating the arc length along the 100 isobath by numerical integration from the source position. If this integration, of the arc length, is directed towards the downstream boundary the value of \(B_{k,n}\) is defined positive.

In figure 5 we depict the dispersion in cross and along shelf direction. The dispersion of particles in cross shelf direction is approximately within a region of \(\pm 10\) km relative to the 100 isobath, while the dispersion of particles in along shelf direction stretches from
20 to 160 km at the end of the simulation. The maximum values of $B_{k,n}$ are obtained by particles which are early transported onto the 500 isobath and steered along this isobath, see also figure 4. Strong small scale eddies in the nearer region of the source and in the transition zone restrict an effective drift in along shelf direction for a majority of the particles. The cross shelf dispersion is caused mainly by the eddies located at the transition zone. The main effect of this smaller scale currents is to randomly delay particles in their dominant along shelf drift and to disturb their relative cross shelf position. This combination is source to an effective stirring mechanism particularly near the 100 isobath at the transition zone.

![Graph showing dispersion](image)

Fig. 5. Left panel shows the dispersion, $F_{k,n}$, in cross shelf direction relative the 100 isobath. Right panel shows the dispersion, $B_{k,n}$, in along shelf direction. All 50 particles are depicted. The time $n = 0, 0.25, 0.5 \ldots 360$ hours is individual connected to each particle, starting when the particle is released from the source.

A calculation of the diffusion coefficients in cross and along shelf direction quantify the rate of dispersion (Burrows et al. 1999; Booth 1988). The horizontal diffusion coefficients are defined by

$$K_z = \frac{1}{2} \frac{d\sigma_z^2}{dt}$$  \hspace{1cm} (5)

where $\sigma_z^2$ denotes the variance and $z$ is a subscript for either in cross shelf or along shelf direction.

In figure 6 the result of a centered difference estimate, $dt = 0.25$ hours, of the diffusion coefficients as function of time is shown. A constant diffusion coefficient is suitable if the variance is proportional to the time. The rapid oscillations shown in figure 6 indicate a more complex model of diffusion related to the developments of eddies along the 100 isobath. The diffusivity at peak is approximately $1.6 \cdot 10^2 \text{ m}^2/\text{s}$ and $3.3 \cdot 10^3 \text{ m}^2/\text{s}$ in respectively cross and along shelf direction. These estimates are comparable to the constant diffusions estimate of for example Burrows et al. (1999) and Booth (1988) for the Nordic seas, but in our case the effect is strongly reduced by the rapid oscillation and the strong contractions (negative diffusivity).
Fig. 6. Left panel shows the diffusion coefficient calculated in cross shelf direction. Right panel shows the diffusion coefficient calculated in along shelf direction.

The strong diffusivity in along shelf direction is caused by the combination of a dominant shelf directed current and the development, displacement and fading of small scale eddy structures in the current field. The strong topographic steering of the jet and the small scale of these eddies restrict on the other hand the diffusivity and dispersion in cross shelf direction.

4 Conclusion

The current field and trajectories of particle released show strong topographic steering of the jet. Although the current field is quite complex, due to flow instability and eddy formation, the cross shelf transport is relatively small.

Small scale eddies and perturbation in the dominant current field displace and delay particles in their dominant along shelf drift. This complexity in the current field has the main effect of a small dispersion in cross shelf direction and a larger dispersion in along shelf direction. This dispersion of particles is source to mixing particular in a small region near the 100 isobath.

The maximum estimated diffusion coefficients are approximately $1.6 \cdot 10^2$ m$^2$/s and $3.3 \cdot 10^3$ m$^2$/s in respectively cross and along shelf direction. The estimated diffusion coefficients vary considerable in time due to the eddy activity in the flow. Due to these variations and negative diffusion the dispersion in both cross and along shelf direction is much smaller than what the peak estimates of the diffusion indicate.

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References


