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Derivation of hydropower rules for multi-reservoir systems and its application for optimal reservoir storage allocation

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Abstract:

Hydropower operating rules can guide the reservoir release and storage for maximizing the hydropower generation. However, the optimal hydropower operating rules are usually derived by numerical hydropower simulation methods. This study focuses on the analytical solution to optimal hydropower rules for storage allocation of a multi-reservoir system. An energy function, namely the E -function, is proposed to describe the objective of hydropower generation. The E -function for a multi-reservoir system can be represented as a function of two variables, i.e., the proportional coefficient (α , which represents the proportional relationship between any two reservoirs in a multi-reservoir system) and the total increment in storage. The α -discriminant deduced from the E -function is used to infer the optimal reservoir storage allocation for the maximum hydropower generation. With the Ankang-Danjiangkou Reservoirs as a case study, the E -function and the α -discriminant were compared with the numerical hydropower simulation. The results indicated that (1) the E -function can be used to estimate the hydropower generation for multi-reservoir systems with acceptable hydropower generation estimation errors between the analytical E -function and the numerical hydropower simulation; and (2) the optimal reservoir storage allocation can be explicitly obtained using the α -discriminant. These findings are helpful for understanding the allocation of reservoir storages in multi-reservoir systems.

Key words: Hydropower operating rules; Multi-reservoir system; Energy function; α -discriminant; Reservoir storage allocation.

1. Introduction

Reservoirs are one of the most effective engineering measures for water resources development and management, and reservoir operation systems contain many decision variables and multiple objectives (Liu et al. 2006; Yeh 1985; Zhang et al. 2018; Zhou et al. 2016). Reservoir operating rules usually play an important role in guiding the reservoir storage and release to obtain the optimal benefit of water resources utilization (Oliveira and Loucks 1997; Sigvaldason 1976). However, determining the operating rules for a multi-reservoir system involves complex decision-making issues and it is extremely difficult to obtain appropriate operating rules for multi-reservoir systems (Ashrafi and Dariane 2017; Labadie 2004; Ostadrahimi et al. 2012).

Because of the development and growing use of optimization algorithms, the numerical solutions of optimal operating rules for multi-reservoir systems can usually be obtained by optimization and simulation techniques (Lerma et al. 2013; Tejada-Guibert et al. 1995). The objective functions of an optimization model are usually determined by maximizing benefits (Li et al. 2014; Liu et al. 2011; Ming et al. 2017), minimizing losses (Chandramouli and Raman 2001; Guo et al. 2013; Hui and Lund 2015) or a combination of benefits and losses (Afzali et al. 2008; Haddad et al. 2008; Wang et al. 2005). The derivation of numerical solutions for optimal operating rules contributes greatly to guiding the release for reservoir operation. However, it is still necessary to focus on the theoretical solutions to the optimized operating rules.

There are analytical solutions to operating rules that are supported by conceptual or mathematical deductions. The basic framework of these rules is to establish an index

that determines the optimal order of water storage (or water supply) for multi-reservoir systems, with the purpose of maximizing the benefit or minimizing the loss. (Draper and Lund 2004; Jiang et al. 2016; Johnson et al. 1991; Lund and Guzman 1999; Zeng et al. 2015). Water storage rules, flood control rules and hydropower production rules are several common conceptual rules applied to reservoirs in series (Kelman et al. 1989; Lund and Guzman 1999). For parallel reservoirs, the reservoir operation process can be guided by New York City rules, space rule, flood control balancing rules and hydropower production rules (Clark 1950; Johnson et al. 1991; Nalbantis and Koutsoyiannis 1997; Zeng et al. 2015).

The specific objective of the steady-state hydropower production rule is to maximize hydropower generation during a single time-step when a total storage target for the system is given, and the rule is primarily applied to the drawdown season (Lund and Guzman 1999). This rule prefers to allocate the storage to those reservoirs that create a higher head per unit volume of storage, which is represented as a ratio V_i for each reservoir i in the multi-reservoir system. Then, reservoirs are filled from highest to lowest values of V_i until the total water storage target is met. Based on the ratio V_i , Guo et al. (2009) put forward a new optimal hydropower operating rule for cascade reservoirs operation to obtain the optimized combined reservoir operation curves. However, this rule does not consider possible changes in spilled water caused by changes in reservoir storage during the derivation process.

Lund (2000) derived power production and energy drawdown rules for multi-reservoir systems based on the rule presented by Sheer (1986). The economic value of

hydropower z for a multi-reservoir system contains three parts, i.e., present power generation, the average power generation from the end of current time-step until refill, and the power generation between refill and the beginning of the next drawdown. The marginal economic value of release for each reservoir $\partial z / \partial R_i$ is used as an indicator to guide the storage for multi-reservoir systems. The rule is to draw down reservoirs with the greatest value of $\partial z / \partial R_i$ first, and to refill them in the reverse order.

The discriminant coefficient method (DCM) is the joint operating rule of cascade reservoirs (Jiang et al. 2016). The two main parameters of DCM are the calculated total output N_c and the required total output of the cascade system N_q , respectively, and comparison of the results between N_c and N_q can be used to determine whether the cascade system needs to supply or store water. The discriminant coefficient K_i can then be calculated by considering the extra energy obtainment of unit electricity generation, which is used to guide the releases for each reservoir of the cascade system. Moreover, Jiang et al. (2016) combined DCM with the energy storage operation curves of the cascade reservoirs, and proposed a new DCM based drawing model. In general, the above three hydropower rules only allow decisions regarding the order of priority of fill or drawdown without considering the condition of filling or drawing down multiple reservoirs simultaneously.

Therefore, the overall purpose of this study is to put forward hydropower operating rules based on mathematical analysis that may be used to directly guide the allocation of the reservoir storage in a multi-reservoir system in both flood season and/or refill-drawdown season. The specific aim of this study is to formulate an energy function,

namely the E -function, which describes the objective of hydropower generation, and can be used to estimate the hydropower generation for a multi-reservoir system. Moreover, the alpha-discriminant deduced from the E -function may be used to explicitly infer the optimal reservoir storage allocation scheme for the maximum hydropower generation.

The remainder of this paper is organized as follows. Section 2 describes the establishment of the E -function and the derivation of the alpha-discriminant. Section 3 depicts a case study of the Ankang-Danjiangkou Reservoirs. The results and discussions are presented in Section 4 and Section 5, respectively, and the conclusions are given in Section 6.

2. Methodology

The analytical E -function can be used to directly estimate the total hydropower generation, and the alpha-discriminant is used to obtain the optimal reservoir storage allocation scheme with the purpose of maximizing the hydropower generation. Then, reservoir releases can be given by the water balance equation if change values of the reservoir storage have been given through alpha-discriminant. The assumptions that have been made for deriving the E -function, are given in Section 2.1. The derivation of the E -function for a single-reservoir system and a multi-reservoir system is shown in Sections 2.2 and 2.3, respectively. Furthermore, Section 2.3.3 describes the derivation of the alpha-discriminant for a multi-reservoir system.

2.1 Assumptions

The E -function in this study can also be used to describe the increased benefit of hydropower generation caused by optimal storage allocation scheme if the benefit corresponding to the current scheme is known. The increment in hydropower generation is related to two factors, the change in power head and the possible change in power water volume caused by the change in storage. Two assumptions can be made for the derivation of E -function:

(1) The change in reservoir storage during the time period T is neglected. Time period T refers to the operation time horizon at which the values of reservoir storage are set at the optimal storage allocation scheme, without focusing on the intermediate change process of the reservoir storage, which changes from the current scheme to the optimal scheme.

(2) The relationship between the reservoir water level and the water stored in the reservoir is represented as a power function form (Mohammadzadeh-Habili and Heidarpour 2010), and is shown in Eq. (1) (Appendix S2 shows fitting results for the storage-capacity curve):

$$Z=H(S)=aS^b \quad (a>0, 0<b<1) \quad (1)$$

where Z is the reservoir water level, S represents the water stored in the reservoir, a and b are the parameters of this power function, and $H(\cdot)$ is the function name.

2.2 Derivation of the E -function for a single-reservoir system

2.2.1 Without considering the spilled water

Without considering the maximum power output limits of a generator, the power discharge is equal to the reservoir release. Then, the reservoir hydropower generation per unit time is represented as follows (Zhao et al. 2015):

$$N_t = \eta \cdot \bar{h}_t \cdot q_t \quad (2)$$

$$\bar{h}_t = \frac{1}{2} [H(S_t) + H(S_t + Q_t - q_t)] - h_w(q_t) - h_s(q_t) \quad (3)$$

where η is the comprehensive efficiency coefficient, $\bar{h}_t$ is the average power head at time t , Q_t represents the reservoir inflow at time t and q_t is the reservoir release at time t , Δt is the unit time interval (one hour), $h_w(\cdot)$ represents the tailrace water level, and $h_s(\cdot)$ is the power head loss.

If the increment of the water stored in the reservoir is ΔS_t , the new water stored in the reservoir S_t^* is equal to $S_t + \Delta S_t$. Then, the new reservoir hydropower generation per unit time N_t^* can be represented as follows:

$$N_t^* = f(S_t^*, q_t) = \eta \cdot q_t \cdot \left[\frac{1}{2} H(S_t^*) + \frac{1}{2} H(S_t^* + Q_t - q_t) - h_w(q_t) - h_s(q_t) \right] \quad (4)$$

The increment in the reservoir hydropower generation per unit time $N_t^* - N_t$, and the increment in the reservoir hydropower generation during time period T ($t=1, 2, \dots, T$), which is represented as $E^* - E$, are deduced as Eqs. (5) and (6), respectively.

$$\begin{aligned} N_t^* - N_t &= \frac{\eta}{2} \cdot q_t \cdot [H(S_t^*) + H(S_t^* + Q_t - q_t) - H(S_t) - H(S_t + Q_t - q_t)] \\ &= \frac{\eta}{2} \cdot q_t \cdot (S_t^* - S_t) [H'(S_t) + H'(S_t + Q_t - q_t)] \end{aligned} \quad (5)$$

$$E^* - E = \Delta E = \frac{\eta}{2} \cdot \Delta t \cdot \sum_{t=1}^{t=T} q_t \cdot (S_t^* - S_t) \cdot [H'(S_t) + H'(S_t + Q_t - q_t)] \quad (6)$$

where E and E^* are the present and new hydropower generation, respectively, and $H'(\cdot)$ is the first derivative of $H(\cdot)$. In addition, the term “present (current)” means the parameter values corresponding to the current scheme, while the term “new” refers to the optimal scheme.

Based on the first assumption, the relationship between the new and present water stored in the reservoir during time t could be deduced as Eq. (7).

$$S_t^* - S_t = S_1^* - S_1 \quad (7)$$

where S_1^* and S_1 are the new and present water stored in the reservoir at the beginning of the time period T , respectively, and q_i^* represents the release corresponding to the optimal scheme. The increment in the reservoir hydropower generation ΔE is represented as Eq. (8).

$$\begin{aligned} \Delta E &= \frac{\eta}{2} \cdot \Delta t \cdot (S_1^* - S_1) \cdot \sum_{t=1}^{t=T} q_t \cdot [H'(S_t) + H'(S_t + Q_t - q_t)] \\ &= \eta \cdot \Delta t \cdot (S_1^* - S_1) \cdot \sum_{t=1}^{t=T} q_t \cdot H'(S_t) + \frac{\eta}{2} \cdot \Delta t \cdot (S_1^* - S_1) \cdot \sum_{t=1}^{t=T} q_t \cdot (Q_t - q_t) \cdot H''(S_t) \end{aligned} \quad (8)$$

$$\text{where } \sum_{t=1}^{t=T} q_t \cdot H'(S_t) = H'(S_1) \cdot (q_1 + q_2 + \dots + q_T) + H''(S_1) \cdot \sum_{t=2}^{t=T} q_t \cdot \sum_{k=1}^{k=t-1} (Q_k - q_k) \quad ,$$

$$\sum_{t=1}^{t=T} q_t \cdot (Q_t - q_t) \cdot H''(S_t) = H''(S_1) \cdot \sum_{t=1}^{t=T} q_t \cdot (Q_t - q_t), \text{ and } H''(\cdot) \text{ is the second derivative}$$

of $H(\cdot)$. Based on the first assumption and water balance equation, the E -function for a single-reservoir system can be simplified as Eq. (9) if spilled water is also ignored.

$$E^* = E + \eta \cdot (S_1^* - S_1) \cdot H'(S_1) \cdot \sum_{t=1}^{t=T} q_t \cdot \Delta t = E + \eta \cdot \frac{W}{T_H} \cdot [H(S_1^*) - H(S_1)] \quad (9)$$

where W is the reservoir inflow volume, and T_H is a parameter used for dimensional

transformation between the left side and the right side of Eq. (9). The value of T_H is 3600 s/h.

2.2.2 Considering the spilled water

If the constraint of the maximum power output limit of a generator is considered into the derivation process, then spilled water needs to be included. Furthermore, the increment of the water stored in the reservoir ΔS_t would cause the decrement of the spilled water volume ΔW . It is assumed that the reservoir power output $N_{\max}$ is one of the points in the curve for the maximum power output limit of a generator, and then $N_{\max}$ can be represented by $N_{\max} = \eta \cdot H(S_1) \cdot q = \eta \cdot H(S_1^*) \cdot q^*$. Therefore, the increment in the power discharge can be deduced by

$$\Delta q = q^* - q = \frac{H(S_1)}{H(S_1^*)} \cdot q - q = \frac{H(S_1) - H(S_1^*)}{H(S_1^*)} q, \text{ and the decrement of the spilled water}$$

volume ΔW is as follows:

$$\Delta W = \frac{H(S_1) - H(S_1^*)}{H(S_1^*)} (W_{IN} - W_{SP}) \quad (10)$$

Where W_{IN} is the reservoir inflow volume, and W_{SP} is the spilled water volume corresponding to the present water stored in the reservoir S_1 . Thus, the E -function for a single-reservoir system, which has taken the spilled water into consideration, can be determined as follows:

$$E^* = E + \eta \cdot \frac{(W_{IN} - W_{SP})}{T_H} \cdot \frac{H(S_1)}{H(S_1^*)} \cdot [H(S_1) - H(S_1^*)] \quad (11)$$

2.3 Derivation of the E -function for a multi-reservoir system

The two-reservoir system is taken as an example to address the derivation process

of the E -function and the alpha-discriminant for a multi-reservoir system, because the derivation results of the E -function can be easily expanded to a multi-reservoir system that contains more than two reservoirs.

2.3.1 Without considering the spilled water

Without considering the spilled water, the new total hydropower generation E_{Total}^* for two reservoirs in series or in parallel can be deduced as follows:

$$E_{Total}^* = E_{Total} + \eta_1 \cdot \frac{W_{IN1}}{T_H} \cdot [H_1(S_{1,1}^*) - H_1(S_{1,1})] + \eta_2 \cdot \frac{W_{IN2}}{T_H} \cdot [H_2(S_{2,1}^*) - H_2(S_{2,1})] \quad (12)$$

where E_{Total} is the present total hydropower generation, η_i is the comprehensive efficiency coefficient of the i th reservoir ($i = 1, 2$), W_{INi} is the inflow volume of the i th reservoir, $S_{i,1}$ and $S_{i,1}^*$ are the present and new water stored in the reservoir, respectively, of the i th reservoir at the beginning of the time T , and $H_i(\cdot)$ is the reservoir water level of the i th reservoir.

It is assumed that the relationship between the total increment in available storage for flood control for a multi-reservoir system ΔV_{Total} and the increment in flood storage for the upstream reservoir in series (or the left reservoir in parallel) ΔV_1 can be represented as Eq. (13).

$$\Delta V_1 = \Delta V_{Total} \cdot \alpha \quad (13)$$

where α is the proportional coefficient. Then the increment in flood storage for an additional reservoir ΔV_2 is equal to $\Delta V_{Total} \cdot (1 - \alpha)$. Thus, the E -function for a multi-reservoir system that does not take spilled water into consideration can be given by:

$$E_{Total}^* = E_{Total} + \eta_1 \cdot \frac{W_{IN1}}{T_H} \cdot [H_1(S_{1,1} - \Delta V_{Total} \cdot \alpha) - H_1(S_{1,1})] + \eta_2 \cdot \frac{W_{IN2}}{T_H} \cdot \{H_2[S_{2,1} - \Delta V_{Total} \cdot (1 - \alpha)] - H_2(S_{2,1})\} \quad (14)$$

2.3.2 Considering the spilled water

When the constraint of the maximum power output limit of a generator is considered, the new total hydropower generation E_{Total}^* can be deduced as follows:

$$E_{Total}^* = E_{Total} + \lambda_1 \left[1 - \frac{H_1(S_{1,1})}{H_1(S_{1,1} - \Delta V_{Total} \cdot \alpha)} \right] + \lambda_2 \left[1 - \frac{H_2(S_{2,1})}{H_2(S_{2,1} - \Delta V_{Total} \cdot (1 - \alpha))} \right] \quad (15)$$

where $\lambda_i = \eta_i \cdot \frac{W_{INi} - W_{SPi}}{T_H} \cdot H_i(S_{i,1}) > 0$, W_{INi} is the inflow volume of the i th reservoir, and W_{SPi} is the spilled water volume of the i th reservoir corresponding to the present water stored in the reservoir $S_{i,1}$.

2.3.3 Alpha-discriminant for reservoir storage allocation

If the present water stored in the reservoir, the present total hydropower generation, and the present power water volume have been given to describe the present state of a multi-reservoir system, then α and ΔV_{Total} are two variables of the E -function for multi-reservoir systems (Eqs. (14) and (15)). Then, the E -function can be represented by Eq. (16).

$$E_{Total}^* = F(\Delta V_{Total}, \alpha) \quad (16)$$

Spilled water is usually produced during the hydropower generation operation process, and consequently the E -function that considered the spilled water is chosen to deduce the relationships between E_{Total}^* and α (or ΔV_{Total}). Because α and ΔV_{Total} are independent variables, the analysis for searching for the maximum E_{Total}^* can be separated into two parts. Part one is to analyze the relationship between E_{Total}^*

249 and ΔV_{Total} , and part two is to analyze the relationship between E_{Total}^* and α .

250 Eq. (17), which is deduced from Eq. (15), indicates that E_{Total}^* decreases when

251 ΔV_{Total} increases, and that ΔV_{Total} has a diminishing marginal contribution to E_{Total}^* .

252 The relationship between E_{Total}^* and ΔV_{Total} is in accordance with the conflicting

253 relationship between water conservation and flood control.

$$254 \quad \begin{cases} \frac{\partial E_{Total}^*}{\partial \Delta V_{Total}} < 0 \\ \frac{\partial^2 E_{Total}^*}{\partial \Delta V_{Total}^2} < 0 \end{cases} \quad (17)$$

255 Eq. (18) is deduced from Eq. (15). The term $L(\alpha)$ (refer to Eq. (19)) is used to

256 analyze where the maximum E_{Total}^* can be obtained. The coefficient of proportionality

257 α has a diminishing marginal contribution to E_{Total}^* , but the sign of $\frac{\partial(E_{Total}^*)}{\partial \alpha}$ depends

258 on the value of α .

$$259 \quad \begin{cases} \frac{\partial(E_{Total}^*)}{\partial \alpha} = -\Delta V_{Total} \cdot L(\alpha) \\ \frac{\partial^2(E_{Total}^*)}{\partial \alpha^2} < 0 \end{cases} \quad (18)$$

$$260 \quad L(\alpha) = \lambda_1 \cdot S_{1,1}^{b_1} \cdot b_1 \cdot (S_{1,1} - \Delta V_{Total} \cdot \alpha)^{-b_1-1} - \lambda_2 \cdot S_{2,1}^{b_2} \cdot b_2 \cdot [S_{2,1} - \Delta V_{Total} \cdot (1-\alpha)]^{-b_2-1} \quad (19)$$

261 The value of α is between 0 and 1 because it reflects the flood storage allocation

262 of a multi-reservoir system. A summary of the relationship between the maximum

263 E_{Total}^* and α can be given as follows, which is called alpha-discriminant:

264 (1) If $L(0) \cdot L(1) \geq 0$ and $L(0) \cdot \Delta V_{Total} \geq 0$, then E_{Total}^* of the E -function (Eq.

265 (15)) can acquire the maximum value $E_{Total}^* = E_{Total} + \lambda_2 \left[1 - \frac{H_2(S_{2,1})}{H_2(S_{2,1} - \Delta V_{Total})} \right]$ where

266 $\alpha = 0$.

(2) If $L(0) \cdot L(1) \geq 0$ and $L(1) \cdot \Delta V_{Total} \leq 0$, then E_{Total}^* of the E -function (Eq.

(15)) reaches the maximum value $E_{Total}^* = E_{Total} + \lambda_1 \left[1 - \frac{H_1(S_{1,1})}{H_1(S_{1,1} - \Delta V_{Total})} \right]$ where

$\alpha = 1$.

(3) If $L(0) \cdot L(1) < 0$, E_{Total}^* of the E -function (Eq. (15)) obtains its maximum

value $E_{Total}^* = E_{Total} + \lambda_1 \left[1 - \frac{H_1(S_{1,1})}{H_1(S_{1,1} - \Delta V_{Total} \cdot \alpha^*)} \right] + \lambda_2 \left\{ 1 - \frac{H_2(S_{2,1})}{H_2[S_{2,1} - \Delta V_{Total} \cdot (1 - \alpha^*)]} \right\}$

when a value of alpha $\alpha^* \in (0,1)$ satisfies equation $L(\alpha^*) = 0$, and the value of α^*

would be different when ΔV_{Total} varies.

The operating rules put forward in Section 2 can be applied to guiding reservoir storage for multi-reservoir systems during refill-drawdown seasons if the time period is set as a unit time interval (e.g., one hour, one day). This rule can also be used for optimizing the flood storage allocation during flood seasons because assumptions made for this rule are satisfied for this operation period, i.e., the ending storage is equal to the initial storage which corresponds to the available storage for flood control.

3. Case study

Fig. 1 shows that the Hanjiang River is located at $106^\circ 15' \sim 114^\circ 20'$ east longitude, $30^\circ 10' \sim 34^\circ 20'$ north latitude in China. The Hanjiang River is the largest tributary of the Yangtze River, and has a basin area of about 159,000 km². The river crosses most of Hubei Province, and flows from the northwest to the southeast (Yang et al. 2016). The Ankang and Danjiangkou Reservoirs are adjacent reservoirs in the main stream of the Hanjiang River, and the Ankang-Danjiangkou reservoirs in series are used for a case study in this study. The flood season of these two reservoirs is from late June to middle

October, and it is divided into two stages: (1) the summer flood season is from late June to late August, and (2) the autumn flood season is from early September to mid-October. The flood storage allocation schemes for the summer and autumn flood seasons deduced by the E -function are referred to as Scheme A1 and Scheme B1, respectively, while the corresponding flood storage allocation schemes obtained by the numerical hydropower simulation are referred to as Scheme A2 and Scheme B2, respectively.

[Insert Figure 1 here]

4. Results

The results for deriving the optimal flood storage allocation scheme through the E -function and the alpha-discriminant are shown in Section 4.1. The numerical hydropower simulation methods for the Ankang-Danjiangkou Reservoirs are shown in Appendix S1. Section 4.2 describes comparison results derived by the analytical E -function and the numerical hydropower simulation.

4.1 Derivation of flood storage allocation using the E -function

4.1.1 Summer flood season (Scheme A1)

The new average annual hydropower generation during the summer flood season $\overline{E_{Total}^*}$ for the Ankang-Danjiangkou Reservoirs system was estimated by Eq. (15), and the characteristic parameter values for Scheme A1 are shown in Table 1. For the Ankang and Danjiangkou Reservoirs, the storage-capacity curves were represented by $Z_1 = a_1 S_1^{b_1} = 248.931 S_1^{0.0884}$ and $Z_2 = a_2 S_2^{b_2} = 78.091 S_2^{0.1361}$, respectively (see Appendix S2). Then, the E -function for Scheme A1 can be represented as follows:

$$\overline{E_{Total}^*} = 8.14 + \lambda_1 \left[1 - \frac{20.75^{0.0884}}{(20.75 - \alpha \Delta V_{Total})^{0.0884}} \right] + \lambda_2 \left\{ 1 - \frac{198.2^{0.1361}}{[198.2 - (1 - \alpha) \Delta V_{Total}]^{0.1361}} \right\} \quad (20)$$

where $\lambda_1 = \eta_1 \cdot \frac{\overline{W_{IN1}} - \overline{W_{SP1}}}{T_H} \cdot H_1(S_{1,1}) = 10.51 \times 10^8 \text{ kW h}$ and $\lambda_2 = 13.69 \times 10^8 \text{ kW h}$.

According to the alpha-discriminant, the maximum $\overline{E_{Total}^*}$ was obtained using Eq. (21) where $\alpha = 0$, because $L(0) \cdot L(1) \geq 0$ and $L(0) \cdot \Delta V_{Total} \geq 0$.

$$\overline{E_{Total}^*} = 8.14 + \lambda_2 \left\{ 1 - \frac{198.2^{0.1361}}{[198.2 - (1 - \alpha) \Delta V_{Total}]^{0.1361}} \right\} \quad (21)$$

where $\lambda_2 = 13.69 \times 10^8 \text{ kW h}$. The results indicate that if the flood storage increases, then ΔV_{Total} should be totally allocated to the downstream reservoir, and the loss of hydropower generation would be minimized.

Fig. 2 shows the results derived from the E -function for the summer flood season (Scheme A1). The vertical ordinate represents $\overline{E_{Total}^*}$, the horizontal ordinate is ΔV_{Total} , and different lines are corresponding to different values of α . Three points can be made:

(1) The term $\overline{E_{Total}^*}$ decreases when ΔV_{Total} increases, which is in accordance with the conflictual relationship between power benefit and flood control.

(2) The maximum $\overline{E_{Total}^*}$ is acquired where $\alpha = 0$ because the alpha-discriminant $L(0) \cdot L(1) \geq 0$ and $L(0) \cdot \Delta V_{Total} \geq 0$. The higher the proportional coefficient α , the smaller the $\overline{E_{Total}^*}$.

(3) There exists a boundary of ΔV_{Total} when $\alpha = 0.3 \sim 1.0$ because the flood storage of the upstream reservoir (Ankang Reservoir) reaches its upper limit (The upper limit for flood storage of the Ankang Reservoir is 2.20 billion m^3 when all the storage between the normal pool water level and the dead water level is used as flood storage).

[Insert Table 1 and Figure 2 here]

4.1.2 Autumn flood season (Scheme B1)

The new average annual hydropower generation during the autumn flood season $\overline{E_{Total}^*}$ for the Ankang-Danjiangkou Reservoirs was obtained using Eq. (15), and the characteristic parameter values for Scheme B1 are shown in Table 1. The E -function for Scheme B1 was represented as follows:

$$\overline{E_{Total}^*} = 6.08 + \lambda_1 \left[1 - \frac{22.186^{0.0884}}{(22.186 - \alpha \Delta V_{Total})^{0.0884}} \right] + \lambda_2 \left\{ 1 - \frac{228^{0.1361}}{[228 - (1 - \alpha) \Delta V_{Total}]^{0.1361}} \right\} \quad (22)$$

where $\lambda_1 = 8.59 \times 10^8$ kW h and $\lambda_2 = 5.93 \times 10^8$ kW h.

Because the alpha-discriminant for Scheme B1 satisfies $L(0) \cdot L(1) \geq 0$ and $L(0) \cdot \Delta V_{Total} \geq 0$, the maximum $\overline{E_{Total}^*}$ was obtained using Eq. (23) where $\alpha = 0$.

$$\overline{E_{Total}^*} = 6.08 + \lambda_2 \left\{ 1 - \frac{228^{0.1361}}{[228 - (1 - \alpha) \Delta V_{Total}]^{0.1361}} \right\} \quad (23)$$

where $\lambda_2 = 5.93 \times 10^8$ kW h.

Fig. 3 shows the results derived from the E -function for the autumn flood season. The vertical ordinate represents $\overline{E_{Total}^*}$, the horizontal ordinate is ΔV_{Total} , and different lines are corresponding to different values of α . Three similar points as in the Fig. 2 can be concluded from Fig. 3.

[Insert Figure 3 here]

4.2 Comparisons of E -function and numerical hydropower simulation

4.2.1 Summer flood season (Scheme A1 and Scheme A2)

Fig. 4 shows the comparison of the E -function (Scheme A1) and the numerical

hydropower simulation (Scheme A2). Four main points can be drawn from the comparison of Scheme A1 and Scheme A2:

(1) For both Scheme A1 and Scheme A2, the maximum average annual hydropower generation during the summer flood season is obtained where $\alpha = 0$.

(2) For both Scheme A1 and Scheme A2, there is a boundary for the total increment in flood storage ΔV_{Total} , because the flood storage of the upstream reservoir (Ankang Reservoir) reaches its upper limit.

(3) Compared with Scheme A1, Scheme A2 considers the constraints of flood control, and thus there is a risk boundary for Scheme A2. If ΔV_{Total} is totally allocated to the downstream reservoir, flood risk would occur when ΔV_{Total} is greater than 0.5 billion m^3 .

(4) The hydropower generation estimation errors between Scheme A1 and Scheme A2 are acceptable, which are shown in Table 2. The estimation errors are caused by two assumptions made in Section 2.1. Because the storage-capacity curve fits well based on the second assumption (see Appendix S2), the main reason for the production of

estimation errors is that two terms $H''(S_1) \cdot \sum_{t=2}^{t=T} q_t \cdot \sum_{k=1}^{k=t-1} (Q_k - q_k)$ and

$\sum_{t=1}^{t=T} q_t \cdot (Q_t - q_t) \cdot H''(S_t)$ are neglected according to the first assumption (referred to Eq.

(8)).

[Insert Figure 4 and Table 2 here]

4.2.2 Autumn flood season (Scheme B1 and Scheme B2)

The comparison of the E -function (Scheme B1) and the numerical hydropower

simulation (Scheme B2) for the autumn flood season is shown in Fig. 5. Main summaries concluded regarding the comparison of Scheme B1 and Scheme B2 are similar to that of the comparison of Scheme A1 and Scheme A2, except for two points as follows:

(1) Compared with Scheme B1, Scheme B2 considers the constraints of flood control, and thus there is a risk boundary for the conditions where $\alpha = 0.7 \sim 1.0$. For example, if $\alpha = 0.7$, flood risk would occur when ΔV_{Total} is greater than 1.3 billion m^3 .

(2) The estimation errors for hydropower generation between Scheme B1 and Scheme B2 are acceptable, which are shown in Table 3.

[Insert Figure 5 and Table 3 here]

5. Discussions

The value of the alpha-discriminant $L(\alpha)$ is influenced by two main factors, i.e., the proportion of flood storage between two reservoirs and the proportion of inflow volume between two reservoirs. Two scenarios were assumed to analyze the effects caused by the proportion of flood storage between the two reservoirs (Scenario 1 in Section 5.1) and the proportion of inflow volume between the two reservoirs (Scenario 2 in Section 5.2), respectively. Both Scenario 1 and Scenario 2 make assumption about the change of parameter values based on Scheme A1.

Compared with the E -function (Scheme A1 and Scheme B1), the numerical hydropower simulation (Scheme A2 and Scheme B2) considers the constraint of flood control. The effects of the flood risk constraint will be discussed in Section 5.3.

5.1 Proportion of flood storage between the two reservoirs

For the purpose of analyzing the effects caused by the proportion of flood storage between these two reservoirs in the case study, the flood storage of Ankang Reservoir in summer is assumed to be 4.76 billion m^3 (i.e., $S_{1,1} = S_{2,1}$), and the relationship between the reservoir water level and the water stored in the reservoir is assumed to be represented as $Z_1 = H(S_1) = a_1 S_1^{b_1} = 216.5 S_1^{0.0884}$. Then, the alpha-discriminant $L(0) \cdot L(1) < 0$, and the maximum value of E_{Total}^* can be obtained when there is a value of alpha $\alpha^* \in (0,1)$ to satisfy equation $L(\alpha^*) = 0$. Fig. 6 indicates that the value of α^* changes when ΔV_{Total} increases, and the value of α^* ranges from 0.35 to 0.52. There is very little difference among the values of E_{Total}^* when the values of α are equal to 0.3, 0.4, or 0.5 (see Fig. 7). Thus, in Scenario 1, the value of α could be 0.45 when the value of ΔV_{Total} is between 0.1 and 0.5 billion m^3 , and the value of α could be 0.35 when the value of ΔV_{Total} is between 0.5 and 3.5 billion m^3 .

In Scheme A1, the slope of the storage-capacity curve for the Ankang Reservoir dZ_1/dV_1 is larger than that of the Danjiangkou Reservoir dZ_2/dV_2 when $dV_1 = dV_2$. Then, the decrement of the power head for the Ankang Reservoir would be larger than that for the Danjiangkou Reservoir when $dV_1 = dV_2$. Thus, it would be better to operationally allocate ΔV_{Total} to the Danjiangkou Reservoir. However, the value of dZ_1/dV_1 in Scenario 1 changes, which influences the relationship between dZ_1/dV_1 and dZ_2/dV_2 , and causes changes to the value of $L(\alpha)$. Therefore, in Scenario 1, the value of the alpha-discriminant can be influenced by the proportion of flood storage between these two reservoirs.

[Insert Figure 6 and Figure 7 here]

5.2 Proportion of inflow volume between the two reservoirs

The proportion of inflow volume between these two reservoirs represents the distribution of the water resources volume used for hydropower generation. The water resources distribution could influence the flood storage allocation for this multi-reservoir system. In the case study, the inflow of the downstream reservoir includes two parts, i.e., the release from the upstream reservoir and the interval inflow.

In Scenario 2, without considering the change of the spilled water, $\overline{W}_{IN1}$ changes to 4.40 billion m^3 and $\overline{W}_{IN2}$ is assumed to be unchanged. Thus, $L(0) = -0.00448 \text{ kWh/m}^3$, $L(1) = -0.00415 \text{ kWh/m}^3$. Because $L(0) \cdot L(1) \geq 0$ and $L(1) \cdot \Delta V_{Total} \leq 0$, the maximum $\overline{E}_{Total}^*$ was obtained where $\alpha = 1$ (see Fig. 8). Because $\overline{W}_{IN1}$ in Scenario 2 is less than that of Scheme A1, the value of λ_1 decreases compared with that of Scheme A1. This may result in $L(1) < 0$. If the proportion of inflow volume between two reservoirs $\overline{W}_{IN1}/\overline{W}_{IN2}$ is large enough, the decrement of hydropower generation caused by the increment in the flood storage for the 1st reservoir may be larger than that caused by the same increment in the flood storage for the 2nd reservoir. Therefore, it may be more appropriate to put all the increment in the flood storage into the 2nd reservoir. Thus, it can be seen in Scenario 2 that the value of alpha-discriminant can be influenced by the proportion of inflow volume between these two reservoirs.

[Insert Figure 8 here]

5.3 Constraint of flood control

In the case study, the total increment in the flood storage ΔV_{Total} cannot be negative when considering the constraint of flood control, because the present total flood storage is regarded as the minimum value when considering flood risk. When verifying the change of the total flood storage for satisfying the constraint of flood control, the maximum water level and the maximum release under the present designed standard are regarded as the upper-limit values for flood control operations. Fig. 9 shows the results for the E -function without considering the constraint of flood control, and the ΔV_{Total} can be a negative value. The maximum $\overline{E_{Total}^*}$ was obtained when $\alpha = 1$ for a negative ΔV_{Total} , and when $\alpha = 0$ for a positive ΔV_{Total} . The results in Fig. 9 indicate that dE_1/dV_1 is less than dE_2/dV_2 in this case study ($dE_1/dV_1 < 0$ and $dE_2/dV_2 < 0$). A positive ΔV_{Total} indicates that the benefits of hydropower generation would be satisfied. In this case, it would be better to allocate ΔV_{Total} to the 2nd reservoir (Danjiangkou Reservoir) to minimize the satisfaction of hydropower generation. In contrast, if the value of ΔV_{Total} is negative, it would be better to allocate ΔV_{Total} to the 1st reservoir (Ankang Reservoir) with the purpose of maximizing the increment in benefits.

[Insert Figure 9 here]

6. Conclusions

This study focuses on the analytical solution to optimal hydropower rules for reservoir storage allocation of a multi-reservoir system. An energy function, the E -function, is proposed to describe the objective of hydropower generation. The E -

function was deduced to estimate the new hydropower generation when the available storage for flood control changes. Two conditions were considered during the derivation process, i.e., with and without considering the spilled water. The alpha-discriminant was deduced from the E -function for a multi-reservoir system and can be used to infer the optimal storage allocation scheme for the maximum hydropower generation. The Ankang-Danjiangkou Reservoirs were used for a case study, and the E -function and the alpha-discriminant were applied to the derivation of the optimal flood storage allocation. Two main findings were made as follows:

(1) The E -function can be used to estimate the hydropower generation for multi-reservoir systems with acceptable estimation errors for hydropower generation between the analytical E -function and the numerical hydropower simulation. In this study, the E -function is regarded as the function of two variables in a multi-reservoir system (regardless of whether the reservoirs are in series or parallel). These two variables are (1) the proportional coefficient α , which represents the proportional relationship between any two reservoir storages in the multi-reservoir system, and (2) the total increment in the flood storage. Moreover, if the storage capacities for any two reservoirs are similar (or at least if the difference between the storage of any two reservoirs is not as large as that of the Ankang and the Danjiangkou Reservoir), the E -function can also be represented by the function of two variables, i.e., the total reservoir storage (but not the increment in the total reservoir storage) and the proportion of reservoir storage between any two reservoirs.

(2) The optimal reservoir allocation scheme can be explicitly obtained using the

alpha-discriminant, rather than by deriving the optimal reservoir storage allocation scheme through the numerical hydropower simulation. The alpha-discriminant can be applied to guiding reservoir storage for multi-reservoir systems during refill-drawdown seasons if the time period set as a unit time interval (e.g., one hour, one day). This rule can also be used for optimal flood storage allocation during flood seasons.

In this study, the E -function itself does not consider the constraint of flood control. Therefore, the optimal flood storage allocation scheme derived from the E -function needs to be validated for the constraint of flood control. The E -function can be applied to dynamic operations of multi-reservoir systems when combining hydrological forecasting techniques. For example, if the future inflow is predicted to be very large in a multi-reservoir system, then the total reservoir storage needs to decrease. Under this condition, the E -function could be used to derive an optimal reservoir storage allocation scheme with the purpose of minimizing the satisfaction of the hydropower generation. Furthermore, the E -function could also be applied to multi-reservoir systems that contain more than two reservoirs. This can be achieved by dividing the multi-reservoir system into several two-reservoir systems based on the aggregation-decomposition method. Otherwise, new parameters are added to represent the proportion of flood storage among these reservoirs, e.g., the proportional coefficients can be represented by α , γ and $1-\alpha-\gamma$ for a three-reservoir system.

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Tables

Table 1 Characteristic parameter values for Scheme A1 and Scheme B1.

Parameters	value		Parameters	value	
	Scheme A1	Scheme B1		Scheme A1	Scheme B1
η_1	8.5	8.5	η_2	8.73	8.73
$S_{1,1}$ (10^8m^3)	20.75	22.186	$S_{2,1}$ (10^8m^3)	198.2	228
$\overline{W_{IN1}}$ (10^8m^3)	56.16	53.93	$\overline{W_{IN2}}$ (10^8m^3)	38.46	14.95
$\overline{W_{SP1}}$ (10^8m^3)	42.48	42.82	$\overline{W_{SP2}}$ (10^8m^3)	3.26	0.0
$\overline{E_1}$ (10^8kW h)	2.67	2.27	$\overline{E_2}$ (10^8kW h)	5.47	3.81

Note: parameters with subscripts 1 and 2 stand for ... and ..., respectively.

Table 2 Hydropower generation estimation errors between Scheme A1 and Scheme A2.

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
SSE (%)	0.01	0.09	0.05	0.09	0.06	0.04	0.03	0.03	0.03	0.02	0.02
MIE (%)	0.01	0.04	0.06	0.09	0.08	0.06	0.04	0.03	0.02	0.01	0.00
MAE (%)	0.27	0.67	0.51	1.05	1.03	0.96	0.91	0.89	0.86	0.84	0.82

SSE stands for the sum of squared errors, MIE stands for the absolute value of the minimum error, and MAE stands for the absolute value of the maximum error.

Table 3 Estimation errors for hydropower generation between Scheme B1 and Scheme B2.

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
SSE (%)	0.27	0.02	0.06	0.06	0.03	0.03	0.03	0.03	0.03	0.03	0.02
MIE (%)	0.04	0.00	0.30	0.51	0.22	0.04	0.08	0.16	0.21	0.25	0.29
MAE (%)	1.50	0.51	0.96	0.56	0.62	0.69	0.74	0.77	0.78	0.82	0.82

SSE stands for the sum of squared errors, MIE stands for the absolute value of the minimum error, and MAE stands for the absolute value of the maximum error.

Figures

Fig. 1. Location of the Hanjiang basin and gauge stations.

Fig. 2. *E*-function results for the summer flood season (Scheme A1).

Fig. 3. *E*-function results for the autumn flood season (Scheme B1).

Fig. 4. Comparison of the E -function (Scheme A1) and the numerical hydropower simulation (Scheme A2) for the summer flood season: Solid lines represent the results derived by the E -function while dotted lines represents the results derived from the numerical hydropower simulation. bc (A1) and bc (A2) represent the boundary curves of Scheme A1 and Scheme A2, respectively.

Fig. 5. Comparison of the E-function (Scheme B1) and the numerical hydropower simulation (Scheme B2) for the autumn flood season: Solid lines represent the results derived by the E-function while dotted lines represents the results derived from the numerical hydropower simulation. bc (A1) and bc (A2) represent the boundary curves of Scheme A1 and Scheme A2, respectively.

Fig. 6. Value of alpha for optimal reservoir storage allocation in Scenario 1.

Fig. 7. Results for E-function in Scenario 1.

Fig. 8. Results for the E-function in Scenario 2.

Fig. 9. Results of the E-function without considering the constraint of flood control.