Lower slenderness limits for braced, end-loaded r.c. compression members

Jostein Hellesland

Professor, Mechanics Division, Department of Mathematics, University of Oslo, P. O. Box 1053 – Blindern, NO-0316 Oslo, Norway

ABSTRACT

Second order effects of axial forces on displacements in compression members are small and can be neglected in a great many structures, which may then be designed for forces obtained by conventional first order theory. Extensive nonlinear analysis results are presented to document slenderness limits below which this is so for braced compression members and major factors that affect lower slenderness limit predictions are investigated. A new lower slenderness limit formulation is presented for compression members braced against sidesway. The slenderness is defined in terms of a so-called normalised slenderness that, in addition to the geometrical slenderness, is a function of axial force and reinforcement. The limit itself is a function of the first order end moment ratio. The formulation is rational and reliable and may replace, or used as an alternative to, existing lower limit formulations.

KEYWORDS

Columns (supports); Compression members; Non-slender members; Reinforced concrete; Lower slenderness limits; Second order effects; Slenderness effect; Nonlinear analysis; Moment gradient; Non-sway members; Braced members.
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1 Introduction

The effect of axial forces on the displacements in slender compression members (columns, struts, etc.), will affect sectional forces, and in particular the moments. These second order (or secondary) effects, are small and can be neglected in a great many structures. In a survey carried out in conjunction with the 1971 revision of ACI 318 code, it was found that the lower slenderness limits in that revision allowed second order effects to be neglected in as many as 90% of the columns in braced frames, and in 40% of the columns in unbraced frames (MacGregor, Breen and Pfang 1970). These findings demonstrate the significant usefulness of giving lower limits. And indeed, most design codes and standards for reinforced concrete structures give lower slenderness limits for compression members, and allow structures to be designed for forces obtained by conventional first order theory when these limits are not exceeded.

The use of effective lengths in such limits, to account for different boundary conditions, have been standard for a very long time. For braced members in the 1971 revision of the ACI 318 code, such limits appear for the first time also to be given as a function of the first order moment gradient along the member. This allowed for a significantly more rational assessment of such members in all states of single and double curvature than before. The proposed limit found wide acceptance and was adopted in many codes and standards internationally, in the exact same form, or in very similar forms (DIN 1978; CEB 1978).

Several efforts have been made to develop more refined limits that also included the effect of important parameters such as axial load (Menegotto 1983; EC2 (CEN 1991); CEB-FIP MC90 (CEB 1993); MacGregor 1993) and axial load as well as reinforcement (McDonald 1986; Bridge and Seevaratnam 1987; Hellesland 1987, 1990, 1993; EC2 (CEN 2002); Mari and Hellesland 2002). Several of these have been adopted in national standards: the Menegotto proposal by the Danish DS 411 (DIF 1984), the Bridge and Seevaratnam’s proposal by the Australian AS3600 (AS 1988), the Hellesland proposal by the Norwegian NS 3473 (NSF 1989), and the MacGregor proposal by the Canadian A23.3 (CSA 1994).

Despite the inclusion of additional parameters, these limits give rather different results for some combinations of influencing parameters. This may not strengthen the confidence in their use as documentation of when second order effects may be neglected. A disadvantage of axial force independent limits (e.g., ACI limits) is that they tend to be very conservative at lower axial force levels and may even be very unconservative at higher levels (MacGregor 1993). The degree of conservativeness or unconservativeness is also a function of the reinforcement level. Codified lower slenderness limits for compression members are very important in
that they are accepted as documentation of when second order effects may be ignored. Thus, it is important that they are reliable.

The main objective of the present paper is to present nonlinear analysis results for compression members braced against sidesway, and present lower slenderness limit formulations for such members that accounts for moment gradient, axial force and reinforcement, and to document their applicability. In this pursuit, some existing limits will be reviewed and some major factors that affect lower slenderness limit predictions of braced compression members investigated. The factors considered include
- moment gradient;
- end restraints and their reflection through effective lengths;
- criterion for lower slenderness limit;
- alternative slenderness parameter (normalised slenderness);
- full range of axial forces and reinforcement amounts;
- sustained loading.

Numerical data from linear elastic and nonlinear analyses are presented in the paper to document some aspects, and existing nonlinear analysis data, available in the literature, are employed to document sustained load effects.

The study is limited to members with symmetrical reinforcement and uniform cross section, reinforcement and axial load along the member axis. Compression members that are not braced against sidesway and braced compression members with transverse loading are considered in a separate “companion” report (Hellesland 2002).

2 Problem definition

Fig. 1 illustrates typical displacement shapes and first order moment distributions (full lines) for braced members with a) unrestrained (hinged) ends and b) rotationally restrained ends. Case c), with a controlled relative end displacement, behaves similar to case b). Also shown are total moment distributions (dashed lines) that include second order (slenderness) effects sufficient to produce maximum total moments, \( M_{\text{max}} \), that exceed the numerically largest first order end moments, \( M_{\text{g}} \). In restrained members, also the end moments are affected by slenderness effects. Due to the development of maximum moment between ends, a member’s load capacity will become less than it would have in the absence of slenderness effects. The choice of slenderness limits is to be made such that this capacity reduction does not exceed an acceptable amount.
A braced member is here defined as one whose ends are not free to sway sideways (translate) relative to each other. The displacement of one end relative to the other is prevented completely, or restricted to a certain amount (Fig. 1(c)) by bracing elements. This could be shear walls, etc., or it could be columns in moment resisting frame that have sufficient lateral stiffness to prevent (“control”) the lateral translation to a limited amount. In such frames one has columns that are being “braced”, and columns that provides the “bracing” (lateral stiffness).

3 Overview of lower slenderness limits

An overview of selected slenderness limits, below which it is acceptable to neglect second order (slenderness) effects, is given in Table 1. The limits are for reinforced concrete compression members that are braced against sidesway. In the table, $\lambda$ is the slenderness and $\mu_o$ the first order end moment ratio defined by

$$\lambda = \frac{L_e}{i} = \frac{\beta L}{i} ; \quad \mu_o = \frac{M_{01}}{M_{02}}$$

where $L_e = \beta L$ is the effective length (buckling length), $\beta$ the effective length factor (buckling length factor), $L$ the member length and $i$ the radius of gyration of the cross section.

The moment ratio is between the smaller ($M_{01}$) and the larger ($M_{02}$) factored design end moments. The ratio is to be taken positive for members bent in single curvature by these moments, and negative for members in double curvature.
Table 1: Overview of selected lower slenderness limits

<table>
<thead>
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<th>No.</th>
<th>Source</th>
<th>Braced member limits</th>
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<tr>
<td>1</td>
<td>ACI 318-71 (1971)</td>
<td>( \lambda = 34 - 12 \mu_o )</td>
</tr>
<tr>
<td>2</td>
<td>DIN 1045 (1978)</td>
<td>( \lambda = 45 - 25 \mu_o )</td>
</tr>
<tr>
<td>3</td>
<td>MC78 (CEB 1978)</td>
<td>( \lambda = 50 - 25 \mu_o )</td>
</tr>
<tr>
<td>4</td>
<td>McDonald(1986)</td>
<td>( \lambda = \sqrt{\frac{225-200\mu_o}{N_f/N_o}} )</td>
</tr>
<tr>
<td>5</td>
<td>AS 3600 (1988) Bridge et al. (1987)</td>
<td>( \lambda = 60(1 - \mu_o)(1 - \frac{N_f}{0.6N_o}) \geq 25 )</td>
</tr>
<tr>
<td>6</td>
<td>NS 3473-1989, Hellesland (1987,1990)</td>
<td>( \lambda_N = 18 - 8\mu_o ) ; ( \lambda_N = \lambda \sqrt{\nu/(1 + k_i\omega_i)} )</td>
</tr>
<tr>
<td>7</td>
<td>“EC2” (Eurocode 2) (CEN 1991)</td>
<td>( \lambda = \frac{15}{\nu} \geq 25(2 - \mu_o) )</td>
</tr>
<tr>
<td>8</td>
<td>“MC90” (CEB 1993)</td>
<td>( \lambda = \frac{7.5}{\nu}(2 - \mu_o) \geq 12(2 - \mu_o) )</td>
</tr>
<tr>
<td>9</td>
<td>CSA A23.3 (1994), MacGregor (1993)</td>
<td>( \lambda = \frac{25-10\mu_o}{\sqrt{N_f/f_{yAk}}} \ ; \mu_o \geq -0.5 )</td>
</tr>
<tr>
<td>10</td>
<td>ACI 318-95, ...-02</td>
<td>( \lambda = 34 - 12\mu_o \leq 40 )</td>
</tr>
<tr>
<td>11</td>
<td>“EC2” (Eurocode 2), draft (CEN 2002)</td>
<td>( \lambda = 20\frac{ABC}{\sqrt{\nu^2}} \ ; A = \frac{1}{1+0.2\phi_{ef}} \ ; B = \sqrt{1+2\omega_i} \ ; C = 1.7 - \mu_o )</td>
</tr>
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- Notation here may deviate from that used in original sources
- \( N_o = 0.85f_{ck}A_c + f_{yA_{st}} \) • \( \nu = N_f/(f_{cd}A_c) \) • \( \omega_i = (f_{yA_{st}})/(f_{cd}A_c) \)
- Definition of \( f_{cd} \) varies: NS 3473 \( f_{cd} = f_{cm}/\gamma_c \) ; EC2 (1991) and MC90 \( f_{cd} = f_{ck}/\gamma_c \) ; EC2 (2002) \( f_{cd} = \alpha_{ce}f_{ck}/\gamma_i \).
- \( k_i=4 \) for “corner” reinforced rectangular section; \(-8/3 \) otherwise.
- \( \phi_{ef} \) = effective creep coefficient; if not known \( \phi =0.7 \) may be used.
These end moments are normally those obtained from a conventional first order analysis with recommended stiffness assumptions and based on the intended geometry, i.e., without unintentional eccentricities or imperfections included. In the proposed EC2 limit (CEN 2002), the end moments to be used is to be adjusted for imperfections (accidental eccentricities). To deal with uncertainties at small moments in NS 3473, it was chosen to reflect effects of unintentional eccentricities by choosing a conservative moment gradient relationship (Hellesland 1990) and to require the end moment ratio to be taken equal to unity when the maximum first order moment is less than a specified minimum value.

In these expressions, the radius of gyration \((i)\) is generally taken as that of the gross section \((i_g)\) or, in a few cases, as that of the net concrete section \((i_c)\). The difference between these is academic since \(i_c\) is approximately equal to \(i_g\) for practical steel percentages. The length \(L\) is generally either the unsupported \((L_u,\) as in the ACI code), the system length or something in between. With the \(i\)-definition above, \(\lambda\) is simply a geometrical slenderness parameter.

The design compressive strength of concrete \(f_{cd}\) is defined somewhat differently in the different sources, as indicated in the footnotes to the table.\(^1\) Further details, including numerical values for material factors, etc., see Hellesland (2002).

Although not always explicitly stated in the various sources, the given limits have generally been obtained for members with uniform cross-section and reinforcement is along the member. Care should therefore be exercised when applying the to other cases.

A limit proposed by Menegotto (1983) for unbraced columns, and adopted in the Danish standard DS 411 (1984) for compression members in general, does not include a moment gradient parameter and is not included in the table. It is reviewed in Hellesland (2002).

Criteria adopted in the derivation of the various slenderness limits are reviewed reasonably detailed in Hellesland (2002). Briefly summarized, ACI 318-02, NS 3473, McDonald, and CSA A23.3 relate their limits to a 5% capacity reduction or a 5% moment increase, and DS 411 (Menegotto), MC90 and EC2 to a 10% criterion. The AS 3600 limit, apart from the lower cut-off (25), is on the other hand based on a fit to numerical results for 0% reduction in axial load capacity (under constant eccentricity) of unrestrained columns with and without out-of-straightness imperfections (Bridge and Seevaratnam 1987).

The MC90 limit is derived for minimum reinforcement. No other comments are

\(^1\)Note that in NS 3473, \(f_{ck}\) is used to denote cube strength. For cylinder strength, \(f_{ckc}\) is used.
given as to how it was derived. Introduction of negative $\mu_o$ in the lower limit expression is allowed only if the column is designed for a moment from a minimum load eccentricity ($e/h=0.05$) and the restraints at the column ends are able to resist this moment so that a double curvature deflection can develop. The same requirement applies in EC2 of 1991. Similar restrictions apply to the MC78- and DIN 1045– expressions ($e/h=0.2$ and 0.1, respectively).

The prestandard version of EC2 of 1991 has been in the process of being revised for some time (CEN (1999), first revision draft). Several proposals have been presented and been commented on (Hellesland 2002). The slenderness limit in the “final” draft, Table 1, Case 9, is reasonably similar to that of NS 3473 except that it is formulated in terms of the geometrical slenderness parameter. A major difference is the inclusion of creep in the proposed EC2 limit.

4 Moment gradient and boundary conditions—Elastic column

Elastic theory can conveniently be used to clarify some aspects of importance in the nonlinear study. Below, the moment gradient effect is considered. Since the maximum total moment and the maximum first order moment will occur at different sections (Fig. 1), the moment gradient tends to delay the development of a maximum moment in excess of the largest first order end moment. Therefore, slenderness limits will increase with increasing gradient.

In an elastic member, the moment magnifier, $\delta_M$, defined by the ratio between the maximum total moment, $M_{t_{\text{max}}}$, and the numerically largest first order end moment, $M_{o_2}$, gives a measure of the second order effects. In the general case of a linear elastic, initially straight member with constant cross-sectional bending stiffness $EI$ along the length, small deformations (rotations) and negligible shear deformations, it can be expressed by

$$\delta_M = \frac{M_{t_{\text{max}}}}{M_{o_2}} = \left| \frac{\sqrt{1 + \mu_i^2 - 2\mu_i \cos(L(P/EI)^{1/2})}}{\sin(L(P/EI)^{1/2})} \right| \cdot \frac{M_{o_2}}{M_{o_2}}$$

where $N$ is the axial load and $\mu_i = M_{o_1}/M_{o_2}$ is the ratio between the total moments (slenderness effects included) at end 1 and 2. The ratio is to be taken as positive when the total moments at each end act in opposite directions (clockwise and counter-clockwise, respectively), and negative otherwise. The maximum moment above is derived from the differential equation for the elastic curve (e.g., Galambos (1968)).
In restrained members (Fig. 1(b)), also the total end moments are a function of \( L(N/EI)^{1/2} \). For given restraints, they can be computed using a stiffness relationship (e.g. slope-deflection equations) that includes standard stability functions (e.g., Galambos 1968). In unrestrained members (Fig. 1(a)), moment magnifiers can be obtained directly from Eq. 2 since total end moments are equal to the applied first order moments in such members (e.g., \( M_{12} = M_{02} \)).

Using the outlined approach, slenderness values that give a moment magnification of 5% \( (\delta M = 1.05) \) are shown in Fig. 2(a) in terms of \( (L_c/\pi)(N/EI)^{1/2} \). This parameter is identical to the square root of the stability index \( (=N/N_{cr} \), where \( N_{cr} \) is the critical (buckling) load). The abscissa is the **first order end moment ratio** \( \mu_o = M_{01}/M_{02} \). Curve a is for a member with unrestrained (hinged) ends, and thus with an effective length factor of \( \beta = 1 \). (Curve a is also valid for an arbitrary member or member segment if the the ratio \( \mu_o \) is replaced by that of the total moments at member or segment ends, \( \mu_t \).) Curves b and c are for members that are unrestrained at one end and restrained, relative to the member, by a very stiff beam at the other end. For these “hinged/clamped” cases, \( \beta = 0.7 \). The two curves coincide at \( \mu_o = 1 \) and -1. The lower curve b results when the largest first order end moment \( (M_{02}) \) acts at the end with the smallest restraint (the hinge), and the upper curve c when it acts at the end with the very stiff restraint. At a stiff restraint, there will be a stronger end moment relief (moment transfer from the member to the restraint) with increasing \( L(N/EI)^{1/2} \) than at a more flexible restraint. This explains why the most unfavourable slenderness effects is obtained in case b with no moment relief at the hinged end 2.

The curves (a, b and c) are reasonably close near \( \mu_o = M_{01}/M_{02} = 1 \). Thus, for reasonably uniform first order moment distribution, use of the effective length in the slenderness parameter has accounted reasonably well for differences in boundary conditions. (Ordinate values at \( \mu_o = 1 \) are 0.197 for curve a, and 0.185 for b and c. In comparison, by assuming a sine curve for the deflected shape, a value of 0.218 is obtained). For increasingly non-uniform moment distribution, the effective length alone can be seen not to be sufficient in accounting for end restraints. In choosing approximations, it is therefore necessary to make a sufficiently conservative choice.

Curve b can for all practical purposes be considered a reasonable lower bound on results for any end restraint combination. (In a small region near \( \mu_o = 1, \) a member with stiff restraints at both ends \( (\beta \approx 0.5) \) would produce lower values than given by curve b, but the difference is small). Curve c up to the peak at about \( \mu_o = -0.75, \) and then horizontally to the peak of curve a at \( \mu_o = -1, \) represents an upper bound.

The load at the peaks, at ordinate values of unity, is the critical (buckling) load
Figure 2: Slenderness limits vs. moment gradient – a) Exact elastic results with $\delta_M=1.05$ for different end restraints, and approximate curves (ordinate values at $\mu_o=1.0$: curve $a=0.197$, curve $b$ and $c=0.185$, approximate curves=0.218); – b) Comparison of exact elastic results with $\delta_M=1.05$ and $\delta_M=1.10$.

$\left( N = N_{cr} = \pi^2 EI/(L_e)^2 \right)$. The peaks are associated with a sudden change in curvature distribution in the sense that central part of the column “snaps” out. The restrained member $c$ unwraps from double curvature towards the buckling mode with $\beta = 0.7$. Similarly, the unrestrained (hinged) member $a$ unwraps (unwinds) from double curvature towards the single curvature buckling mode with $\beta = 1.0$. Considering that a column with high moment gradients may actually have loads close to the critical load, and have considerable second order effects even though the maximum total moment may not be greater than the larger first order end moment, there are strong reasons for caution at $\mu_o$-values at which unwrapping may take place. As seen in Fig. 2(b), the relative increase with increasing moment gradient decreases significantly with increasing $\delta_M$. Also this calls for some caution when selecting lower limit approximations.

The rather significant difference between curves $b$ and $c$ is due to the strong difference between the rotational end restraints at the two ends. Infinite and zero end restraint stiffnesses represents a theoretical case that in practice is not easily attainable. For more practical differences the curves will be closer together. Curve $a$, for hinged ends, is reasonably representative also for cases with other, but reasonably similar end restraints.
The straight line approximation labelled \( d \) in Fig. 2, located between curve \( a \) and curve \( c \) (the lower bound), should be reasonably representative for practical cases. It will account for a realistic difference in end restraint, and will avoid the problem of unwrapping. The same is the case for the more conservative, lower bound approximation labelled \( \epsilon \). Both of these will be considered further relative to nonlinear results.

A comparable approximation can be obtained using the approximate moment magnifier expression (e.g., ACI 2002),

\[
\delta = \frac{C_m}{1 - N/N_{cr}} \quad ; \quad C_m = 0.6 + 0.4\mu_0 \geq 0.4
\]

(3)

from which

\[
\frac{L_e}{\pi} \sqrt{\frac{N}{EI_d}} = \sqrt{1 - \frac{C_m}{\delta}}
\]

(4)
can be obtained. For \( \delta = 1.05 \), the right hand side of Eq. 4 becomes \( 0.218\sqrt{8.6 - 7.0\mu_0} \) below the cut off (at \( \mu_0 \leq -0.5 \)). This curve is labelled \( f \) in the figure. It will be unconservative for a wide range of end moment ratios for cases in which the maximum first order moment is applied at the end with the smaller restraint stiffness.

## 5 Criterion

For reinforced concrete members, an objective lower slenderness criterion can most appropriately be related to an acceptable percentage reduction in load carrying capacity, rather than to some moment increase beyond maximum first order moments.

A general lower slenderness limit is defined as that at which second order load effects (slenderness effects) do not reduce a member’s load carrying capacity to values less than a specified percentage (95 or 90%) of the cross-sectional capacity (“non-sler (short) member strength”). Non-negligible detrimental effects of sustained (long term) loading must, if relevant, be included in capacity reduction assessments.

This definition opens up for several possible interpretations (sub-criteria). These include

a) 5% (or 10%) reduction \((\Delta M)\) in moment capacity \((M_d)\) for a constant axial load ;

b) 5% (or 10%) reduction \((\Delta N)\) in axial load capacity \((N_d)\) for an applied constant
Figure 3: Slenderness limits according to various definitions (based on the moment magnifier) for a medium reinforced member (5% red., ω = 0.6)

axial load eccentricity (implies also a 5% (or 10%) reduction in moment capacity);
c) 5% (or 10%) reduction (ΔN) in axial load capacity (N_a) for an applied constant moment.

These criteria lead to significantly different results at high axial loads for members, such as unbraced members, for which the total moment results at the same section as the maximum first moment (Hellesland 2002). For end loaded columns with moment gradients, as considered in this paper, the difference between the criteria are significantly reduced with increasing first order moment gradient at high axial loads. This can be seen in Fig. 3. The load applications that corresponds to the three sub-criteria listed above are defined by the correspondingly labelled inserts (a, b and c) in the figure. The results were obtained using the approximate moment magnifier approach (Eq. 4) in conjunction with a N – M diagram for a typical medium reinforced cross section (ω = 0.6, h'/h = 0.8, ε_y = 0.0024). For details, see Hellesland (2002). The lower terminations of the Case c curves correspond to an applied moment (M_{02}) approximately equal to the pure design moment capacity. Case c, as defined, is not relevant for larger moments. Criterion a gives more conservative limits than criterion b except at lower relative axial load levels (less than approximately the balanced load at 0.43), where the difference in any case is rather small. At higher axial loads, the difference can become quite substantial. In particular in the rather academic case of a member with uniform first order moment (μ_0 = 1 and C_m=1.0). However, in the more practical single curvature case with μ_0 = 0.8 (C_m=0.92), the difference is significantly reduced. As the member bends into a double curvature states, the curves approach that based on criterion a.
This points to the use of criterion $a$ as a single, conservative criterion. However, creep effects due to sustained loading are not considered explicitly above and need further consideration. Possible creep effects are strongest in highly compressed members with smaller load eccentricities. For such members, for which the axial load capacity may be the most relevant strength parameter, criteria such as $b$ or $c$ may be considered the most appropriate. The applicability of the various criteria is discussed in more detail in Hellesland (2002).

In the present nonlinear analyses, criterion $a$ will be used in combination with short term material properties. Effect of creep on lower limits will be checked in a separate section.

6 Normalised slenderness

The sensitivity to second order bending effects in an elastic compression member with negligible shear effects is a function of boundary conditions (end restraints) and the combined parameter $L(N/EL)^{1/2}$ rather than of the individual parameters entering the expression ($L$, $N$ and $EI$). With $L$ replaced by the effective length ($L_e$) to account in an approximate manner for different boundary conditions, such a combined parameter represent also a rational choice in slenderness limit formulations of reinforced concrete members. By assuming a representative sectional bending stiffness, a suitable parameter for reinforced concrete can be defined by

$$\lambda_n = \lambda \sqrt{\frac{\nu}{1 + k_i \omega_i}}$$  \hspace{1cm} (5)

in which

$$\lambda = \frac{L_e}{i_g} \enspace ; \enspace \nu = \frac{N_f}{f_{cd} A_g} \enspace ; \enspace \omega_i = \frac{f_{yd} A_{st}}{f_{cd} A_g}$$  \hspace{1cm} (6)

is the non-dimensional factored design axial load and the total mechanical reinforcement ratio, and $k_i$ is a reinforcement contribution factor (constant for given cross-section and reinforcement) that may be approximated by

$$k_i = \frac{4.3}{1000 \varepsilon_y} \left( \frac{i_s}{i_g} \right)^2$$  \hspace{1cm} (7)

The product $k_i \omega_i$ can alternatively be given directly by

$$k_i \omega_i = \frac{4.3 E_s A_{st}}{1000 f_{cm} A_g} \left( \frac{i_s}{i_g} \right)^2$$  \hspace{1cm} (8)

Unlike $k_i$, this product is not dependent on steel quality. It is a question of preference which form to use.
Figure 4: Typical radii of gyration of gross cross-sections and of symmetrical reinforcing bar arrangements.

The slenderness parameter, \( \lambda_n \), is expressed completely in terms of non-dimensional quantities. It is labelled normalised slenderness to distinguish it from other slenderness parameters. The basis for the parameter is discussed in more detail in Hellesland (2002). It is similar to the \( \lambda_N \) parameter in NS 3473, and there labelled “axial load dependent slenderness”.

Above, \( f_{yd} \) is the design steel yield strength, \( \varepsilon_{yd} = f_{yd} / E_{ad} \) the design yield strain, \( f_{cd} \) the design concrete compressive strength, defined as the peak stress in the design stress-strain diagram for concrete. In many codes (CEB MC90, etc.) the safety philosophy do not call for reducing the steel modules. Thus, \( E_{sd} \) above is replaced by \( E_s \) in those codes.

Further, \( i_g = (I_g / A_g)^{1/2} \) and \( i_s = (I_s / A_{st})^{1/2} \) are the radii of gyration of the gross section \( (A_g) \) and the total reinforcement \( (A_{st}) \), respectively, both about the centroidal axis, and \( I_g \) and \( I_s \) are the corresponding second area moments. Values of \( i_g, i_s \) and \( i_s / i_g \) for typical cross-sections and reinforcement arrangements are given in Fig. 4. In practice, the last case is considered rather unlikely.

Some simplifications of Eq. 7 that involve some conservativeness can be obtained for instance by introducing either \( i_s / i_g = 1.48 h'/h \) with \( h'/h = 0.8 \), or \( \varepsilon_{yd} = 0.0024 \), or both of these, respectively. Doing this yields the three following alternatives:

\[
k_t = \frac{6}{1000 \varepsilon_{yd}} \quad ; \quad k_t = 1.8 \left( \frac{i_s}{i_g} \right)^2 \quad \text{or} \quad k_t = 2.5
\]
7 Nonlinear analyses

Method

Slenderness values are computed that correspond to specified reductions in first order moment capacities of reinforced concrete compression members subjected to constant axial loads. The members considered are symmetrically reinforced and subjected to external loads (axial load and moments) applied at member ends. As is the normal practice for reinforced concrete members, the members were assumed to be initially straight prior to loading. Further, the members had uniform section details and material properties along the length.

The members are either a) statically determinate with rotationally unrestrained (hinged) ends (Fig. 1(a)), or b) statically indeterminate with one end unrestrained and the other rotationally restrained by an extremely stiff beam. The maximum first order end moment is applied at the unrestrained end. As seen in the elastic study, the latter case (b) will give lower bound results (comparable to curve b in Fig. 2).

A computer program, based on the finite difference approach, was tailor made for the problem and included both nonlinear material and nonlinear geometric effects (Aasrum 1992). An overview of the major steps in the iterative analysis is given in the Appendix. A member may become unstable either due to primary material failure (exhaustion of the cross-section capacity) or primary instability failure prior to material failure. The basis for the analysis was the relationship between the curvature ($\kappa$) and the nominal moment resistance at that curvature ($M_{d,\kappa}$). Such $M_{d,\kappa}-\kappa$ relationships for a given section, reinforcement and nominal axial load were computed from the equilibrium equations for the axial load and the moment (about the centroidal axis),

$$N_f = N_d = \int_{A_c} \sigma_c \, dA_c + \int_{A_{st}} \sigma_s \, dA_s$$

$$M_{d,\kappa} = \int_{A_c} \sigma_c z \, dA_c + \int_{A_{st}} \sigma_s z \, dA_s$$

The standard assumptions of plane sections remaining plane (Bernoulli-Navier's hypothesis), full bond between steel and concrete, neglect of the concrete tensile strength and neglect of the favourable tension stiffening (giving increased stiffness between cracks) were incorporated. The commonly adopted parabola-rectangle diagram, Fig. 5 (left), was chosen for concrete in compression and a standard elasto-plastic stress-strain diagram, Fig. 5 (right), for reinforcing steel in tension and compression. The peak concrete compression stress, $f_{cd}$, is the nominal structural ("in situ") compressive strength that is also used in the normalised
slenderness definition of Eq. 5. The effect of unloading that may take place in some fibres is small and was not accounted for. Non-mechanical concrete strains (creep, shrinkage) were not included. Consequently, results obtained are so-called “short term” results.

The material factors \( \gamma_m (\gamma_c, \gamma_s) \) were assumed to be the same at all sections. This is a common, but conservative approach. At sections outside the the most strained (critical) region, where failure is initiated, use of a somewhat higher factors would have been justified. Compared to such a double-factor approach, use of the same, low factors for all sections will give larger displacement and somewhat too small slenderness values. An approximate correction for this difference will be discussed later.

**Parameters**

Slenderness values at which the first order moment capacity (i.e., the member’s capacity for carrying first order moments caused by external loads) is reduced by 5 and 10%, below the cross-sectional moment capacity, have been computed for a wide range of parameters. Unless otherwise mentioned specifically, where relevant, the results presented are for members with a section labelled \( RC \), defined by rectangular cross-section and reinforcement in two opposite layers perpendicular to the plane of bending or equivalent (e.g., corner reinforcement), \( h'/h = 0.8 \), \( \varepsilon_{yd} = 0.0025 \), \( \varepsilon_u = 0.010 \), \( \varepsilon_{co} = 0.002 \), \( \varepsilon_{cu} = 0.0035 \) and \( k_t \) defined by Eq. 7. For this “standard” case, \( k_t = 3.3 \). In \( \lambda_m \) presentations, the elastic effective (buckling) length factors are 1.0 (case a) and 0.7 (case b) for the two kinds of member restraints considered.

Results are included for both a small amount, \( \omega_t = 0.2 \), and a large amount, \( \omega_t = 1.0 \), of reinforcement. These mechanical reinforcement ratios covers a wide range from approximately minimum reinforcement to an, in practice, upper limit.

**Moment gradient, axial force and reinforcement**

The three major factors affecting the slenderness limits are moment gradient, ax-
Figure 6: Slenderness limits vs. moment gradient – Effect of different axial load levels and reinforcement ratios (at 5% capacity reduction) and comparison with elastic results (at $\delta_M=1.05$)

Axial load and reinforcement. The relative effect of moment gradients in reinforced concrete members are stronger than that found in the elastic analyse, as seen in Fig. 6. This is an expected result. The difference can be quite substantial, and the greatest difference is obtained for low axial load and low reinforcement ratio, i.e. for cases with moment–curvature relationships that deviate most from the linear elastic one. The effect on geometrical slenderness ($\lambda = L_e/i_g$) limit results of axial load and reinforcement are seen more clearly in Fig. 7(a) for a member with a small moment gradient of $\mu_o=0.8$. The axial force is the more important of these two factors, but it can be seen that reinforcement effects also are quite substantial. The axial loads of 80% of the “squash load” $(0.8(1+\omega_i))$ are shown by the vertical lines in the figure.

Normalised slenderness versus axial load

Fig. 7(a) is replotted in Fig. 7(b), top, in terms of the normalised slenderness. In Fig. 7(b), bottom, similar results are shown for a uniform moment case, $\mu_o=1.0$. The marked change, as the load level increases from a “tension controlled” to a “compression controlled” case at the balanced point (at about $\nu=0.4$), will be less pointed with distributed reinforcement and will be reduced with increasing moment gradient.
Figure 7: a) Geometric slenderness and b) Normalised slenderness vs. axial load.

To “normalise” results (into horizontal lines) completely, a considerably more complex \( k_t \)-factor would be required. The curves for different reinforcements amounts are seen to be reasonably close together. The chosen \( k_t \) is a compromise value for load levels above and below the balanced load. For further discussion of the \( k_t \)-definition (Eq. 7), including its capability of reflecting the influence of various section details (such as \( h' / h \), reinforcement arrangement, \( \varepsilon_{yd} \), concrete quality and section shapes), see Hellesland (2002).

Had tension stiffening effects been included in the computations, the slenderness results would be increased somewhat in particular at the lower load levels, and helped to linearise results. As seen, by comparing the two cases with \( \mu_o=1.0 \) and \( \mu_o=0.8 \) in Fig. 7(b), a small increase in moment gradient (from \( \mu_o=1.0 \) to \( 0.8 \)) is sufficient to cause a rather significant increase (lift) of results. A uniform moment distribution (\( \mu_o=1.0 \)) represents a rather academic case. A practical “upper uniformity” is probably represented by a ratio of about \( \mu_o=0.8 \).

Normalised slenderness versus moment gradient

In Figs. 8 to 9, normalised slendernesses are plotted versus the first order end moment ratios (“moment gradients”). In the hinged member case (a), these are given directly by the applied moments. In the rather extreme, statically indeterminate case (b), the maximum moment is applied at the hinged end and a rotation is imposed at the other end (with the very stiff restraint) to give a desired moment at that end. In this case, the ratio that corresponds to a slenderness
Figure 8: Normalised slenderness vs. moment gradient for lightly reinforced member with low ($\nu=0.2$) and intermediate ($\nu=0.6$) load level
\[ \nu = 1.2 \quad \omega_t = 1.0 \quad h/h = 0.8 \]

Rectangular section with distributed reinforcement (RD, \( k_t = 2.2 \))

Figure 9: Normalised slenderness vs. moment gradient for hinged, strongly reinforced member with high (\( \nu = 1.2 \)) and very high (\( \nu = 1.5 \)) load level. The value will be dependent on the nonlinear material properties. It is in terms of such ratios that results are plotted.

This is emphasised, since in design, moment ratios are computed using elastic stiffness assumptions. For instance, whereas half of a moment applied at the hinged end would be distributed to the other, clamped end in the elastic case, more than half would be so in the nonlinear case (due to the higher stiffness in the column portion with the smaller moments). Therefore, in this particular case, use of the elastic ratio would be conservative. The correct slenderness would be obtained by entering the figures with a smaller (more negative) \( \mu_o \) than the elastic \( \mu_o = -0.5 \). Although this complicates any direct use of the presented results for case \( b \), they still serve as a good indication of a lower bound on results.

Normalised slenderness results for circular sections and rectangular sections with distributed reinforcement (RD) are very similar to those above. Results for latter case (with \( k_t = 2.2 \)) are shown in Fig. 9. Hinged member results, which are not included in the figure, will, to the same extent seen before, be located above those shown.

The effect of ultimate concrete strains are shown in Fig. 8 (top, right). Use of the lower \( \varepsilon_{cu} = 0.003 \) will allow a somewhat greater slenderness limit than the higher \( \varepsilon_{cu} = 0.0035 \). However, the difference is at most about 5% at low reinforcement levels and less at higher levels.
At $\mu_o=1.0$, slenderness values at 10% capacity reduction is roughly about 40% greater than those at 5% capacity reduction. This difference decrease with increasing moment gradient.

**Comparisons with selected approximate limits**

The present ACI limit (Table 1, Case 10) is included in Fig. 7(a) and Figs. 8 – 9. Generally it is seen to be very conservative at lower axial load levels. At high load levels, it may become unconservative, as seen for the heavily reinforced case in Fig. 9. For a lightly reinforced case, the degree of unconservativeness would be considerably greater than shown. More so for single than for double curvature cases because of a rather modest moment gradient effect in the ACI limit.

Also shown in Fig. 7(b) and Figs. 8 – 9 are approximate limits given by

$$\lambda_n = 24 - 14\mu_o$$  \hspace{1cm} (12)

It is discussed further in the next sections.

The NS 3473 limit $\lambda_N = 18 - 8\mu_o$ (Table, Case 6) differs from the one above by having $k_t$ values that, in the cases considered, are about 21% greater and, more importantly, by having a considerably more modest moment gradient effect. The latter is due to two reasons (Hellesland 1990). First, documentation was not as extensive at the time the Norwegian limit was proposed and some prudence was called for. For instance, only elastic data was available on which to base the gradient effect. Second, it was chosen conservatively in order to allow indirectly for some unintentional eccentricities through a reduced gradient effect. The moment ratio that may be based on nominal values. Thus, a direct comparison is of little value. NS 3473 gives slightly greater values than Eq. 12 near $\mu_o = 1$, but increasingly smaller values than Eq. 12 with increasing gradient.

The proposed EC2 limit (Table 1, Case 11) is very similar to that in NS 3473 and in the present study. This can be best seen by rewriting it as

$$\lambda \sqrt{\frac{\nu}{1 + 2\omega_t}} = \frac{14}{1 + 0.2\phi_{ef}} (2.43 - 1.43\mu_o)$$  \hspace{1cm} (13)

EC2 has a constant $k_t=2$, rather than the section dependent values in NS 3473 (4 and $8/3=2.67$) and in the present study. Further, it has a stronger moment gradient effect than NS 3473. This makes sense since effects of unintentional eccentricities is incorporated into NS 3473 by reducing the moment gradient effect, while such eccentricities are accounted for explicitly by including them in the end moments themselves in EC2. Finally, EC2 reflects creep explicitly.

The gradient effect in the proposed EC2 limit is seen to be very similar to that of Eq. 12, which can further be seen to be a reasonably limit for the 5% reduction.
results. However, in conjunction with a 10% capacity reduction, which EC2 relates its limit to, the gradient effect is, seen in isolation, too strong. As seen also in Fig. 2(b), for elastic results, the relative increase with increasing moment gradient decreases significantly with increasing $\delta_M$. The gradient effect can be seen in the left hand figures of Figs. 8 – 9 where predictions by Eq. 13, without creep, is included. They compare reasonably well with the unrestrained member results (curves labelled a), but are quite unconservative with respect to the results for members with different end restraints. By including an effective creep factor of $\phi_{ef}=2$, the EC2 predictions will be lowered to 0.7 times those shown. This will bring the predictions below the nonlinear analysis results even for 5% capacity reduction, and very close to that given by Eq. 12 for a RD section (Fig. 9) and more conservative than Eq. 12 for a RC section.

8 Sustained loading effects

![Figure 10: Effect of sustained load on slenderness limits for 5 and 10% reduction in axial load capacity (deduced from results given in Manuel (1966))]}

A study on sustained loading of framed columns reported by Manuel and MacGregor (1967) can be used to provide a check on the adequacy of approximate limits. Here, Eq. 12 is considered with respect to creep effects relative to criterion c (Fig. 3). This study is of special interest as it deals with frames that include columns subjected to very high axial forces, at, or even above, 0.8 times the pure axial load capacity. The columns were either symmetrically restrained and bent
in single, uniform first order curvature ($\mu_0 = 1$), or had unequal restraints and bent in double curvature with an elastically obtained $\mu_0 = -0.5$. The frame considered in the latter case is illustrated in the lower insert in Fig. 10.

The full beam load ("moment loading") was applied first and then sustained throughout the load history. This is unfavourable. It would have been more appropriate to split it up into sustained and live load portions. Axial loads, applied at column ends to represent stories above, were either incremented continuously to obtain short-time failure loads, or incremented to specified sustained load ($N_s$) that were maintained for 25 years before a continued short-time incrementation to failure was carried out. Creep factors for the sustained duration were estimated to $\phi_{25yr}=3.5$ to 3.8 for stress levels of $\sigma_c/f_{cm} = 0.4$ to 0.5 (from the pertinent figure in the paper). These are very high. The columns (12 in. (305 mm) square) had totally 2% symmetrical reinforcement in two faces and $h/h' = 0.8$. The beams (12 in. (305 mm) wide and 17.5 in. (445 mm) deep) had tension reinforcement of 3% of cross-sectional area. Other data: $f_{cm} = 0.85f_{ck}$, $f_{ck} = 4ksi$, $f_y = 50ksi$, $\varepsilon_y = 0.0017$, $\varepsilon_{co} = 0.0018$, $\varepsilon_{cu} = 0.0038$, $k_t=4.94$ (Eq. 7), $\omega_t=0.294$. Partial material coefficients were equal to 1.0 in the study.

In the context of slenderness provisions, it is the reduction below the nominal cross-section strength ($N_d$) that is of interest, as illustrated in the upper insert in Fig. 10. For this reason, Manuel's results were reexamined in terms of nominal sectional strengths rather than in terms of computed short time failure loads from the nonlinear analysis given by Manuel. First order end moments ($M_{02}$) due to the beam loading were computed based on the gross concrete section stiffnesses for the columns and half the gross section stiffness for the beams. Beam and column lengths were taken equal to the system lengths. Nominal axial load strengths $N_d$ corresponding to the numerically largest first order end moments ($M_{02}$) were read from $N - M$ diagrams given by Manuel (1966). From diagrams for failure loads versus column slenderness ratios $L/h$, slenderness ratios corresponding to failure loads of $0.95N_d$ and $0.90N_d$ were estimated for different sustained load levels.

Results, such obtained for columns in double curvature, are shown in Fig. 10. For sustained load levels of $N_s/N_d = 0.5$ to 0.6, axial load strengths of $0.95N_d$ could be developed at about $\lambda_n=32$ to 30 (or $\lambda=44$ to 47), respectively. If the full gross section stiffness had been used for the beams, instead of half, estimated $\lambda_n$-limits would be reduced by about 1.0. For the columns in uniform curvature, slenderness limits were found at $0.95N_d$ at about $\lambda_n = 40$ ($L/h = 22$, elastic $\beta = 0.80$, $\lambda = 61$) for short term loading, and of $\lambda_n = 23$ ($L/h = 12$, $\beta = 0.87$, $\lambda = 36$) for a sustained load level of $N_s/N_d=0.6$. The nominal load eccentricity for the latter column was about $\epsilon_{02}/h=0.09$.

It has been seen (Fig. 3) that criterion $c$ must be expected to give considerably
higher slenderness values relative to criterion \( a \) in the uniform curvature case than in the double curvature case. This is confirmed by the present results. The approximate prediction of \( \lambda_n = 10 \) (Eq. 12) is very conservative for the uniform curvature case, despite a significant reduction in slenderness limits due to sustained loading (of about 40%). For the double curvature case, the approximate prediction of \( \lambda_n = 31 \) is close to the numerical result for \( P_s/P_n = 0.55 \). Considering the very high creep factor, and the unfavourable beam loading, one might have expected smaller numerical slenderness values. That this is not so is due to a favourable moment transfer that took place from column ends to the restraining beams. In isolation, this tends to strengthen a column. For a lower, more practical creep factor, the numerical results would increase and the approximation become more conservative.

Other relevant results, for both unrestrained and restrained members, are presented in Mari and Hellesland (2002). They were obtained by numerical nonlinear analysis (FEM) that considered both material and geometric nonlinearities, including cracking and tension stiffening. The columns were first subjected to a period (50 years) of sustained loading at a specified fraction \( (\psi) \) of the axial load and moment capacities. Subsequently they were loaded to failure in a short time load application conforming to one of the 3 load applications defined in conjunction with the sub-criteria \((a, b, c)\) discussed previously. Selected 5% capacity reductions results for \( \psi = 0.6 \), a creep factor of \( \phi_{50y} = 2 \), small and very small nominal eccentricities (mostly \( e_{ax}/h = 0.1 \)) and low and high reinforcement are compared in Fig. 11 to the approximate limit given by Eq. 12. For unrestrained columns \( (\beta = 1) \), the approximate limit is generally conservative at all considered end moment ratios \((\mu_0 = 1.0, 0, -0.9)\). For columns hinged at one end and clamped at the other \( (\beta = 0.7) \), and with an elastic \( \mu_0 = -0.5 \), the limit is normally less conservative but still acceptable. For the criterion \( b \) and \( c \) results, there is still room for additional creep (greater \( \phi \)) relative to the approximate limit.

This review of sustained load effects at small eccentricities documents reasonably well that the approximate Eq. 12 allows for practical creep effects at sustained loads as high as 50-60% of nominal sectional capacities. It should also be noted that, in accordance with normal practice, no concrete strength increase due to aging was included in the studies above. This is conservative. It is also conservative, as mentioned previously, that the same (low) material factors were used also at sections outside the failure region. These “reserves” would allow for increased creep effects.
9 Proposal

Introductory comments

A limit giving predictions below or about equal to the nonlinear results obtained for the “clamped–hinged” case \( b \) in the figures, is given by \( \lambda_n = 22 - 12\mu_0 \) (but not shown in the figures). In Fig. 2(a) for elastic results, the corresponding limit is labelled \( c \). Completely “clamped” and “hinged” restraints are almost impossible to obtain in practice, and is therefore somewhat academic. A limit providing an approximate lower bound on cases with more practical end restraints, is that given by Eq. 12. It corresponds to the limit labelled \( d \) in the elastic case (Fig. 2(a)).

At lower axial load levels, Eq. 12 may seem to be somewhat unconservative relative to the 5% capacity reduction results for members with moment distributions close to the academic uniform one (e.g., for \( \nu = 0.2 \) in Fig. 8, near \( \mu_0 = 1 \)). However, already at the only slightly non-uniform distribution corresponding to \( \mu_0 = 0.8 \), the correspondence is good. Also, had tension stiffening effects been included in the computations, the nonlinear slenderness results would be increased somewhat, in particular at the lower load levels in question.

Eq. 12 has been found to reflect important parameters well. It is given indepen-
dent of creep, but appears sufficiently conservative to allow for normal creep in unrestrained and restrained members.

Standards and codes have traditionally allowed the end moment ratio to be computed with first order nominal moments, without including unintentional eccentricities to allow for imperfections, uncertainties in calculated nominal first order end moments etc. Are such uncertainties, that have the strongest consequence for members with small eccentricities (high axial load and small moments), adequately covered by material factors and generally conservative design assumptions? Or is additional caution called for when dealing with such members, in particular considering that slenderness limits may increase by a factor of about 4 when the end moment ratio changes from 1 to -1? No efforts are made to conclude on these questions here. However, possible adoptions to allow for additional uncertainties, if that should be considered necessary, is discussed.

**Lower limit. Alternative 1**

The normalised slenderness $\lambda_n$ is defined by Eq. 5 with $k_I$ given by Eq. 7, or alternatively, with $k_1w_1$ defined by Eq. 8. Simplifications of $k_I$ that may be considered acceptable are defined by Eq. 9.

For compression members braced against sideways, it is permitted to ignore slenderness effects if

$$\lambda_n \leq 24 - 14\mu$$

where $\mu$ is taken as the conventional first order ratio $\mu_0 = M_1/M_2$ (as previously defined), or alternatively, as a first order, "imperfection (or uncertainty) adjusted" end moment ratio $\mu_1$.

Several approaches may be suitable in allowing for additional uncertainties, if that should be considered necessary.

1) In conjunction with $\mu = \mu_0$, an added restraint to Eq. 14 might be that the end moment ratio is to be taken equal to unity when the maximum first order moment is less than some specified lower value. This is required in NS 3473. In ACI 318-02, it is given as an option in the computation of the moment gradient factor $C_m$ at small moments, but is not mentioned in conjunction with the lower slenderness limit. It has the disadvantage that it introduces a discontinuity, and a strong one in columns with high moment gradients, as $M_{02}$ "passes" the lower value, and it does not cover uncertainties at eccentricities in excess of the minimum one.

2) In conjunction with $\mu = \mu_1$, one approach is to allow explicitly for uncertainties of various kinds by adding an "unintentional" uniform moment distribution.
(\(N_f \Delta e\)) to the nominal distribution. Thus,

\[
\mu_1 = \frac{M_{01} + N_f \Delta e}{M_{02} + N_f \Delta e} = \frac{e_{01} + \Delta e}{e_{02} + \Delta e}
\]

(15)

where the larger moment \(M_{02}\) (eccentricity \(e_{02}\)) is always to be taken positive. \(M_{01}\) (\(e_{01}\)) is to be taken positive when it bends the member into single curvature and negative when it bends the member into double curvature. This approach will not alter the line of axial thrust and would seem like a rational approach. It is illustrated in Fig. 12 for different \(e_{02}\)-values when \(\Delta e = 0.05h\).

This \(\mu_1\)-definition is also a suitable instrument in allowing, if desired, for additional creep effects in members with moment gradients, beyond those already allowed for. The effect on the slenderness limit of a chosen “creep addition” (to \(\Delta e\)) is like that of creep. It is strongest at smaller eccentricities, and tapers off with increasing eccentricity. Also, the effect decreases with increasing uniformity of the moment distribution (when creep effects can be significant and still not be of concern relative to criterion \(\beta\) or \(c\)).

**Lower limit. Alternative 2**

Another approach is to reflect uncertainties implicitly through adoption of a conservative moment gradient relationship. It may be derived on the basis of predictions by Eq. 14 and 15. Based on the case illustrated in Fig. 12, for \(\Delta e = 0.05h\), a sufficiently conservative conclusion might be to allow slenderness effects to be ignored if

\[
\lambda_n \leq 20 - 10\mu_o
\]

(16)

Similarly, for the smaller \(\Delta e = 0.02h\), it can be concluded that slenderness effects may be ignored if

\[
\lambda_n \leq 22 - 12\mu_o
\]

(17)

The lowest \(e_{02}\) in the comparison in the figure is 0.1\(h\). This value corresponds approximately to an upper axial load limit of 0.8 times the pure axial load capacity. The corresponding moment capacity represents close to the smallest moment capacity a member will be provided with.

An advantage of this approach is that it is simple. The disadvantage is that it will have the same conservative impact also at larger eccentricities (or axial loads).
10 Summary and conclusions

Factors affecting lower slenderness limits have been discussed and results presented of extensive nonlinear analyses of braced compression members without transverse loads between ends (i.e., end loaded members).

A new slenderness limit is proposed in terms of a rational and reliable slenderness measure, labelled normalised slenderness, that includes effects of axial force and reinforcement. The limit is considered to be applicable for all concrete qualities, including LWA concrete. It will allow member slenderness effects to be ignored in many more cases than in simpler limits that may be independent of axial force or reinforcement, or both. It is felt that reduced design efforts due to this, and the added reliability of the formulation, more than compensates for the added complexity of limits of the kind proposed here. The complexity is still rather reasonable. The inclusion of the additional parameters also has the advantage of adding focus on and awareness of parameters that is important in design of compression members. An added advantage is that the formulation may be used actively at the design start, when sectional dimensions and reinforcement assumptions are first assumed, to get a priori information on what choices will lead to negligible or non-negligible second order effects.

At a preliminary design stage, conservative assumptions, such as minimum reinforcement and axial load due to an unfavourable load case, may be introduced.
for the new parameters. If necessary, checking with more refined values can be performed at a more advanced stage in the design process. Conservative assumptions may also be introduced in the present proposal for the purpose of obtaining simplified limit formulations if that should be considered desirable.

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NOTATION

\( A_c \) = area of concrete section; can normally be taken as \( A_g \)
\( A_g, A_{st} \) = area of gross section and of total reinforcing steel
\( I_g, I_s \) = second moment of area about centroidal axis of gross section and of total reinforcing steel
\( L \) = length of compression member
\( M_u, M_f \) = nominal moment capacity and factored moment ("ultimate")
\( M_{0f} \) = factored moment ("ultimate")
\( M_{01}, M_{02} \) = smaller and larger factored first order end moment
\( N_u, N_f \) = nominal axial load capacity and factored axial load ("ultimate"), positive as compressive
\( f_{cd} = f_{cn}/\gamma_c \) = design compressive strength of concrete
\( f_{ck}, f_{cn} \) = cylinder and nominal structural compressive strength of concrete
\( f_y \) = yield strength of reinforcing steel
\( f_{yd} = f_y/\gamma_y \) = design yield strength of reinforcing steel
\( \gamma_c, \gamma_y \) = material factors for concrete and reinforcing steel
\( h, h' \) = section depth and distance between reinforcement in opposite faces
\( \beta \) = effective (or buckling) length factor of compression member
\( i = \text{radius of gyration of cross section, normally taken as } i_g \)
\( i_g, i_s \) = radii of gyration of gross concrete section and of total reinforcing steel
\( \delta_M \) = exact moment magnifier
\( \delta \) = approximate moment magnifier
\( \varepsilon, \sigma \) = strain and stress
\( \gamma_m \) = material factor
\( \mu_0 = M_{01}/M_{02} \) = first order end moment ratio
\( \mu_t = M_{t1}/M_{t2} \) = end moment ratio for total moments
\( \nu = \text{relative factored axial load} \)
\( \omega_t = \text{total mechanical reinforcement ratio} \)

References


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28


**Appendix**

The differential equation, $v'' = -\kappa$, where $v''$ is the second derivative of the displacement, $\kappa = 1/R$ the curvature and $R$ the radius of curvature, was discretised using the central difference expression

$$
(v_{i+1} - 2v_i + v_{i-1})/(\Delta x)^2 = -\kappa_i \quad i = 1, 2, \ldots, m - 1
$$

(18)

for the member divided into $m-1$ elements of equal length $(\Delta x)$. Section $i = 0$ is at end 1 and $i = m$ at end 2. Subject to the forced boundary conditions $v_0 = v_m = 0$, Eq. 18 results in a stiffness relationship from which the displacements at all sections can be computed iteratively. It becomes

$$
\begin{bmatrix}
2 & -1 & & & \\
-1 & 2 & -1 & & \\
& \ddots & \ddots & \ddots & \\
& & -1 & 2 & -1 \\
& & & -1 & 2
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_{m-2} \\
v_{m-1}
\end{bmatrix}
= 
\begin{bmatrix}
(\Delta x)^2 \cdot \kappa_1 \\
(\Delta x)^2 \cdot \kappa_2 \\
\vdots \\
(\Delta x)^2 \cdot \kappa_{m-2} \\
(\Delta x)^2 \cdot \kappa_{m-1}
\end{bmatrix}
$$

(19)

**A. Determine slenderness limits**

This description gives the main steps of only the two cases defined by 1) both ends being unrestrained (hinged) and 2) one end unrestrained and the other (end 2) restrained by a very stiff restraint. Details are given in Aasrum (1992). The program is very fast, and a fine subdivision is adopted (normally $\Delta x = h/30$).

1. Choose cross-section and reinforcement (symmetrical).

2. Choose axial load $(N_d = N_f)$ and compute the moment-curvature relationship $(M_{dk} - \kappa)$ of the section. The maximum moment value, i.e., the cross-sectional moment capacity, is denoted $M_d (= \max M_{dk})$. It is obtained at predefined ultimate strains ($\varepsilon_{cu}$ or $\varepsilon_{su}$ (= 0.010)).

3. Set the larger first order end moment $M_{02}$ (at end 2) equal to a specified fraction (0.95 or 0.90) of $M_d$. Since the end is unrestrained (hinged), $M_{02} = M_{02}$.
4. At end 1 with the smaller first order end moment (absolute value), choose a value for the total end moment $M_{11}$, and compute (by iteration as described below in routine B) the largest member length, $L_{\text{max}}$, for which it is possible to establish equilibrium. Continue the procedure with this length and the corresponding displacements $v_i$.

If the member is unrestrained at both ends, set $M_{01} = M_{11}$ and go to Step 7. Otherwise, go to next step.

5. Set the first order rotation at end 1 equal to the total rotation, $\theta_1 = \theta_{11} = (2v_1 + (\Delta x)^2 \kappa_0)/2\Delta x$, as the rotation at the very stiff restraint is insignificantly affected by second order effects.

6. Compute the first order end moment $M_{01}$ that corresponds to this $\theta_1$ (by iteration as described below in routine C).

7. Compute the first order end moment ratio $\mu_0 = M_{01}/M_{02}$ and slenderness parameters $\lambda = L_e/i_\beta$ or $\lambda_n = \lambda\sqrt{v/(1 + k_i\omega_i)}$. Adopted effective lengths are taken equal to elastic effective lengths of members with constant $EI$: $\beta = 1.0$ for a member hinged at both ends; $\beta = 0.7$ for a member hinged at one end and fixed the other.

8. Repeat from step 1, 2, 3 or 4 (as desired).

**B. Maximum L (at instability), iteration scheme**

1. Assume $L$, compute total moments at all discrete sections from $M_{i,i} = M_{i1} + 2(M_{i2} - M_{i1})i\Delta x/L + Nv_i$ (set $v_i = 0$ in the first iteration) and determine corresponding curvature values from the $M - \kappa$ relationship in step A2.

2. Solve the equilibrium equations, Eq. 19, for the displacements $v_i$, and repeat steps 1 to 2 until there is no significant change in the $v_i$’s from two consecutive iterations (adopted tolerance 0.01%).

3. Repeat from Step 1 with new $L$ until the largest length $L_{\text{max}}$ at which equilibrium can be established is obtained (adopted tolerance 0.1%). This length corresponds to stability failure (at or prior to material failure).

**C. End moment $M_{01}$, iteration scheme**

1. Assume $M_{01}$, compute first order moments at all discrete sections from $M_{0,i} = M_{01} + (M_{02} - M_{01})i\Delta x/L$ and determine corresponding curvatures from the applicable $M - \kappa$ relationship (step A2).

2. Solve the equilibrium equations, Eq. 19, for first order values of the displacements $v_i$, and compute the end rotation $\theta_1 = (2v_1 + (\Delta x)^2 \kappa_0)/2\Delta x$.

3. Repeat from Step 1 with a new $M_i$ until the difference between the calculated $\theta_1$ in the step above and $\theta_1$ in step A5 is negligible (adopted tolerance $\Delta \theta_1 = 0.00001$).