Maternal Support of Children's Math Learning in Associations Between
Family Income and Math School Readiness

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Abstract

This study examined maternal support of children's math learning at 36 months ($n = 140$) as a mediator of the association between early childhood family income and children's counting and calculation skills at 4½ and 6-7 years. Family income was associated with this measure of children's math school readiness, but the association was almost entirely indirect and mediated by observed maternal support of children's numerical skills at 36 months. Maternal support of children's spatial concept and general learning (cognitive stimulation and sensitivity) were not significant mediators of this association. Results suggest that income-based gaps in counting and calculation skills at school entry may be due in part to the constraints that low family income places on early numerical learning support.

Keywords: math development, maternal support, family income, school readiness, math achievement

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Children's early math skills are important indicators of their readiness for school with large and persistent differences in these skills evident between children in low- and middle-income families from early childhood through the schooling years (Duncan et al., 2007; Duncan & Magnuson, 2011). While considerable work has been dedicated to uncovering the mechanisms that help explain achievement differences associated with family income, generally speaking (e.g., Conger, Conger, & Elder, 1997; Yeung, Linver, & Brooks-Gunn, 2002), less attention has been focused directly on the mechanisms underlying income-related disparities in early math skills. The goal of this study was to examine maternal support of children's math learning—in numerical and spatial concepts domains—as potential mediators of the association between family income and children's math school readiness.

Disparities in Early Math Skills

Children's readiness for success in school has generally been assessed with attention to early literacy, math, and self-regulatory skills (Duncan & Magnuson, 2011). Among these skills, there is highly-cited evidence that early math may be a particularly powerful predictor of later achievement in both math and reading (Duncan et al., 2007). Across six U.S. datasets, including the one used in the present study, the predictive power of early math skills held true across gender and socioeconomic groups, and has been replicated in a population-based study of Canadian children (Duncan et al., 2007; Pagani, Fitzpatrick, Archambault, & Janosz, 2010). This correlational work suggests that starting school with a solid understanding of foundational math skills may set children up for initial and later trajectories of academic success (Duncan et al., 2007; Duncan & Magnuson, 2011; Galindo & Sonnenschein, 2015). An important caveat is that

“fadeout” patterns in recent intervention research indicate that early math skills are by no means an inoculation against later academic struggles (Bailey et al., 2016). Moreover, mediation analyses demonstrate the critical role of interconnections between early math and language development for later learning (Purpura, Logan, Hassinger-Das, & Napoli, 2017). Nonetheless, early math skills appear to be related to lasting achievement differences between socially advantaged and disadvantaged children.

When examining what accounts for individual differences in children’s math school readiness, comparisons of genetic and environment contributions have suggested that environmental factors explain at least half of the variance (Lemelin et al., 2007). And, family income appears to be one contributor to this variance (Duncan & Magnuson, 2011). Children from higher income families have more advanced math skills at school entry across multiple domains, including numerical and spatial skills (Duncan & Magnuson, 2011; Galindo & Sonnenschein, 2015; Saxe, Guberman, & Gearhart, 1987; Verdine, Irwin, Golinkoff, & Hirsh-Pasek, 2014; Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2017). Indeed, children from low-income families enter kindergarten approximately one year behind their advantaged peers in math skills, creating a gap that persists over the schooling years (Duncan & Magnuson, 2011). Given that income-related differences in math skills are evident prior to school entry, the early home environment likely plays an important role, and more specifically parental investments into children’s math learning are likely critical.

Parental Support of Children’s Early Math Skill Development

In understanding the ways that early support of math learning from parents may underlie income-related disparities in math achievement, two lines of theoretical and empirical evidence are noteworthy. The first represents a large body of theory and research that has sought to

understand the general processes by which low-income can create pressures within families that confer disadvantage, namely through the contributions of physical materials, learning opportunities, and parenting qualities that support children's cognitive development, such as cognitively stimulating and sensitive interactions with children (Bradley & Corwyn, 2004; Conger et al., 1997; Galindo & Sonnenschein, 2015; McLoyd, 1998; Yeung et al., 2002).

Theoretically, this line of inquiry has been focused on the ways low-income constrains parents' abilities to invest in their children and creates exceptionally high levels of stress that disrupt family functioning and healthy child development. While research from these perspectives has provided critical evidence that cognitive stimulation and responsive parenting in the home is a key mediator of income effects on children's development, it has given little direct attention to income disparities in aspects of the home environment that are specific to math. This gap stands in contrast to a more robust literature examining income-related differences in the home language environment (see, for example, Hart & Risley, 1995; Rowe & Goldin-Meadow, 2009).

A complimentary line of research from the field of cognitive development has examined the nature and contributions of parental supports of early math skills. Much of this work has focused on parental support of children's numerical skills, specifically looking at the frequency, range of concepts, and qualities of the parent-child interactions centered around numerical concepts. Findings from this research indicate, for instance, that greater frequency of parental talk about numerical concepts is associated with children's understanding of cardinality (Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010) and knowledge of a range of numerical concepts (Ramani, Rowe, Eason & Leech, 2015). The complexity and precisely what that talk includes, however, is also critical (Casey et al., 2018; Levine et al., 2010; Gunderson & Levine, 2011; Mix, Sandhofer, Moore & Russell 2012; Ramani et al., 2015; Saxe et al., 1987;

Skwarchuk, 2009). For example, when examining the types of numerical concept supports that parents offer young children, labeling the set size of visible objects (a precursor to understanding the cardinal principle) has proven particularly powerful in predicting later math knowledge development (Casey et al., 2018; Gunderson & Levine, 2011; Ramani et al., 2015).

Alongside numerical skills, spatial skills have shown to be essential for children's math abilities as early as preschool (Gunderson, Ramirez, Beilock, & Levine, 2012; LeFevre et al., 2013; Levine, Ratliff, Huttenlocher, & Cannon, 2012; Mix et al., 2016; Verdine et al., 2014; 2017; Zhang & Lin, 2017). Spatial skills involving the mental representation of images, maintenance of spatial information in working memory, and the transformation of mental images have been related to math readiness through early number line and calculation skills (Gunderson et al., 2012; LeFevre et al., 2013; Mix et al., 2016; Zhang & Lin, 2017). The ability to visualize the relation between numbers along a mental number line, for example, may be a key mechanism linking early spatial and math skills (Gunderson et al., 2012).

Compared with numerical support, less research has focused on parental support of spatial concepts. Existing research points to the importance of parents' frequency of spatial concept language (such as spatial features and properties of objects), spatial explanations, and encouragement of children's spatial visualization skills. Specifically, measures of parental spatial support have been linked with children's use of spatial language themselves (Ferrara, Hirsh-Pasek, Newcombe, Golinkoff, & Lam, 2011; Pruden, Levine, & Huttenlocher, 2011) as well as children's later spatial mental transformation skills (Levine et al., 2012; Pruden et al., 2011) and math problem solving abilities (Lombardi, Casey, Thomson, Nguyen, & Dearing, 2017).

Parental Support as a Contributor to Disparities in Children's Math School Readiness

While parental math support across areas of math-related development has been linked with children's early math abilities, as described above, only a handful of studies have examined the relation between income and parental math support. This research has documented significant correlations between family income and measures of numerical and spatial concept support (Casey et al., 2018; Levine et al., 2010; 2012; Lombardi et al., 2017; Saxe et al., 1987; Vandermaas-Peeler et al., 2009). In the only study that we know of to examine parental math support as a mediator of the relation between family income and children's math skills, Levine et al. (2010) found that parental number talk partially explained the association between family socioeconomics (SES), a combined measure of family income and maternal education, and preschool-aged children's understanding of the cardinal principle. No other known work has examined how parental math support, within either numerical or spatial skill domains, contributes to income-based disparities in children's math school readiness.

The Present Study

Therefore, the goal of the present study was to examine the degree to which parental support of children's math learning explains the association between family income and children's math school readiness. Specifically, we tested maternal support of children's numerical and spatial concept skills when children were 36 months old as mediators of the relation between early childhood family income and children's counting and calculation skills at school entry while separately estimating the role of maternal human capital characteristics (education level and verbal skills) and maternal general learning support (cognitive stimulation and sensitivity), both known to be positive contributors to children's achievement (Bradley & Corwyn, 2004). Based upon theory and prior literature (Bradley & Corwyn, 2004; Conger et al., 1997; Levine et al., 2010; Yeung et al., 2002), we expected that maternal math support would

explain some, but not all, of the association between family income and children's school entry counting and calculation skills.

In addition to providing needed evidence on the contribution of maternal math support to the association between income and children's math school readiness, this study also contributes to the existing literature in following ways. First, it examines the quality and quantity of math support across two conceptual domains underlying children's early math skill development, numerical and spatial concept support, thus simultaneously testing and accounting for the influence of each. Second, measurement of math support has been primarily comprised of frequency-based measures of parent talk and less often captured the quality or style of the interaction, language directed at or encouraging of the child to initiate math related work, or efforts to engage the child in joint math learning opportunities. Given evidence that the complexity of concepts has stronger links with children's later math skills (Casey et al., 2018; Levine et al., 2010; Saxe et al., 1987), the present study uses global measures of numerical and spatial concept support designed to capture both maternal use of these constructs as well as efforts to encourage use of these constructs on the part of the child. Third, we disaggregated SES into family income and mothers' human capital characteristics, answering calls to separately examine the unique influence of each in order to identify protective factors and design effective interventions (Elliott & Bachman, 2018; McLoyd, 1998). Drawing on conceptual models of adult human capital formation (Becker, 1964), we measured mothers' human capital characteristics using years of education and verbal skills. Finally, direct assessments of mothers' cognitive stimulation and sensitivity during the same interaction allowed us to isolate maternal general support of children's learning from the support provided that was specific to numerical and spatial concept skills.

Method

Sample

The study data came from the Boston, Massachusetts site of the NICHD Study of Early Child Care and Youth Development (SECCYD). Soon after giving birth in 1991, 140 women and their infants living in the Boston area were recruited to participate in the study using a conditional random sampling method (for complete recruitment and sampling details, see NICHD ECCRN, 2001). The present study investigated early predictors of math school readiness and thus utilized data collected during Phase I (with data collection occurring at 1, 6, 15, 24, and 36 months) and Phase II (54 months and 1st grade). The NICHD SECCYD included videotaped parent-child interactions, longitudinal data on family income and maternal characteristics, and direct assessments of children's academic skills. All of the measures were coded during the original NICHD SECCYD data collection, with the exception of the measures of maternal math support that were coded on the videotaped mother-child interactions at 36 months.

The sample was economically diverse with an income-to-needs over early childhood that ranged from 0-18 with a mean of 3.73 ($SD = 2.66$). Mothers had an average of 14.17 years of education ($SD = 2.62$) with about one-third (30%) having a high school degree or less and one-third (35%) having a Bachelor's or graduate degree. When the observational measures were assessed, mothers were an average of 32 years old ($SD = 5$ years) and over three-quarters (77%) were married. A notable limitation was the lack of racial and ethnic diversity with 85% of the sample being European American, thus constraining the generalizability of the results.

Procedures

Four measures of maternal support of children's learning were observed when children were 36 months of age. These were comprised of two measures of math-related support that we

coded (numerical and spatial concept) and two measures of general learning support that were coded by the original NICHD SECYYD research team (cognitive stimulation and sensitivity). The measurement of maternal support was based on a global analysis of parental supportive behaviors during a joint problem-solving task (Martin et al., 2007; NICHD ECCRN, 1999; 2008). A global rating scale analysis assigns a rating based upon the entire maternal-child interaction (rather than involving frequency counts of specific moment-by-moment behaviors). For these global ratings, specific maternal characteristics are evaluated throughout the entire interaction and the assigned rating provides a value of overall support (Darling & Steinberg, 1993). This type of global rating scale has repeatedly demonstrated predictive utility for academic outcomes (e.g. Crosnoe et al., 2010; Martin et al., 2007; NICHD ECCRN, 1999; 2008).

All measures were assessed during a 15-minute videotaped mother-child interaction that took place in a laboratory room. During the “Three Boxes” task (NICHD ECCRN, 1999), mother-child dyads were presented with three boxes each containing age-appropriate toys designed to facilitate interactions. The first box contained stencils and tracing materials, the second box included a cash register and dress-up clothes, and the third box contained Duplo blocks and a 2-D diagram of a block structure. Mothers were told to have their children play with the toys in each of the 3 boxes in order, for a total of 15 minutes. No other specific instructions were given.

The measures of cognitive stimulation and sensitivity were assessed over the entire 15-minute interaction. For numerical support, only interactions involving boxes 2 (cash register and dress-up clothes) and 3 (Duplo blocks) were assessed due to the affordances provided by these materials for eliciting numerical learning support; box 1 (stencils) offered mothers very few opportunities for supporting numerical learning, with almost no number talk from mothers or

children while playing with the stencils. This resulted in an interaction of approximately 10 minutes long for maternal numerical support. Spatial concept support was assessed during the interaction with box 3 as the blocks and 2-D diagram offered many opportunities for using and supporting spatial concepts. Although we also considered using the stencil task and cash register to examine spatial support, we excluded these boxes based on our observations of there being limited opportunities for spatial concept support with either material. In addition, we are not aware of research assessing spatial concept learning or support during play with stencils or a cash register; in contrast, blocks are empirically established stimuli for studying spatial support and learning (e.g. Verdine et al., 2014). Focusing on play with the Duplo blocks resulted in spatial concept support being assessed on an interaction that was approximately 5 minutes long. Note, that a minimum of 2 minutes of time with the Duplos was required in order to be given a rating. During the initial scale development, this was found to be the minimum length of time necessary for constructive play to begin and for mothers to have an opportunity to demonstrate spatial concepts. As a robustness check, the final analytical models were run on only the cases with a 3-minute interaction length or more and the results replicated.

Measures

Maternal math support. Five raters coded the videotaped observations for each measure of maternal math support. Seven percent of the videos were used for training. The training videos were watched and coded, in batches of 4, separately by all raters. Areas of disagreement were noted and resolved via discussion, ultimately resulting in consensus. We established inter-rater reliability between the five raters who coded 20% of the mother-child interactions for reliability as well as another 10% to address coder drift (i.e., 30% of participants, in total). Reliability among the raters was high, with Spearman-Brown statistics of 0.88 for numerical support and

0.96 for spatial concept support. Homogeneity of variance tests showed that variances among the raters did not differ significantly, and each rater used the entire rating range.

Numerical support: Labeling set sizes. Raters assessed maternal support of three numerical concepts across the interaction involving boxes 2 and 3: identifying numerals, one-to-one counting, and labeling set sizes. In a prior analysis, mothers' use and support of labeling set sizes was found to significantly predict children's math skills at 4 ½ years and first grade while the other two concepts, identifying numerals and one-to-one counting, were not significantly associated with children's later math skills (Authors, 2018). Consequently, for the present study, we used maternal support of labeling sets as a proxy measure of maternal numerical support. Maternal support of labeling sets was assessed on a scale of 0 (none) to 3 (high). Labeling set sizes was defined as using a number word to refer to a specific quantity of present physical objects (e.g., using "two" to refer to a set of two pennies). Mothers who did not label any sets of objects, respond to the child's labeling of sets of objects, or encourage the child to label any sets of objects received a rating of none ("0"). Mothers who labeled a set of objects once or twice during the interaction (e.g., "You have 3 pennies in that pile"), but did not respond or encourage the child to use the concept received a rating of low ("1"). A moderate ("2") rating was given to mothers who labeled sets of objects at least twice and responded to the child's use of labeling sets sizes in an effort to encourage continued use of the concept, but did not elicit the child's use of numerical concepts. Mothers who labeled sets of objects at least twice, responded to the child's use of the concept in an effort to encourage its use, as well as elicited the child's use of labeling set sizes (e.g., "How many pennies are there?"), were given a high ("3") rating. The full scale and examples are shown in Table S1.

Spatial concept support. Spatial concept support was a measure of the quality of maternal support of spatial concept learning used in relation to the Duplo blocks. Broad categories of spatial concepts were taken from Cannon and colleagues (Cannon, Levine & Huttenlocher, 2007): spatial dimensions, shapes, locations and directions, orientations and transformations, continuous amount, spatial features and properties, and patterns. Ratings were made on the mothers' support of spatial concept learning on a scale of 0 (none) to 3 (high). Mothers who did not use or support spatial concepts at all during the play with the Duplo blocks received a rating of none ("0"). Mothers who verbally used spatial concepts in a few (1 to 3) instances received a low rating ("1"). A moderate rating ("2") was given to mothers who both used spatial concepts in a few instances as well as prompting the child to use spatial concepts. Prompting the child to use spatial concepts occurred when mothers provided tasks and suggestions for the child that utilized spatial concepts. Mothers who received the high rating on spatial concept learning ("3") used spatial concepts, prompted the child to use spatial concepts, and also provided enriched explanations, gestures, or actions, including strategies for approaching the spatial task more effectively. A prior investigation found that this measure of spatial concept learning was predictive of children's math achievement at 54 months, but not at first grade (Authors, 2017). The full scale and examples are shown in Table S2.

Maternal general learning support. Maternal cognitive stimulation and sensitivity were measured during the same observation at 36 months with the three boxes. The videotaped interactions were assessed by a different team of trained coders in the original SECCYD study who had participated in intensive training, were blind to information about the families, and who met frequently to prevent drift. At each assessment, inter-rater reliability was examined by comparing the ratings of 20% of the tapes by two coders (NICHD ECCRN, 1999).

Cognitive stimulation. Maternal cognitive stimulation assessed the degree to which mothers tried to foster their child's overall cognitive and mental development on a scale from 1 to 7. Mothers scoring high on this scale helped their child learn new skills through frequent and developmentally appropriate behaviors and gestures to encourage learning during the interaction, such as helping the child to focus on a task or object, pointing out unique features of a play material, and responding to and elaborating on a child's verbalizations and non-verbal gestures. Mothers scoring low on this scale exhibit few or none of these stimulating behaviors. The interclass correlation between coders on this measure was 0.78 and it was significantly correlated with family income, maternal education and PPVT, and math achievement at both time points.

Maternal sensitivity. A measure of sensitive parenting was based on three ratings, all measured on 7-point scales, designed to assess the mother's capacity to interact in a sensitive, warm, and stimulating manner: (a) supportive presence, (b) hostility (reversed), and (c) respect for autonomy ($\alpha = .80$). Higher scores on the sensitivity composite reflected behaviors that were child-centered, engaged, warm, and stimulating. In contrast, lower scores reflected behaviors in which the mother rarely responded appropriately to the child's cues and did not shown an awareness of the child's needs. Based on independent coding of 20% of the tasks, interrater reliability was good (.84) and the measure was also found to significantly correlate with family income, maternal education and PPVT, and math achievement at both time points.

Children's math school readiness. Children's early mathematics achievement was a direct assessment of children's counting and calculation skills, assessed when children were 54 months and in first grade (6 to 7 years of age) using the Applied Problems subtest of the Revised Woodcock-Johnson Psycho-Educational Achievement Tests (WJ-R ACH, Woodcock and Johnson, 1989). The WJR is a comprehensive assessment of children's cognitive skills and

achievement that has been shown to have good reliability and validity (McGrew, Werder, & Woodcock, 1991). The split-half reliability for the Applied Problems test is .93 (McGrew et al., 2001). To increase the applicability of the WJ-R, test and cluster content was aligned with core curricular areas and domains specified in federal legislation. The Applied Problems test showed strong concurrent validity at both 54 months, correlating from .54 to .68 with the Numbers and Shapes subtest of the Early Arithmetic, Reading and Learning Indicators (EARLI), and at first grade, in which it was strongly correlated ($r = .79$) with the Number Knowledge Test.

The Applied Problems subtest measured quantitative reasoning, math achievement, and math knowledge. It required children to analyze and solve practical mathematics problems presented orally while being shown pictured objects. Solving the problem required that the child listen to the problem while look at the picture, recognize the procedure to be followed, and perform simple counting and calculations exercises. Some problems included extraneous information, requiring the child to identify both the appropriate mathematical operations and which numbers to include. Items were given to children in order of increasing difficulty, scored either 0 (incorrect answer or non-response) or 1 (correct answer), with established basal and ceiling levels. The same assessment was used at each time point (54 months and 1st grade). Vertically equated W-scores were used, which are ideal when utilizing the assessment at multiple time points due to the statistical values having the same mathematical meaning at all levels and in all areas of measurement (Mather & Jaffe, 2002; McGrew et al., 1991).

Child input. To adjust for the influence of children's behaviors on maternal support, three measures of child input during the interaction were assessed. First, to adjust for children's input that was most likely to prompt maternal numerical support, children's numerical language during the interaction was assessed with a frequency count of the number of instances in which

the child identified a numeral, demonstrated one-to-one counting, or labeled a set of objects. Second, to adjust for children's input that was most likely to elicit maternal spatial concept and general learning support, children's enthusiasm and persistence were assessed with 7-point global rating scales ranging from very low (1) to very high (7). Enthusiasm was a measure of the child's positive affect and engagement over the interaction. Persistence rated the child's level of involvement, lack of distraction, and sustained attention with the materials.

Maternal human capital characteristics. When the children were one month old, mothers reported their years of education. When children were 36 months old, mother's receptive vocabulary skills were directly assessed using the Peabody Picture Vocabulary Test-Revised (PPVT-R). These two characteristics of mothers were included due to being central to our study hypotheses, having demonstrated associations with young children's math achievement in previous research, and comprising conceptual models of human capital skills (Becker, 1964).

Family income. Family income-to-needs was assessed at five time-points prior and concurrent to the measures of maternal math support: 1, 6, 15, 24, and 36 months. At each of the five observations, mothers reported their total household income (annualized) and family size. A ratio of family income-to-needs was computed at each observation by dividing total family income by the poverty threshold for the appropriate family size.

Statistical Analyses

We estimated structural equation models (SEM) in Stata 16 to examine direct and indirect associations between family income, maternal human capital characteristics, maternal support, and children's math school readiness. First, we examined a measurement model (i.e., confirmatory factor model) in which our measured variables were estimated as indicators of the proposed latent variables with no path constraints. Second, we estimated the hypothesized

structural model. Model fit was assessed using the chi-square statistic, the Tucker-Lewis Index (TLI), the comparative fit index (CFI), and the root mean square error of approximation (RMSEA; Browne & Cudeck, 1993; Hu & Bentler, 1999). For both models, all latent constructs were allowed to correlate. Although, SEM is not a solution for all sources of measurement error, one of its strengths is the statistical adjustment for unreliability caused by inter-rater or inter-item measurement errors (DeShon, 1998); estimated associations among latent constructs are disattenuated to correct for measure unreliability.

Missing Data

Within the analytic sample, there was a moderate amount of missing data (i.e., 34% across all participants and variables). On the income, maternal, and outcome measures, missing rates ranged from 0% (maternal education) to 27% (math achievement at first grade). On the measures of maternal support and child input, 15% of the data were missing due to attrition (i.e., families that did not participate in the observation at 36 months). In addition, of the mother-child dyads who participated, 23% did not have sufficient time to play with the third box containing Duplos during the time allotted for the interaction. Thus, the combined effects of attrition and lack of time resulted in 38% missing data on the spatial concept support measure. Based on Little's chi-square statistic for examining missing data, missingness in the present study appeared to be missing at random (MAR; Little, 1988). Following recommendations for working with moderate amounts of data that are MAR, we estimated our structural equation modeling using full information maximum-likelihood (FIML). This method has been shown to produce unbiased estimates and valid model fit information (in comparison to listwise deletion and other imputation techniques) even with moderate amounts of missing data (Enders & Bandalos, 2001).

Results

Descriptive Statistics

Table 1 provides descriptive statistics for each of the study variables [Table 1 here]. In addition, we present zero-order correlations among the study variables in Table S3. Among the maternal support observed indicators, labeling set sizes was significantly correlated with spatial concept support ($r = .21$) and cognitive stimulation ($r = .26$), however was not linked with sensitivity. Conversely, spatial concept support was significantly correlated with sensitivity ($r = .21$) but not with cognitive stimulation. Cognitive stimulation and sensitivity were significantly correlated ($r = .59$), a correlation which was significantly stronger in comparison to all of the others.

As expected, family income-to-needs (averaged from 1 to 36 months) was significantly correlated with children's math skills at 54 months ($r = .36$) and in first grade ($r = .39$). It was also significantly correlated with maternal labeling set sizes ($r = .20$) and spatial concept support ($r = .22$), as well as cognitive stimulation ($r = .25$) and sensitivity ($r = .28$). Family income was not significantly correlated with any of the measures of child input.

Turning to the maternal characteristics, the pattern was similar for maternal verbal skills. That is, maternal PPVT scores were significantly correlated with both measures of children's math school readiness (54 months $r = .28$; first grade $r = .33$) and maternal support of spatial concepts ($r = .22$), as well as cognitive stimulation ($r = .25$) and sensitivity ($r = .30$). One difference is that maternal verbal skills were not significantly correlated with labeling set sizes, though the correlation did not statistically differ from the correlation between family income and labeling set sizes. Also, maternal verbal skills were significantly correlated with child numerical language during the interaction ($r = .19$).

In contrast, while maternal education was positively and significantly associated with children's math school readiness (54 months $r = .27$; first grade $r = .38$) and general learning support (cognitive stimulation $r = .24$; sensitivity $r = .33$), it was not significantly correlated with either measure of maternal math support. However, there was not a statistically significant difference in the correlations between maternal education and math support and between those with family income and maternal verbal skills. Maternal education was also not significantly correlated with any measures of child input during the interaction.

Finally, maternal support of labeling set sizes was significantly correlated with math skills at both time points (54 months $r = .31$; first grade $r = .39$). These correlations were of similar size to the associations between family income-to-needs and children's math skills. Maternal support of spatial concepts was significantly associated with children's math skills at 54 months ($r = .36$), with positive but non-significant associations at first grade. The measures of maternal general learning support were significantly correlated with children's math skills at both time points (cognitive stimulation 54 months $r = .26$, first grade $r = .28$; sensitivity 54 months $r = .32$, first grade $r = .33$). In relation to child input, child enthusiasm at 36 months was significantly correlated with both measures of math skills (54 months $r = .27$; first grade $r = .33$) and child numerical language was significantly correlated with math skills at 54 months ($r = .21$), while child persistence was not significantly correlated with either measure.

The Hypothesized Models

The two steps of the statistical analysis were to (1) estimate a measurement model for the hypothesized latent constructs (Figure 1) and then (2) estimate the hypothesized structural model (shown with the paths depicted in Figure 2). We hypothesized that numerical and spatial concept support would comprise two latent constructs, rather than one latent construct of maternal math

support, based upon the separate bodies of work that distinguish numerical and spatial concept learning and support. Given that each maternal support measure had a single observed indicator, we took a conventional and relatively conservative approach to estimating measurement variance by assuming for each measure that the corresponding error was equal to 15% of that measure's overall variance (Jöreskog & Sörbom, 1993; Schumacker & Lomax, 2010). We used the same approach in the measurement of the child numerical language latent construct, which was comprised of the one observed indicator of child numerical language during the observation. For the remaining five latent constructs, multiple indicators were observed: five for family income (income-to-needs at 1, 6, 15, 24, and 36 months), two for maternal characteristics (verbal skills and education), two for maternal general learning support (cognitive stimulation and sensitivity), two for children's enthusiasm and persistence (enthusiasm and persistence), and two for children's math school readiness (Woodcock Johnson Applied Problems at 54 months and first grade). Upon estimating this measurement model (Figure 1), we found that all of the factor loadings were above a .60, with most over a .70, and the global fit statistics indicated a good fit to the data ($\chi^2(76) = 84.46$, $p = .237$; RMSEA = .028; CFI = .992; TLI = .987) [Figure 1 here].

The Structural Model

The resulting structural model examined the direct and indirect pathways of association between the family income, maternal characteristics, maternal numerical support, maternal spatial concept support, and maternal general learning support with children's math school readiness, while also controlling for children's numerical language and enthusiasm and persistence (shown in Figure 2). We compared this structural model with the null model (i.e., the measurement model depicted in Figure 1). The resulting structural model found that the data fit well, $\chi^2(83) = 92.80$, $p = .217$; RMSEA = .029; CFI = .990; TLI = .986. Figure 2 presents

standardized estimates for the path coefficients; significant ($p < .05$) paths are depicted with a solid line while trend ($p < .10$) and null paths are shown with a dotted line [Figure 2 here]. In total, the estimated structural model accounted for 59% of the variance in children's math school readiness. Table 2 depicts unstandardized coefficients, standard errors, and standardized estimates for each of the direct paths between the predictors and mediators with children's math school readiness. Table 3 presents the total and indirect effects, computed using the product of coefficients [Tables 2 and 3 here].

Findings from the structural model indicated that greater family income was associated with greater maternal numerical support ($\beta = .33, p < .05$) and greater maternal numerical support was, in turn, predictive of children's math school readiness ($\beta = .40, p < .01$). In contrast, family income was only marginally associated with spatial concept support and this support was not a significant predictor of children's math school readiness. There were also no significant links between family income and maternal general learning support, contrary to expectations. Note, however, that the strength of the pathways between family income and each of the three latent constructs of maternal support did not significantly differ from one another.

As hypothesized, maternal human capital characteristics were significantly associated with maternal general learning support ($\beta = .49, p < .01$) and greater maternal general learning support was linked with children's math school readiness ($\beta = .31, p < .05$). Surprisingly, there were no significant links between maternal human capital characteristics and either latent construct of maternal math support (however, these pathways also did not differ statistically in comparison to the pathway with maternal general learning support). After accounting for the role of the maternal support latent constructs, the paths between family income and maternal human capital characteristics with children's math school readiness were not significant. Moreover, the

confidence intervals for the pathway from family income to maternal numerical support (i.e., 95% CI = .027- .250) versus the pathway from maternal human capital to maternal numerical (i.e., 95% CI = -.029 to .008) were not overlapping, indicating that these pathways differed significantly from each other.

Indirect and total associations. An examination of indirect effects from family income and maternal characteristics to children's math school readiness revealed one significant indirect effect. Shown in Table 3, the indirect path of association between family income and numerical support and between numerical support and children's math school readiness was significant ($\beta = .13, p < .05$). The indirect path between maternal human capital characteristics and maternal general learning support and between maternal general learning support and children's math school readiness was also significant ($\beta = .15, p < .05$). All of the other indirect paths were non-significant. The total effects (bottom of Table 3) showed that family income had a marginal total effect on children's math school readiness that did not meet the standards of statistical significance ($\beta = .21, p < .10$) while the total effect of maternal human capital was significant ($\beta = .31, p < .05$).

Additional specifications. Following an examination of the main model, we conducted several additional specifications. First, given that three of the latent constructs only had one observed indicator and we assumed that each had an error equal to 15% of that measure's overall variance, we also estimated a model in which each of the factor loadings for these three measures was set to 1. Here, we also found the results to be the same (Figure S1).

We also examined the results of our main structural model on three subsamples to test the robustness of the results within these groups. Given that there was missing data on the spatial concept measure due to lack of time with Box 3, we reran the model on the subset of cases with

complete data on this measure ($n = 91$). We also reran the model on sample of children who had at least 3 minutes of time with Box 3 ($n = 75$). In both cases, we found the pattern of results to be the same as was found with the full sample. Finally, due to evidence of different patterns in the relations between parental support and children's math school readiness across racial and ethnic groups (Sonnenschein & Galindo, 2015), we ran the structural model on only the White children in the sample ($n = 117$) and found the results replicated. Due to lack of sufficient numbers of children from other racial and ethnic groups in the sample, we were unable to examine the pattern of results within other groups.

Sensitivity analysis. Despite the strength of the analytical modeling strategy, the results are subject to potential omitted variable bias given the non-experimental design. There may be unmeasured factors that influence both maternal support and children's math skills, rather than support per se influencing math skills. To better understand the robustness of the results to potential omitted variable bias, we conducted a causal mediation sensitivity analysis (Dearing & Zachrisson, 2019; Imai, Keele, & Tingley, 2010; Imai, Keele, & Yamamoto, 2010). Using methods developed by Imai and colleagues (Imai, Keele, & Tingley, 2010), we examined how strong a set of confounders would need to be in order to nullify the: 1) direct association between numerical support to math skills and 2) mediation effect from family income to numerical support to math skills. In brief, the direct association was highly robust to potential omitted variable bias, but the indirect effect was somewhat less robust. To nullify the direct association from numerical support to math skills, omitted variables would need to explain at least 24% of the variance in math skills, and result in a negative correlation of $-.60$ between the error terms for the predictor and outcome. Thus, to nullify the result, omitted variables would need to explain at least 50% as much variance in math skills as was explained by all variables in the model,

combined (i.e., $R^2 = .59$). Moreover, omitted variables that are negatively correlated with numerical support and positively correlated with math skills, or vice versa, are difficult to identify through the literature or conjecture. The indirect effect of income to numerical support to math skills, however, was somewhat less robust. To nullify the indirect effect, omitted variables would need to explain at least 11% of the variance in math school readiness and result in a correlation of at least .40 between the error terms for the mediator and outcome. These numbers fall between values that have been identified as robust versus sensitive to omitted variable bias (e.g., Imai et al., Keele, & Tingley, 2010; Imai, Keele, & Yamamoto, 2010). Given the robustness of the association between numerical support and math skills, the link between income and numerical support appeared most sensitive to potential omitted variable bias.

Discussion

The goal of this study was to examine parental math support as a mediator of the association between early childhood family income and children's math school readiness. In doing so, we aimed to contribute to the literature uncovering the contributions of the home environment to income-based gaps in children's early math skills. Existing literature has examined broad measures of cognitive stimulation as key predictors of achievement and specific aspects of parental support that contribute to children's math skills, yet little research has sought to delineate the specific mechanisms through which disparities in school entry math skills are explained by differences in parental math support. One important contribution of the present study was the use of direct observations of maternal math support in two key conceptual domains theorized to underlie children's developing math skills – numerical and spatial concept skills. Including observations of general learning support during the same interaction allowed us to disentangle broad support of children's learning from math-specific support.

Findings revealed that maternal support of children's numerical skills significantly mediated the association between family income and children's counting and calculation skills at school entry. Unexpectedly, maternal human capital characteristics were unrelated to maternal math support, however we found evidence of their influence on this measure of children's math school readiness through general learning support. Maternal spatial concept support played a weaker role, however it positively contributed to the 59% of variance in children's math skills that was explained by the model. Taken together, findings suggest that maternal numerical support is a unique mechanism through which economic disadvantage transfers to school-entry disparities in math school readiness, assessed through counting and calculation skills. Below, we contextualize the results in more detail.

The first pattern of results relates to the roles that family income and maternal human capital characteristics played in predicting maternal support and, in turn, math school readiness. On the one hand, family income was indirectly associated with children's math school readiness via numerical support; higher income predicted greater numerical support (i.e., mothers more frequently labeling set sizes and encouraging, prompting, and responding to their child's labeling of set sizes) and, in turn, greater maternal numerical support predicted higher math school readiness scores. Maternal human capital, on the other hand, was significantly linked with mothers' general learning support and, in turn, higher levels of this support were predictive of children's school entry counting and calculation skills. Unexpectedly, however, there were no links between maternal human capital characteristics and either measure of maternal math support. It is important to note, however, that these associations did not differ significantly from one another, except for the associations between family income and maternal numerical support versus maternal human capital and maternal numerical support.

Importantly, the direct associations of family income and maternal human capital characteristics on children's math school readiness were non-significant, most of their associations with math readiness were accounted for by the three latent constructs of maternal support. These results are consistent with the hypothesis that parental support of specific cognitive competencies may be key to understanding income- and SES-based gaps that emerge during early childhood (Bradley & Corwyn, 2004; Conger et al., 1997; Galindo & Sonnenschein, 2015; Yeung et al., 2002). Although there is a rich body of research on these mechanisms when it comes to children's early language skills (e.g. Hart & Risley, 1995; Rowe & Goldin-Meadow, 2009), there is less knowledge in relation to math. We found evidence of specific types of support that contribute to math skills through the separate latent constructs of maternal numerical, spatial concept, and general learning support. Given that the vast majority of research has utilized broad measures of home quality, these findings suggest that disentangling components of these supports could yield a better understanding of the specific drivers of individual differences in development.

The primary question that drove our study was whether maternal math support mediated the relation between family income and children's math school readiness. We found evidence to support this hypothesis in relation to maternal numerical support, but not in relation to maternal spatial concept support. Specifically, mothers' support of labeling set sizes explained a significant portion of the association between early childhood family income and children's counting and calculation skills at school entry. These findings coincide with that of Levine and colleagues (2010), who found that parental number talk partially explained the association between family SES and preschool-aged children's understanding of the cardinal principle, and suggest that this finding may extend to school entry math skills. While in our analyses spatial

concept support played a positive, yet non-significant mediating role, there is strong evidence that early spatial skills are foundational for math learning (LeFevre et al., 2013; Levine et al., 2012; Mix et al., 2016; Verdine et al., 2017; Zhang & Lin, 2017). The weaker link we found in this study for spatial support may be due to the fact that the math school readiness measures focus on early numerical skills; we suspect that spatial support would more strongly predict later math skills, as these advanced skills increasingly benefit from spatial reasoning (Casey et al., 2015). We see this work as a promising avenue for further research into the contributions of spatial skill support in income-based achievement gaps more broadly.

One specific contribution of our study to the cumulative knowledge on early math learning was the disaggregation of SES into family income and maternal human capital characteristics. Existing research has often used a composite measure of SES, which benefits from the combined and correlated power of these demographic factors but also conflates what could be distinct causes of achievement gaps. Parental education has been implicated in SES-based math skill gaps via the effects of parental math cognitions – expectations, attitudes, and beliefs about the importance of math – on parent math support (Elliott & Bachman, 2018; Ramani et al., 2015). Surprisingly, however, we found that maternal human capital characteristics, including education level, was associated with general cognitive stimulation and support, but not math-specific support. Instead, family income was predictive of individual differences in the math support that children received from their mothers.

Although beyond the scope of the present study, we suspect that our findings for family income were due to the psychosocial repercussions of low-income for parents. The extant literature indicates that low-income families are likely to face multiple barriers when it comes to supporting their children's early math learning, including lack of resources to purchase materials

that might promote math, lack of time to engage children in math supporting activities, and economic stress undermining parents' cognitive and emotional capacities during parent-child interactions (Bradley & Corwyn, 2004; Conger et al., 1997; Yeung et al., 2002). Our findings could prove to be best explained by this last mechanism; economic stress may interfere with parents' capacities to support math learning during parent-child interactions, perhaps even more so than it does general learning support. There is also a potential role in regards to the materials in children's homes. Specifically, it is possible that the cash register and Duplos blocks were more familiar to higher-income mothers and children, thus prompting more stimulating interactions. Other materials more familiar to low-income families may have elicited a different pattern of findings. While these speculations are a matter for future research, as is the unexpected lack of relation between maternal human capital characteristics and math support, the findings in our study highlight the value of disaggregating SES components and the importance of studying both general and math-specific learning supports.

Limitations, Conclusions, and Implications

Our study adds to the cumulative knowledge seeking to identify mechanisms through which the early home environment contributes to income-based gaps in children's achievement. Yet, in interpreting the significance and implications of the results, it is essential to acknowledge the limitations. Each measure of math support was assessed during a brief observation session. Although a brief parent-child interaction is a paradigm that has been used effectively (e.g. Devine, Bignardi, & Hughes, 2016; Martin et al., 2007; NICHD ECCRN, 1999; 2008), it may have introduced measurement error into the maternal support ratings and should be replicated using longer observations. Relatedly, the short interaction resulted in higher rates of missing data on the spatial concept measure. While we used best practices for addressing moderate amounts

of missing data and replicated the analyses using listwise deletion, these missing data are a potential source of bias. It should also be noted that while we measured maternal support within two conceptual domains that comprise early math development (numerical and spatial concept), we used only one observed measure for each construct. Despite conservative approaches to estimating the latent constructs, future research utilizing multiple observed indicators is needed. We also used the same approaches to estimating error in both of the single-item latent constructs, despite one being twice as long in length as the other, emphasizing the need for longer observations with multiple observed indicators. Finally, we did not have a direct measure of children's spatial skills or maternal math skills. Specifically, while our measure of children's math skills was a highly reliable and valid assessment of counting and calculation skills, future research is needed examining predictors of children's spatial abilities. Relatedly, a key ingredient missing in our measure of human capital, and missing from many studies on this topic, was maternal math skills. There is a critical need for more work that directly measures parents' mathematical skills. Human capital that incorporates math and spatial skills may prove to be a stronger predictor of early math related learning support.

Although our sample was relatively large within the domain of comparable parental support observational research, it was still constrained in terms of statistical power. Further, it was not racially, ethnically, or culturally diverse and the observational measures were collected in 1994 prior to expansions in publicly-funded pre-kindergarten and increases in income inequality. Relatedly, while it included two measures of math support, it did not include math support by other family members or examine other ways that families support children's math skills outside of a dyadic mother-child interaction, both of which are likely contributors to sociocultural differences in children's early math environments that differ between racial and

ethnic groups (Sonnenschein & Galindo, 2015). These questions need to be asked in a larger sample that is more contemporary and representative, and within different sociocultural groups, in order to fully understand the ways that modern and diverse families support children's early math and the role of this support in explaining achievement gaps.

It is also important to note that despite the strengths of the data and statistical modeling, these results are correlational for which a concern is potential omitted variables. Based on our sensitivity analyses, unmeasured confounders are a concern particularly for the relation between income and numerical support. Results for the path between numerical support and math skills were robust to potential omitted variable bias. One reason for this may be that we parsed apart the unique contributions of maternal numerical and spatial support from more general parenting quality, having included the latent indicator comprised of mothers' cognitive stimulation and sensitivity levels. The indirect effect from family income to numerical support to math skills appeared less robust, however. It may be that there are unmeasured factors that are causally related to both income and maternal numerical support. For example, it is possible that mothers with better math skills earned more income and provided better numerical support. Nonetheless, the robustness of the numerical support to math skills link is critical indicating that whether or not it is income per se that leads to better numerical support, versus some other factor correlated with income, we have robust evidence that maternal numerical support is the proximal mechanism explaining differences in skills. Thus, even if unmeasured factors were the cause of better support, it is of importance for developmental science and practice to know that maternal numerical support remains the critical proximal mechanism linked with children's math skills.

In the context of these cautions, our findings suggest that maternal support of children's numerical skills during early childhood explain a significant portion of the income-based gap in

children's school-entry counting and calculation skills. Given existing evidence that similar measures of early math skills are highly predictive of children's later achievement across domains and that income-based gaps do not converge after school entry (Duncan et al., 2007; Duncan & Magnuson, 2011; Pagani, et al., 2010), these results have important implications. Interventions seeking to target low-income children's opportunities to receive parent support in labeling set sizes may be beneficial to children's math school readiness. Findings also point to policy remedies that target family incomes in order to influence children's home learning environments. Future research replicating, refining, and contextualizing these associations will enable specific remedies to be designed, thereby illuminating opportunities to reduce math achievement gaps.

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Table 1
Descriptive statistics for observed variables

Variable	<i>M(SD)</i>	Min.	Max.
Family Income			
Income-to-Needs, 1 month	3.26(2.28)	0	11.77
Income-to-Needs, 6 months	4.11(3.09)	0	18.00
Income-to-Needs, 15 months	4.08(3.34)	0	18.37
Income-to-Needs, 24 months	3.88(2.72)	0	14.83
Income-to-Needs, 36 months	3.93(2.97)	0	14.67
Maternal Human Capital Characteristics			
Verbal Skills (PPVT-R)	98.48(16.62)	46	143
Education (years)	14.17(2.62)	8	21
Maternal Math Support, 36 months			
Labeling Set Sizes	0.70(0.83)	0	3
Spatial Concept	1.51(1.06)	0	3
Maternal General Learning Support, 36 months			
Cognitive Stimulation	4.66(1.26)	2	7
Sensitivity	17.40(2.39)	10	21
Child Input, 36 months			
Numerical Verbal Frequency	3.24(3.57)	0	18
Enthusiasm	4.95(1.06)	1	7
Persistence	5.13(1.23)	2	7
Child Math School Readiness			
WJ Applied Problems, 54 months	428.82(16.92)	356	454
WJ Applied Problems, 1st grade	469.87(15.60)	438	505

Note. *M* = Mean; *SD* = Standard Deviation; PPVT-R = Peabody Picture Vocabulary Test - Revised; WJ = Woodcock Johnson, Standard Score.

Table 2

Direct associations with children's math school readiness

	Children's Math School Readiness		
	Unst. Coeff.	Std. error	Std. Coeff.
Family Income	.97	.90	.15
Maternal Human Capital	.21	.15	.22
Numerical Support	6.22**	1.70	.40
Spatial Concept Support	1.81	1.59	.15
General Learning Support	4.42*	1.78	.31

Note: * $p < .05$, ** $p < .01$.

Table 3
Indirect and total effects of family income and maternal human capital
characteristics on children's math school readiness

	Indirect effect		
	Unst. Coeff.	Std. error	Std. Coeff
<u>Indirect paths of association</u>			
1. Family income → maternal numerical support → children's math school readiness	.87*	.43	.13
2. Family income → maternal spatial concept support → children's math school readiness	.18	.22	.03
3. Family income → maternal general learning support → children's math school readiness	-.17	.27	-.03
4. Maternal human capital → maternal numerical support → children's math school readiness	-.07	.06	-.07
5. Maternal human capital → maternal spatial concept support → children's math school readiness	.01	.03	.01
6. Maternal human capital → maternal general learning support → children's math school readiness	.15*	.07	.15
	Total effect		
	Unst. Coeff.	Std. error	Std. Coeff
<u>Total effects on children's math school readiness</u>			
1. Family income	1.84+	.98	.29
2. Maternal human capital	.30*	.15	.31

Note: + $p < .10$, * $p < .05$, ** $p < .01$.

Figure 1. Measurement model loadings and model fit ($\chi^2(76) = 84.46, p = .237$; RMSEA = .028; CFI = .992; TLI = .987).

Figure 2. Structural model of direct and indirect associations linking family income, maternal human capital characteristics, and maternal support with children's math school readiness ($\chi^2(83) = 92.80, p = .217$; RMSEA = .029; CFI = .990; TLI = .986). Solid lines indicate statistically significant associations ($p < .05$); dashed lines indicate null associations. Standardized coefficients are reported, note however that the significance levels represent the significance levels of the unstandardized coefficients and standard errors. + $p < .10$, * $p < .05$, ** $p < .01$.