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Hanbury Brown and Twiss effect demonstrated for sound waves from a waterfall: An experimental, numerical, and analytical study

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Hanbury Brown and Twiss determined the angular size of a visible light source (the star Sirius) by studying how the cross-correlation in intensity fluctuation recorded by two detectors changes with the distance between the detectors. We find that this principle works equally well for sound waves from a waterfall. This is remarkable, since sound is a completely different kind of wave from the HBT case. The frequency of the waves differs by a factor \(\sim 10^{12}\) and the wavelength as well as the angular extension of the source seen from the observer’s position differ by a factor \(\sim 10^7\). Our analysis is based on the general properties of broadband waves. We start with broadband waves at the amplitude level (not at intensity level) and demonstrate a HBT-like effect. We follow up with an explanation and demonstrations showing how the effect also manifests itself at the intensity level, providing a bridge to the original HBT work. We use the same reasoning in our numeric and analytical treatments, as well as in the experimental work, with identical results. The presentation is simple enough to be introduced even for second year bachelor students. Computer programs (in Matlab), including software for time-resolved frequency analysis, as well as original sound files are available from the authors. © 2022 Published under an exclusive license by American Association of Physics Teachers.

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I. INTRODUCTION

In the 1950s, Hanbury Brown and Twiss (HBT) developed a method where the angular extent of objects in the sky could be determined from time variation in intensity measurements at two nearby radio telescopes. When the telescopes were close to each other, the signals detected by the two telescopes were highly correlated, but the correlation dropped considerably as the distance between the telescopes increased. The angular extent of the source could be calculated from these data using a classical statistical wave description.

Soon after, the method was also used at optical wavelengths for the measurement of the angular diameter of the star Sirius. The fact that the HBT effect also arises at optical wavelengths, where the quantum nature of the electromagnetic field should be important, was initially resisted by physicists and astronomers. However, after Glauber derived the quantum theory of optical coherence, many physicists now consider the HBT effect to be a true quantum phenomenon arising from “photon bunching.” Since the quantum as well as the semi-classical explanations of the phenomenon (see, e.g., Ref. 6) are rather complex, both conceptually and mathematically, the effect is considered unsuitable for the undergraduate physics curriculum. While the HBT method is no longer widely used in its original context, it is commonly used in high energy physics, providing motivation for introducing it to students in courses treating wave phenomena.

Numerical modeling has lately been introduced at the undergraduate level. It often makes complex physics topics easier to handle than theory alone, making such topics more relevant and interesting for students. In this paper, we use numerical modeling to study the HBT effect. We believe that this leads to a better understanding, not only of the HBT effect, but of broadband waves in general. By “broadband waves” we mean waves that have a broad continuous frequency distribution, in contrast to simple harmonic waves.

II. BROADBAND WAVES STUDIED BY TIME–FREQUENCY ANALYSIS (TFA)

A broadband wave can be generated by a single source, or it may be the sum of radiation from many independent sources, such as in a star. In this section, we first imagine a single source broadband wave that is recorded by a detector at a sampling frequency well beyond twice the maximum frequency in the wave. Thus, the time-varying amplitude is recorded.

This may seem to be a strange choice, since the HBT experiment on Sirius was performed at the intensity level,
because the time resolution of the detectors and associated instrumentation was far too low compared to the frequency of visible light. However, waves always add at the amplitude level, and we will show that the HBT effect is fundamentally an amplitude level effect. Since there is a close connection between amplitudes and intensities, it will also be possible to use it at the intensity level, as was done historically. We will do both in this paper.

We model a broadband wave numerically by generating a Fourier spectrum where each of the frequency components has a random amplitude (limited by a Gaussian shape) and random phase. The center frequency is $f_0$, and the full width at 1/e of max is chosen to be $f_0$. This relative width is similar to the frequency distribution for visible light from Sirius, as well as the filtered sound wave signals used in our experiments. The wave amplitude vs. time is obtained by an inverse Fourier transform, and the resulting wave can be described by

$$W(t) = \sum_{j=1}^{N} a_j e^{i\omega_j t} + c.c.,$$

(1)

where $\omega_j$ is the angular frequency of the $j$th component, and the coefficients $a_j$ are complex numbers with absolute values chosen randomly between zero and a maximum value given by a Gaussian frequency distribution

$$a_j = p \cdot e^{-(\omega_j - \omega_0)^2/2\sigma^2},$$

where $p$ is a random complex number with an absolute value between 0 and 1.

In the numerical description, time is also discretized, but we leave out this detail in our expressions to simplify the notation.

A short piece of such a wave is shown in Fig. 1(a). It is an irregular train of overlapping “wavelets” with a temporal duration and spectral width limited by the time-bandwidth product. The dominant frequency of each wavelet varies within the spectral width of the overall signal. It is difficult to visualize the statistical characteristics of a broadband wave by only displaying amplitude vs time. Instead, we found it very useful to use a time–frequency analysis (TFA), as shown in Fig. 1(b). Time–frequency analysis is now available in a Matlab toolbox and in Python libraries. We chose the increasingly popular scalograms of the continuous wavelet transform (CWT) based on Morlet wavelets and used the version of this analysis described by Vistnes.

The relative frequency resolution is constant for all frequencies in the CWT scalogram. This is a nice feature since it makes it easier to optimize the choice between resolution in time vs. resolution in frequency. A parameter $K$ measures the approximate number of oscillations in the wavelet, independent of the frequency. Low (high) $K$-values give high (low) time-resolution and low (high) relative frequency resolution. This detail is important since time and frequency are tightly connected by the time-bandwidth product ($\Delta f \Delta T \geq 1$), where $\Delta f$ is the spectral width (in frequency space) and $\Delta T$ is the temporal duration of each wavelet. This is the classical analogy to Heisenberg’s uncertainty principle.

Within each yellow patch in the time–frequency diagram, it is possible to give an approximate time of appearance, frequency, amplitude, and phase of the wave. However, all of these parameters change from one yellow patch to another.

It is therefore impossible to define a global phase or a global frequency or wavelength for broadband waves. The summation of broadband waves leads to non-correlated results from time-slot to time-slot. Each independent broadband wave is unique.

The pattern in the TFA-diagram works like a fingerprint of a broadband wave. Every time we generate a new independent wave, the pattern differs from previous ones, even if the frequency distribution is identical.

Two important tools can characterize broadband waves: the autocorrelation function of a single wave, and the cross correlation between different broadband waves. The autocorrelation function $ac$ is defined by

$$ac(\tau) = \frac{\sum_{t} W(t) \cdot W(t+\tau)}{\sum_{t} W(t) \cdot W(t)}.$$  

(2)

Figure 4(a) shows the autocorrelation function for our model waves. The pattern is similar to a “damped oscillation” with a characteristic frequency $f_0$ and a decay time of the order the width of the frequency distribution $1/f_0$. The decay of the autocorrelation function tells us roughly how much time it takes from when we have complete knowledge of the wavelet until this knowledge is lost (except for the overall statistical description). The autocorrelation function is exactly the result we get from a Michelson interferometer experiment for any source of light, since we also in that case compare a wave with a delayed version of itself.

![Figure 1](image1.png)

![Figure 4](image4.png)
The cross correlation between two independent broadband waves $W_1$ and $W_2$ is defined as

$$CC = \frac{\sum_i W_1(i) \cdot W_2(i)}{\sqrt{\sum_i W_1(i)^2} \sqrt{\sum_i W_2(i)^2}}.$$  

If we follow two waves for a very long time compared to the inverse bandwidth $(t_{\text{max}} - t_{\text{min}} \gg 2\pi/\omega_0)$, the cross correlation tends to zero, even if the two waves have the same statistical distributions. This result is independent of whether $W_1(t)$ and $W_2(t)$ are calculated at the same time $t$ or if one is time shifted relative to the other. For shorter times, the calculated cross correlation is small and statistically distributed symmetrically around zero for repeated calculations.

III. SUMMATION OF BROADBAND WAVES

In HBT experiments, many uncorrelated broadband waves add to each other at the positions of the detectors. Let us explore by numerical modeling what happens when we add two broadband waves to each other. In Fig. 2, we show a tiny part of two uncorrelated waves generated with identical mean amplitude, spectral width, and center frequency (upper row). Note the differences in the fingerprint patterns. The lower row shows two different summations of the two waves. The summation is interesting in several ways:

- The mean amplitude of the sums is approximately a factor of $\sqrt{2}$ greater (not shown) than the amplitude of either wave, as expected for broadband waves.
- The TFA-diagram of the sum is qualitatively identical to the one for each of the separate waves. The fingerprints are different, but it would be impossible from the sum-TFA-diagram alone to judge if the wave were generated by a single source or by several sources.
- More important for this discussion, the fingerprint pattern of a sum wave is not a simple sum of the TFA-diagrams of the two original signals. The reason is that the two waves interfere constructively during some time segments, but destructively during others. The sum depends on the detailed variation of frequency, amplitude, and phase during the span of the diagram.
- The sum changes if the two waves are time-shifted relative to each other. If the time shift is much less than half the period $T_m$ of the dominant frequency, the sum wave is not changed much. However, the changes get more and more dramatic as the time shift is increased to $T_m/2$ and beyond. The effect is more complex than what we are used to for harmonic waves.

IV. SUMMATION OF WAVES IN A TYPICAL HBT EXPERIMENT

We will now apply our basic knowledge about the summation of broadband waves to an HBT experiment. Typically, many independent but identical sources of broadband waves are confined to a limited area in space. For simplicity, let $2n$ sources be positioned along a straight line of length $L = 2L'$ (see Fig. 3). Two identical detectors, A and B, are placed a distance $D$ from the source line, in the symmetrical manner shown in the figure. The distance $d = 2d'$ between the detectors can be varied. Waves from all $2n$ sources will reach both detectors, and the resulting wave at either detector is the sum of all $2n$ source waves. Let $W_A(t)$ and $W_B(t)$ be the waves detected by A and B at time $t$, and let $W_k(t)$ be the wave emitted from source $k$ at time $t$. We assume that $D \gg d$ and $D \gg L$ and neglect the change of amplitude due to the variation of the source-detector distance.

In order to calculate $W_A(W_B)$, we need to know the time of flight for each component wave, which is proportional to the path length from the source to detector. Let $a_k$ be the path length from the source $k$ to detector A for the left half of $L$. For sources on the right half of $L$, we use the index $k'$ and denote the path lengths $a_{k'}$.

![Fig. 2](image2.png)

![Fig. 3](image3.png)
Note that \( k' = k \).

The time of propagation from source element \( k \) to detector \( A \) is \( t_{kA} = a_k / c \), where \( c \) is the speed of the wave. Waves from the left part of the source are denoted by \( W \) and from the right part \( W' \), and we sum symmetrically, that is \( k = k' \), and get

\[
W_A(t) = \sum_k [W_k(t - a_k / c) + W_k'(t - a_k c / c)]
\]

or

\[
W_A(t) = \sum_k [W_k(t' + k\delta / c) + W_k'(t' - k\delta / c)], \tag{4}
\]

where \( k = 1, 2, ..., n \), \( t' = t - X / c \), and \( \delta = L'd' / Dn \). Similarly,

\[
W_B(t) = \sum_k [W_k(t' - k\delta / c) - W_k'(t' + k\delta / c)]. \tag{5}
\]

In an HBT experiment, the cross-correlation between \( W_A \) and \( W_B \) is measured as a function of the distance between the detectors. From Eq. (3), the cross-correlation is

\[
CC = \frac{\sum_{t} W_A(t') \cdot W_B(t)}{\sqrt{\sum_{t} W_A(t') \cdot W_A(t')} \sqrt{\sum_{t} W_B(t') \cdot W_B(t)}} = \frac{\sum_{t} W_A(t') \cdot W_B(t)}{N}, \tag{6}
\]

where the denominator is the product of the average amplitudes of the two waves and is a constant. From Eqs. (4) and (5), the cross correlation will consist of 4n² terms, where \( 4n^2 - 2n \) of them are cross-correlations of waves emitted by different sources. Since these component waves are independent, these terms will be zero. The 2n surviving terms in the numerator are

\[
\sum_{t} \sum_{k} W_k(t' - k\delta / c)W_k(t' + k\delta / c)
+ \sum_{t} \sum_{k} W_k(t' + k\delta / c)W_k'(t' - k\delta / c).
\]

Note that for \( W_A \), the wave from source \( k \) arrives before the wave from source \( k' \), while it is the other way around for \( W_B \). It follows that the TFA-diagrams for detectors \( A \) and \( B \) will not differ significantly if \( n\delta / c \) is much less than half the period of the dominant wavelength, \( T_m / 2 \).

Interchanging the summations over time and wave source, we find

\[
\sum_{k} \sum_{t'} W_k(t')W_k(t' + 2k\delta / c)
+ \sum_{k} \sum_{t'} W_k'(t')W_k'(t' - 2k\delta / c).
\]

According to Eq. (2), this is just proportional to the sum of autocorrelation values. Since the autocorrelation function is symmetric around zero, we simply get

\[
CC \propto \sum_{k} a_k(2k\delta / c), \tag{7}
\]

which is equivalent to an integral of the autocorrelation function over an interval from 0 to a maximum value \( 2n\delta / c = (Ld / 2D) \) corresponding to a maximum path difference

\[
\Delta = \frac{Ld}{2D}. \tag{8}
\]

Since the denominator in our cross-correlation calculation does not depend on \( k \), we are free to choose a normalization constant so that \( CC = 1.0 \) when the detectors are very close to each other. The cross correlation will then simply be the mean value of the autocorrelation function of the component waves over an interval from zero to \( \Delta \).

Figure 4(a) shows the autocorrelation function for a single broadband wave while Fig. 4(b) shows the normalized cross correlation calculated as a function of the maximum time shift \( \Delta / c \) divided by the period \( T_m \) of the dominant frequency, or

\[
\frac{\Delta / c}{T_m} = \frac{\Delta}{\lambda_m} = \frac{L}{2D\lambda_m} = \frac{d}{\lambda_m} \tan(\theta / 2), \tag{9}
\]

where \( \lambda_m \) is the dominant wavelength and \( \theta \) is the angle subtended by the source at the location of the detectors. By measuring how the cross correlation changes with the distance between the detectors, it is possible to calculate the ratio \( L / D \), or the angle \( \theta \) subtended by the entire source of waves, assuming we know the dominant frequency of the broadband waves emitted by the source. This is the essence of the original HBT experiment, and we will return to it in Sec. X at the end of this paper.

V. EXPERIMENTAL

To experimentally test our model of the HBT effect, we used sound waves with a frequency spectrum far lower than the sampling frequency of inexpensive, readily available digitizers. This allows us to first work with signal amplitudes and later add an analysis based on intensities.

A waterfall at a dam in Akerselva river (Oslo, Norway) was used as a source of broadband waves. The waterfall had a relatively even flow across all of its width (approximately 17 m). Two microphones were mounted on camera tripods, and the distance between them was varied from 9 to 10 m,
moving each microphone symmetrically around a stationary midpoint. We placed the front of the microphones along the rail of a walking bridge that ran nearly parallel to the waterfall, at a distance of approximately 78 m from the waterfall. See Figs. 5 and 6. Note the similarity between Figs. 3 and 6. The sound level from the waterfall was approximately 60 dB(A), which is approximately the sound level of conversation. Since we carried out the measurements very early in the morning, before road traffic began, there were no other audible sources of sound nearby. The landscape on both sides of the waterfall had no obvious sound-reflecting objects, so the recorded signals were mainly direct sound from the waterfall.

We used Sennheiser MKH 8070 microphones, which had close to a perfectly uniform sensitivity throughout the angular extension of the waterfall (±2 dB over 13°). Two Focusrite Scarlett Solo second generation audio interfaces powered and amplified the microphone signals. A National Instruments USB 6211 (16 channel, 16 bit, maximum sample rate 250 kS/s) multifunction I/O unit digitized the resulting signals, and a Dell XPS laptop computer controlled the digitizer at a sampling rate of 44 150 S/s. The time difference between samples from the two microphones was so small that it did not influence our results at the frequencies used in this work. The data sampling period was 5 s, and the measurements were repeated three times for each chosen microphone distance. The electronic measurement equipment was powered by a Mascot inverter Type 2285 connected to a 12 V battery. The inverter provided 230 V true sine wave to minimize the noise from this source. We found no trace of 50 Hz harmonics in the recorded signal.

We wrote the analysis software using Matlab. Essential functions used were sampling and storing to disk, fast Fourier transform, frequency filtering based on Fourier

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**Fig. 4.** (a) The autocorrelation function for a single broadband wave described in Secs. II and III. x-axis values are fractions of the period of the dominant frequency $T_m$. (b) The “mean value” of the autocorrelation function over an interval from 0 to $\Delta/c$. According to Eq. (7), this is also the cross-correlation of multiple source waves arriving at detectors $A$ and $B$. x-axis values are fractions of $T_m$. (c) The same curve as (b), but with logarithmic ($\log_{10}$) axis. The mean autocorrelation value (the cross-correlation value) drops to 0.5 when $\Delta$, the maximum difference in distances between a detector and the sources, is 0.30 times the dominating wavelength ($\log_{10}(0.30) = -0.52$). Compare this figure with similar figures for the experimental and theoretical data later in this paper.

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**Fig. 5.** Experimental setup with two microphones directed toward the center of the waterfall.

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**Fig. 6.** Geometrical relationship between the waterfall and the line of positions for the microphones. A red (online) dot indicates the symmetry point for the microphones. The 5° offset was compensated for by a systematic and progressively larger time shift of signal $A$ relative to signal $B$ before the cross correlations were calculated.
transform, calculation of the autocorrelation and cross correlation functions, and the time-resolved frequency analysis (TFA) described in Sec. II.

In Sec. IV, we assumed that the distance between the source and the detectors $D$ is very much larger than the width of the source $L$. In our experiment, this assumption is not strictly valid since $D$ is only $\approx 4.5L$, leaving us uncertain about whether our formalism would work well for our experimental work. Fortunately, it can be shown that the potential problem vanishes if the geometry is strictly symmetric. However, the midpoint between the microphones is offset by $5^\circ$ from the midpoint of the waterfall (see Fig. 6). We compensated for this slight asymmetry by a systematic and progressively larger time shift of signal $A$ relative to signal $B$ as the microphone separation was increased, before the cross correlations were calculated. These details play only a minor role.

VI. RESULTS, ANALYSIS OF THE CORRELATIONS AT THE AMPLITUDE LEVEL

The microphones record sound waves at the amplitude level (sound pressure vs time). The frequency spectrum of the recorded sound stretched from approximately 25 to 8000 Hz and showed a broad peak in amplitude around 95 Hz. Figure 7(a) shows only a part of the frequency spectrum.

The broad spectrum makes it impossible to identify a dominant frequency. We therefore passed the signal through a Gaussian filter, using a fast Fourier transform (FFT) and inverse FFT after the filtering. This allowed us to select four frequency bands in our analysis, with center frequencies $f_0 = 100, 300, 600,$ and $1000$ Hz. In each case, the full bandwidth (at $1/e$ of the maximum) is equal to $f_0$. Figure 7(b) shows the spectrum of a filtered signal.

As discussed earlier, a time frequency analysis (TFA) can ease the understanding of the physics underlying the HBT effect, and Fig. 8 provides an example. These diagrams are similar to those obtained earlier by numerical modeling (Fig. 2).

Figure 9(a) shows how the cross correlation between the two microphone signals changes with the distance between the microphones, for the four frequency bands listed above. These data are qualitatively very similar to the results from our numerical modeling shown in Fig. 4(c). In Fig. 9(a), each curve starts close to 1.0 when the microphones are close to each other, but after a characteristic plateau drops sharply to zero. The microphone distances that correspond to a cross correlation of 0.5 are approximately 8.1, 3.2, 1.7, and 1.0 m for the filter frequencies 100, 300, 600, and 1000 Hz. Note that the shapes of the four curves are similar, and that as the filter frequency increases (wavelength decreases) there is a shift toward shorter microphone distances.

In Fig. 9(b), we have “normalized” the distance between the microphones by multiplying the actual microphone distance by 3, 6, and 10 for the frequency bands centered on 300, 600, and 1000 Hz, respectively. Thus, the four curves all correspond to an “effective” wavelength equal to the wavelength for 100 Hz filtering.
The four curves overlap strongly, indicating that the HBT effect depends heavily on the dominant wavelength of the summed signal. A detailed explanation of the collapse of the scaled curves will be given in Sec. VIII.

VII. THE CRUCIAL CONNECTION BETWEEN THE AMPLITUDE LEVEL AND INTENSITY LEVEL

So far, our analysis has used signal amplitudes, whereas the original HBT analysis was based on intensities. Let us look into the connection between these two levels of detection and analysis.

The intensity of a wave is proportional to its amplitude squared. If we choose just two components in the summation of waves, with frequencies \( \omega_1 \) and \( \omega_2 \), the signal intensity will have components at both the sum and difference frequencies. This is easily proved analytically. For a broadband signal, the difference frequencies can appear several decades lower in frequency than the original signal (all the way to 0 Hz). In the original HBT experiment, the signal from each detector was sent to a bandpass filter before analysis. The passband was several orders of magnitude lower than the frequency of light, but still high enough to measure cross correlations within an acceptable recording time.

In our experiments, we recorded very broadband signals and filtered them to obtain a narrower frequency distribution. The relative intensities of these filtered signals were calculated by squaring the amplitude at every point in time. A Fourier analysis of a typical result is shown in Fig. 10. The original signal in this example was filtered using the Gaussian filter centered at 600 Hz with full bandwidth 600 Hz (see Fig. 7(b)), so the intensity frequency spectrum is very wide, and there is an overlap between the difference and sum frequencies. If the original filter had been narrower, the difference and sum frequency spectra would have been more distinct.

Using our waterfall data, we first calculated cross correlations for the complete intensity signal (e.g., such as that shown in Fig. 10), including both difference and sum frequencies. Then, we used a bandpass filter to select parts of the difference frequency signal to study how filtering influences the result.

We first filtered the microphone signal at the amplitude level, using the same Gaussian filters as described above. Squaring the amplitudes point for point along the complete recording gave us the intensity signals. Finally, we calculated the cross correlation between the right and left microphone intensity signals for all 17 microphone distances as before. The results are displayed in Fig. 11(a).

Figure 11(a) looks very similar to the amplitude analysis result in Fig. 9(a), but the curves are slightly shifted toward smaller microphone distances. Moreover, the curves do not go to zero for large microphone distances, but fall to a level of approximately 0.33 (see Eq. (14) below for an explanation for this). Thus, instead of choosing the 0.5 cross correlation point to characterize the drop in correlation, as we did at the amplitude level, we chose the 0.67 cross correlation point as a characteristic parameter. The microphone distances that correspond to this point are 5.8, 2.1, 1.2, and 0.6 m for the signals with dominant frequency 100, 300, 600, and 1000 Hz, respectively. When plotting the cross correlation vs the normalized microphone separation (Fig. 11(b)), the four curves overlap quite well, illustrating once again the wavelength dependence of the correlation.

In the original HBT work, analyses were necessarily performed within a frequency band very much lower than that of light itself (filter passband 5–45 MHz, while dominant frequency of visible light from Sirius is \( \approx 5 \times 10^{12} \) Hz).
our case, however, it was difficult to obtain widely separated frequency bands for the original signal and the analyzed signal, because the frequency of the sound from the waterfall was too low. Nevertheless, we performed several analyses where the full intensity signal was filtered before calculating the cross correlation, to mimic the process used in the original HBT work. The characteristic cross correlation parameter for these filtered intensity signals was equal to that of the full intensity signal within uncertainty. This suggests that we would have obtained the same results, even if we had to rely only on intensity fluctuations within a frequency band far lower than the wave frequency itself, as was the case in the original HBT work.

VIII. THEORETICAL TREATMENT OF OUR SYSTEM

In this section, we provide a theoretical analysis of the experiment, showing how one can explain the scaling of the cross correlation with the normalized detector separation illustrated in Figs. 9(b) and 11(b), as well as the fact that the intensity correlation in Fig. 11 decays to a level different from zero. The model that we use is a direct generalization of the large number of sources model presented in Fig. 3. The difference between the treatment here and that in Sec. IV is that we use a continuous line of sources instead of a discretized one. In this approach, it is easier to derive the scaling relation of the cross correlation, while the treatment in Sec. IV is more directly suitable for numerical implementation. We believe that the ability to switch between representations is a useful skill for students and that repeating the same analysis in both forms is a useful exercise.

Once again, we consider (see Fig. 12) a line of sources with length \( L \). The detectors are separated by a distance \( d \) and the perpendicular distance from the source line to the midpoint between the detectors is \( D \). Unlike the previous treatment, the midpoint of the line of sources is displaced a distance \( s \) from the center line between the detectors, thereby accounting for the actual asymmetry in our experiment.

We assume that there are independent sources of broadband waves at each position \( x \) where \(-L/2 \leq x \leq L/2\) and that these waves reach the detectors with equal amplitude but different time shifts. The quantity \( W_A(t) \) in Sec. IV must now be replaced by a continuous function \( W(x, t) \). The signal received at detector A is then

\[
W_A(t) = \int_{-L/2}^{L/2} dx W(x, t - r_A/c),
\]

where \( c \) is the speed of sound and \( r_A(x) \) is the distance from the source at \( x \) to detector A. A similar expression describes the signal \( W_B(t) \) received by detector B. We then have

\[
r_{A/B}(x) = \sqrt{D^2 + (x + s + d/2)^2},
\]

\[
\approx D + \frac{1}{2D} \left( x + s + \frac{d}{2} \right),
\]

(10)

when \( D \) is large compared to \( x, s, \) and \( d \). The cross correlation of the amplitudes in the two detectors is proportional to

\[
\langle W_A W_B \rangle = \int dx_1 dx_2 (W(x_1, t - r_A(x_1)/c) \times W(x_2, t - r_B(x_2)/c)),
\]

where \( \langle \cdots \rangle \) indicates time averaging. We assume that the sources at different \( x \) are uncorrelated and stationary so that

![Fig. 12. Geometry used in the analysis, including the shift \( s \) of the midpoint of the line of sources.](image-url)
\[ (W(x_1, t_1)W(x_2, t_2)) = \delta(x_1 - x_2)ac(t_1 - t_2), \]

where \( ac(t) \) is the autocorrelation function for the signal from a single source. Using Eq. (10) we get the cross correlation

\[ CC \propto \langle W_A W_B \rangle = \int_{-L/2}^{L/2} dx ac(r_-(x)/c - r_+(x)/c) \]

\[ = \int_{-L/2}^{L/2} dx ac \left( \frac{d}{Dc} (s + x) \right). \]

This is the continuous analog to Eq. (7).

Let us now assume that the sources are broadband with a large number of frequencies contributing. Then the source will produce a signal

\[ W(x, t) = \int d\omega A(\omega) \cos (\omega t + \phi_\omega), \]

where the phases \( \phi_\omega \) are random numbers between 0 and 2\( \pi \) and are independent for each \( \omega \). The amplitudes \( A(\omega) \) either describe the actual spectral distribution of the source, or the characteristics of the filter used in the experiment. The latter choice will be used below. Since we have assumed that all sources are statistically equivalent, the resulting autocorrelation function is independent of \( x \). Using Eq. (2), we get

\[ ac(\tau) = \frac{1}{2T} \int d\omega d\omega' A(\omega)A(\omega') \int_{-T}^{T} dt \cos (\omega t + \phi_\omega) \times \cos (\omega'(t + \tau) + \phi_{\omega'}). \]

The average of the cosine product is zero unless \( \omega' = \omega \), and we get

\[ ac(\tau) = \frac{1}{2} \int d\omega A^2(\omega) \cos \omega \tau. \]

Using a Gaussian filter

\[ A(\omega) = e^{-\frac{(\omega - \omega_0)^2}{2\sigma^2}}, \]

so

\[ ac(\tau) = \frac{1}{2} \int d\omega e^{-\frac{(\omega - \omega_0)^2}{2\sigma^2}} \cos \omega \tau = \frac{\sqrt{\pi}\sigma}{2} e^{-\frac{\tau^2}{\sigma^2}} \cos \omega_0 \tau, \]

where \( \sigma = \omega_0/2\sqrt{2} \) for the filters used in this experiment. We then get for the cross correlation

\[ CC = \frac{Dc}{Ld\omega_0} \int_{-\omega_0+u_0}^{\omega_0+u_0} du e^{-2u^2} \cos u, \tag{11} \]

where we have normalized by requiring that \( CC = 1 \) when \( d = 0 \), and introduced the new variables

\[ u = \frac{d\omega_0}{Dc}(x + s), \quad u_0 = \frac{Ld\omega_0}{2Dc}, \quad u_s = \frac{sd\omega_0}{Dc}, \]

and \( x = \frac{\sigma^2}{4\omega_0^2}. \tag{12} \]

From (11) and (12), we see that the cross correlation depends only on the product \( d\omega_0 \) as long as we keep the ratio \( \sigma/\omega_0 \) fixed, so that \( z \) is the same for all \( \omega_0 \). This explains why the four curves shown in Fig. 9(b) overlap when plotted against the scaled axis \( d\omega_0 \). Using Eq. (8), we see that \( CC \) depends on the ratio \( \Delta/\lambda_m \), where \( \lambda_m \) is the dominant wavelength of the received signal. This implies that the cross correlation can be used to find the subtended angle of the source as in the original HBT experiment.

**IX. ANALYSIS AT THE INTENSITY LEVEL**

The intensity is the square of the amplitude, \( I_{A/B} = W^2_{A/B} \).

The cross correlation of the intensities is

\[ \langle I_{A}I_{B} \rangle = \int dx_1 dx_2 dx_3 dx_4 W(x_1, t - r_+(x_1)/c) \times W(x_1, t - r_+(x_1)/c)W(x_2, t - r_-(x_2)/c) \times W(x_2, t - r_-(x_2)/c)). \]

Since the sources are independent at each position, the coordinates must be pairwise equal, so that either

\[ x_1 = x_1' \quad \text{or} \quad x_1 = x_2 \quad \text{or} \quad x_1 = x_2'. \]

The last two combinations give equal contributions because of symmetry, and we get

\[ \langle I_{A}I_{B} \rangle = \int dx_1 dx_2 (0^2 + \frac{1}{2} \int dx_1 dx_2 ac \left( \frac{d}{Dc} (s + x_1) \right) \times \frac{d}{Dc} (s + x_2)) = \langle I_{A} \rangle \langle I_{B} \rangle + 2CC^2. \tag{13} \]

Thus, we see that there is a close connection between the amplitude and intensity correlations. (For a further discussion of this, see the review of Baym.7) Equation (13) explains why the intensity correlation approaches a non-zero value at large \( d \) and also explains the collapse of the experimental data at the intensity level, as seen in Fig. 11(b).

It is interesting to compare the measured cross correlations at the intensity and amplitude levels with the theoretical predictions. Since we normalized the correlation functions to 1 at \( d = 0 \), we rewrite the relation between cross correlations of the amplitude and intensity, Eq. (13), in terms of normalized correlations as

\[ \langle I_{A}I_{B} \rangle = \frac{1}{3} \left( 1 + \frac{2}{3} CC \right)^2. \tag{14} \]

In Fig. 13 we show the theoretical curves found by numerical integration of (11) and using (14). In the numerical calculations we used the measured width \( L = 17 \) m and distance \( D = 78 \) m of the waterfall, so the curves shown have no fitting parameters. We see that the curves fit well with the observed data, confirming that the observed cross correlation is consistent with the known width of the waterfall. In a real HBT experiment, \( L \) and \( D \) are unknown, so the ratio must be inserted as an adjustable parameter to achieve the best fit to the data.

To connect with the discussion of Fig. 9, we can use the numerical integration of Eq. (11) to find that \( CC = 0.5 \) corresponds to \( \Delta/\lambda_m = 0.30 \) when choosing \( s = 0 \). Using Eq. (14)
we can also find that selecting $\langle I_A I_B \rangle = 2/3$ corresponds to $CC = 1/\sqrt{2}$, which gives the value $\Delta/\lambda_m = 0.22$.

X. SUMMARY

We have in Secs. IV, VIII, and IX determined the expected normalized cross correlation (CC) in a HBT experiment for a given geometry and frequency distribution of the contributing waves. The shapes of the CC curves are all similar and can be characterized by the $\Delta/\lambda_m$-value where the CC curves drop to 0.50 or 0.67 for amplitude and intensity measurements, respectively. The shapes of the CC curves in our experiments are very similar to those obtained by numerical or analytical modeling. This allows us to use our modeling to determine the angular extent of the source from experimental data, as was done in the original HBT work.

Rearranging Eq. (9) gives us the connection between the geometry of the linear HBT setup and $\Delta/\lambda_m$

$$\tan(\theta/2) = \frac{\Delta}{\lambda_m} \cdot \frac{\lambda_m}{d}.$$  \hspace{1cm} (15)

The first term is determined from our modeling: $\Delta/\lambda_m = 0.30$ for amplitude measurements or 0.22 for intensity measurements. The second term comes from our experiments: $d$ is the detector separation where the CC curves drop to 0.50 or 0.67 when the dominant wavelength is $\lambda_m$. The results from our experimental amplitude data are shown in Table I.

We hesitate to specify the accuracy of these numbers, since we were not able to get a perfect Gaussian frequency distribution in our experimental work. The 100 Hz result deviates from the other three. This is not surprising, since $d$ is so large in this case that the asymmetry in our experimental setup plays a more significant role than for the other cases.

The results from our experimental intensity data are shown in Table II.

If we exclude the 100 Hz results, the mean value of the desired angle is $11.6^\circ$ based on the amplitude data and $13.1^\circ$ based on the intensity data. Using direct measurements of $D$ and $L$ (see Fig. 6), we get: $\tan(\theta/2) = 8.5/78$ or $\theta = 12.4^\circ$. That is, the angles determined experimentally from Eq. (15), using either amplitude or intensity data, are both close to the angle determined by direct geometrical measurements. The uncertainty is estimated to be between 5% and 10%. The consistency in our total work is very satisfying.

It is interesting to compare our results with the original HBT work.\textsuperscript{2} Let us assume that their estimate of the angular extent of Sirius, 0.0068 arc sec, is correct. They observed a drop in cross correlation in the intensity of the light, halfway from max to the limiting value (normalized CC = 0.67 in our language), when $d = 8$ m and a $\lambda_m \approx 500$ nm. The calculated $\Delta/\lambda_m$ is then $\approx 0.26$. This is about 20% larger than our result. This difference can probably be ascribed to the difference in geometry of the source for their broadband waves (circular in the 1956 HBT study and linear in our work). In addition, the autocorrelation functions for their waves (blackbody radiation) and our Gaussian frequency distributed waves are not identical, even though we chose our total bandwidth to be similar to their experiment. Their autocorrelation function will in general deviate a bit from our modeling.

The similarity between the original HBT work and our results is remarkable since we are working with completely different kinds of waves: sound waves in air compared to electromagnetic waves in the visual range. The frequencies differ by a factor $10^{12}$, and the wavelengths and apparent

---

Table I. Angle of the source determined from measurements at the amplitude level.

<table>
<thead>
<tr>
<th>$f_m$ (Hz)</th>
<th>100</th>
<th>300</th>
<th>600</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_m$ (m)</td>
<td>3.37</td>
<td>1.12</td>
<td>0.56</td>
<td>0.34</td>
</tr>
<tr>
<td>$d$ (m)</td>
<td>8.1</td>
<td>3.2</td>
<td>1.7</td>
<td>1.0</td>
</tr>
<tr>
<td>$\theta$ (deg)</td>
<td>14.3</td>
<td>12.0</td>
<td>11.3</td>
<td>11.6</td>
</tr>
</tbody>
</table>

Table II. Angle of the source determined from measurements at the intensity level.

<table>
<thead>
<tr>
<th>$f_m$ (Hz)</th>
<th>100</th>
<th>300</th>
<th>600</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_m$ (m)</td>
<td>3.37</td>
<td>1.12</td>
<td>0.56</td>
<td>0.34</td>
</tr>
<tr>
<td>$d$ (m)</td>
<td>5.8</td>
<td>2.1</td>
<td>1.2</td>
<td>0.6</td>
</tr>
<tr>
<td>$\theta$ (deg)</td>
<td>14.6</td>
<td>13.3</td>
<td>11.8</td>
<td>14.2</td>
</tr>
</tbody>
</table>
angular size of the object as judged from the observer differ by approximately a factor of $10^7$.

XI. OUR WORK IN A WIDER PERSPECTIVE

The 1956 HBT paper led to heated discussions about the nature of light and interpretations of quantum mechanics. One can only wonder how the discussions of that time would have been if physicists knew that the HBT effect could also be demonstrated for sound waves. Today, the particle-like photon is the most common mental picture of light, but our description in this paper is based purely on waves, in compliance with Hanbury Brown’s view. The immediate mental pictures we get from concepts like “photon bunching” and “correlated photons” are incompatible with our description of broadband waves and the TFA diagrams shown in Fig. 2.

Even today, there are several different, conflicting views of the nature of light, and numerous physicists have pointed out this problem. In 2013, Schlosshauer et al. wrote: “Quantum theory is based on a clear mathematical apparatus… Yet, nearly 90 years after the theory’s development, there is still no consensus in the scientific community regarding the interpretation of the theory’s foundational building blocks.” While these authors are referring to different interpretations of quantum mechanics, the lack of consensus they describe heavily influences the current array of opinions regarding the nature of light as well.

We believe that it is important to introduce this dilemma to physics students and have some experience doing this at both the high school and university levels. It is our hope that this paper will inspire constructive student discussions about the nature of light. Perhaps the new generation of bright young physicists will be able to correct the current situation by finding a unifying description of light.


\textsuperscript{5}R. J. Glauber, Quantum Theory of Optical Coherence. Selected Papers and Lectures (Wiley-VCH Verlag, New York, 2007).

\textsuperscript{6}L. Mandel and E. Wolf, Optical Coherence and Quantum Optics (Cambridge U.P., Cambridge, 1995); See chapters 9.8–9.11 and references therein.


\textsuperscript{10}A. Kingsley and D. Durfee, Instructional Lab for Undergraduates Utilizing the Hanbury Brown and Twiss Effect, Bachelor Project Report (Brigham Young University, Provo, 2015).


