Digital superresolution in seismic amplitude processing

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SUMMARY

We describe a method for geophysical inversion of seismic amplitude data, we argue that this type of inversion is a natural extension of the processing flow. The methodology is related to digital image-video restoration and single image superresolution. It formulates the inverse problem in term of a regularization and solve it by an augmented Lagrangian approach. It can be seen as an extension of 1D sparse spike inversion to 3D. The approach impose weak geological constraints through the gradient of the earth parameter and is thus particular useful in the exploration setting, and regions with little well control. We test three different regularizations in a synthetic example, and show how this is used for a data set acquired in the Barents sea. We find that the anisotropic total variation regularizer is robust and efficient when it comes to restoring earth models, and outperform the weighted L2 norm when it comes to reproducing discontinuities along smooth edges.

INTRODUCTION

In exploration seismic data is a key driver for regional understanding, creating geological ideas, and generating new plays. Seismic inversion provides a quantification of earth parameters which is used in the risking process. The illposedness of the seismic inverse problem require that additional information must be provided. Recently there has been much attention to methods imposing constraints derived from rock physical relations, see e.g. Gunning and Sams (2018), Fjeldstad and Grana (2018), Grana (2018). In the early exploration setting there is often large uncertainties related to these models, and errors in trends can give erroneous solutions (Rimstad et al. (2012)).

An inversion tuned for exploration should balance the constraints with retaining and enhancing the details and information present in the data, thus we formulate a scheme that fits as a sibling to the typical final stack in data processing, i.e. sharpening and extending the bandwidth of the data. In the inversion framework, we explore regularization based on the gradient of the earth parameter. We use total variation norms and also a weighted L2 norm. The total variation norm can be seen as a natural extension of the geological sparse spike approach in 2D and 3D. Variants of the methodology outlined here is commonly used for digital image restoration, e.g. Yang et al. (2009) and Afonso et al. (2010), thus there have been developed many different algorithms for solving the problem. In this work, we use the augmented Lagrangian approach, as described in Chan et al. (2011).

The geophysical link between earth parameters and geophysical data is often describes by a linear 1D convolution model also when the geological inversion is in a 3D setting, e.g. Buland et al. (2003), Kemper and Gunning (2014). This geo-

physical model is also used in sparse spike methods Liang et al. (2017). The 1D convolutional model have shortcomings, see e.g. Lecomte et al. (2016). In our geophysical model we use a 3D point spread function as presented in Schuster and Hu (2000) as the link between the earth parameter and seismic amplitudes. Together with the 3D constraint this provides an inversion scheme that accounts for 3D features and geometries in the inverse problem.

THEORY

The geophysical signal is linked to the earth parameters through a convolution with a pointspread function,

$$\mathbf{d} = \mathbf{G}\mathbf{m} + \varepsilon. \quad (1)$$

where $\mathbf{d}$ denotes the geophysical data, $\mathbf{m}$ denotes the geophysical model parameter (e.g. impedance), ε denotes the error term, and $\mathbf{G}$ is an operator corresponding to the point-spread function. In a region we analyze we assume that the point-spread function is stationary, thus the operator is diagonalized by the 3D fast Fourier transform. The limited resolution in the point-spread function causes an under-determined system, thus we formulate the problem using a regularization term:

$$\text{minimize: } \frac{\mu}{2} \|\mathbf{d} - \mathbf{G}\mathbf{m}\|_2^2 + \|\mathbf{m}\|. \quad (2)$$

Here $\|\mathbf{d} - \mathbf{G}\mathbf{m}\|_2^2$ is the sum of squares for the residuals, $\|\mathbf{m}\|$ denotes the regularization term, and μ provides a trade off between the two terms. We consider three forms of the regularization. These are,

$$\|\mathbf{m}\|_{WL2} = \sum_{n=1}^N |\mathbf{D}_x \mathbf{m}_n|^2 + |\mathbf{D}_y \mathbf{m}_n|^2 + |\mathbf{D}_t \mathbf{m}_n|^2, \quad (3)$$

$$\|\mathbf{m}\|_{TV} = \sum_{n=1}^N |\mathbf{D}_x \mathbf{m}_n| + |\mathbf{D}_y \mathbf{m}_n| + |\mathbf{D}_t \mathbf{m}_n|, \quad (4)$$

$$\|\mathbf{m}\|_{ATV} = \sum_{n=1}^N \sqrt{|\mathbf{D}_x \mathbf{m}_n|^2 + |\mathbf{D}_y \mathbf{m}_n|^2 + |\mathbf{D}_t \mathbf{m}_n|^2}, \quad (5)$$

where the sum is over all grid cells in the 3D grid, $\mathbf{D}\mathbf{m} = (\mathbf{D}_x \mathbf{m}, \mathbf{D}_y \mathbf{m}, \mathbf{D}_t \mathbf{m})$ is the gradient of the earth parameter, and $\mathbf{D}_j \mathbf{m}_n$ indicates the gradient in the n th location. The norms listed are denoted the weighed L2 norm, the total variation norm, and the anisotropic total variation norm. The general solution of (2) is derived using an augmented Lagrangian approach where we rewrite minimization problem as:

$$\begin{aligned} &\text{minimize: } \frac{\mu}{2} \|\mathbf{d} - \mathbf{G}\mathbf{m}\|_2^2 + \|\mathbf{u}\| \\ &\text{subject to: } \mathbf{u} = \mathbf{D}\mathbf{m}, \end{aligned} \quad (6)$$

where the set of augmented variables are $\mathbf{u} = (\mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_t)$, being defined to match the gradient of the earth parameter, and the norm $\|\cdot\|$ is defined to match the regularization term. The purpose of augmenting the set of unknown is to relax the identity $\mathbf{u} = \mathbf{D}\mathbf{m}$ during the computations and then gradually retrieve

it again. The Lagrangian solution to the relaxed version of expression (6) is the minimization of:

$$\frac{\mu}{2} \|\mathbf{d} - \mathbf{G}\mathbf{m}\|_2^2 + \|\mathbf{u}\| - (\mathbf{u} - \mathbf{D}\mathbf{m})^T \lambda + \frac{\rho}{2} \|\mathbf{u} - \mathbf{D}\mathbf{m}\|_2^2 \quad (7)$$

where λ is the Lagrangian multiplier of the same dimension as $\mathbf{u}$ and ρ provides a slack for the relation $\mathbf{u} = \mathbf{D}\mathbf{m}$. Increasing the value of ρ gives less relaxation on the match. The exact match is theoretically obtained when $\rho = \infty$. The minimum is obtained using an alternating direction method (ADM),

Algorithm 1 Alternating direction method

initialize:

$$\mathbf{u}^0 = \mathbf{0}, \lambda^0 = \mathbf{0}$$

iterate (8), (9), (10) until $\|\mathbf{m}^i - \mathbf{m}^{i-1}\| < \delta \|\mathbf{m}^i\|$:

solve: $\mathbf{m}^i$ as minimum of

$$\frac{\mu}{2} \|\mathbf{d} - \mathbf{G}\mathbf{m}\|_2^2 - (\mathbf{u}^{i-1} - \mathbf{D}\mathbf{m})^T \lambda^{i-1} + \frac{\rho_i}{2} \|\mathbf{u}^{i-1} - \mathbf{D}\mathbf{m}\|_2^2 \quad (8)$$

solve: $\mathbf{u}^i$ as minimum of

$$\|\mathbf{u}\| - (\mathbf{u} - \mathbf{D}\mathbf{m}^i)^T \lambda^{i-1} + \frac{\rho_i}{2} \|\mathbf{u} - \mathbf{D}\mathbf{m}^i\|_2^2 \quad (9)$$

update: λ^i

$$\lambda^i = \lambda^{i-1} - \rho_i (\mathbf{u}^i - \mathbf{D}\mathbf{m}^i) \quad (10)$$

The parameter δ determines how large the relative change is at termination. The parameter ρ_i need to increase during the iterations in order to narrow down the mismatch between the slack variable $\mathbf{u}$ and the gradient $\mathbf{D}\mathbf{m}$. The proposed approach splits the original non-local non-linear problem of expression (2) into one linear de-blurring problem, and one nonlinear local problem. The problem of expression (8) is independent of the form of the regularizer, whereas the problem in expression (9) does not depend on the blurring kernel. The update of λ in expression (10) is standard within the framework of augmented Lagrangian approach, see Hestenes (1969) and Powell (1969).

The first problem can be solved efficiently in Fourier domain, where the explicit solution is,

$$\hat{m}_k = \frac{\mu \bar{\hat{G}}_k \hat{d}_k + \sum_{j=x,y,t} \bar{\hat{D}}_{jk} (\rho \hat{u}_{jk} - \hat{\lambda}_{jk})}{\mu |\hat{G}_k|^2 + \rho \sum_{j=x,y,t} |\hat{D}_{jk}|^2}, \quad (11)$$

here k numerates all atoms in the frequency-wavenumber domain, the hat denotes 3D Fourier transform of $\hat{m}_k, \hat{d}_k, \hat{u}_{jk}, \hat{\lambda}_{jk}$ whereas for $\hat{G}_k, \hat{D}_{jk}$ it should be interpreted as the eigenvalues of the corresponding operators.

The second problem splits directly into problems for each spatial location in geophysical domain. Define the quantities $w_{jn} = \mathbf{D}_j \mathbf{m}_n + \lambda_{jn} / \rho$, and $w_n = \sqrt{\sum_j w_{jn}^2}$, where the sub index j is in the set (x, y, t) , and n numerates the grid cells, we find that:

$$\text{WL2: } u_{jn} = \frac{\rho}{1 + \rho} w_{jn}, \quad (12)$$

$$\text{TV: } u_{jn} = \max \{ |w_{jn}| - 1/\rho, 0 \} \cdot \text{sign}(w_{jn}), \quad (13)$$

$$\text{ATV: } u_{jn} = \max \{ |w_n| - 1/\rho, 0 \} \cdot \frac{w_{jn}}{w_n}. \quad (14)$$

SYNTHETIC EXAMPLE

We illustrate the methodology in a synthetic example. Cross section of the pointspread function and data are shown in Figure 1. The point spread function is limited in frequency content and the angles of illumination and has a rotational symmetry. It has the form discussed in Lecomte (2008) assuming constant background velocity. In that notation, we use a Ricker wavelet as a source wavelet and assume an uniform illumination within a 45° cone. White noise is added to the geophysical data and have a standard deviation of the order of 0.1.

Figure 1: Pointspread function and seismic data.

Figure 2: Results of relative inversion. The top left is the ground truth, top right is the WL2 inversion, bottom left is the TV inversion, and the bottom right is the ATV inversion.

The parameter μ , governs the trade off between data fit and regularization. For a fair comparison of the three methods, we tune this parameter to be the optimal for the three different regularizers. The results of the relative inversion compared to the ground truth is seen in Figure 2. Figure 3, show the same result in the f-k domain, where a 2D Fourier transform is applied to the intersection. The inversions are relative thus the base level of the inversion is lost, however the TV approaches keep the level going away from the main contrasts to a higher degree than the weighted L2 approach. The total variation approaches obtain a bandwidth extension compared with the WL2. For all three inversion the limited angle of illumination is seen, but

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Figure 3: Results of relative inversion in f-k domain. The top left is the ground truth, top right is the WL2 inversion, bottom left is the TV inversion, and the bottom right is the ATV inversion.

only the WL2 estimate have no energy outside the illuminated region. Table 1 show a comparison of the mismatch of the three models together with the residuals obtained after fitting. The root mean square of the prior is just the deviations from a constant level. The constant is also subtracted when comparing the match to the relative inversions. The residuals of the prior model corresponds to the energy in the noisy data. There is a substantial improvement in the fit for the two total variation approaches. This is commonly seen in these types of problems where the variability in the model is dominated by abrupt changes along smooth edges. In contrast to the WL2 and ATV the TV norm is dependent on the direction in the grid, this gives a preference towards the ATV over TV.

Regularization	Model	Residuals
Prior	0.452	0.248
WL2	0.388	0.098
TV	0.261	0.098
ATV	0.260	0.098

Table 1: Root mean square deviation in optimal fit, and residuals. The table show the deviation between the model and the inversion for the optimal choice of the tuning parameter and the root mean square fit to data.

FIELD EXAMPLE

We have applied the methodology on a data set acquired in the Barents sea (Dhelie et al. (2018)), where we have used a generalization of the statistical wavelet estimation to derive the point spread function. The data went through a noise attenuation / conditioning flow prior to inversion. Figure 4 shows

inversion result, where inverted relative impedance has been blended with the inverted reflectivity, thus producing a dual attribute display where geological packages have prominent character, and edges are drawn sharply at their boundaries in order to produce an image for structural and qualitative amplitude interpretation. In the image, white represents lower impedance values. Some distinct features can be noticed in this image, such as (A), two parallel "flat spots" that are not accompanied with any softening of the amplitudes that could be expected, especially in the case of a gas/oil contact. In line with this observation, a dry well indeed drilled through both and proved that they do not represent fluid contacts. We can also see a soft (white) structure (B) below an apparent closure, that was proven by a well to contain gas.

DISCUSSION

The inversion is based on stationary point spread function, which can be limiting in complex settings. The use of a total variation norm rather than the traditional weighted L2 norm shifts the assumptions of the geological constraints from smoothness to blockiness, and thereby achieve digital superresolution as the inversion contain more frequencies than what is seen in the seismic data. The major benefit of the weighted L2 approach is the linear theory of resolution which have an elegant way of quantifying the uncertainty. In the exploration setting the methodology introduced here will mitigate wavelet effects and sharpen the images of the subsurface. We thereby sharpen the eye of the exploration geologist, and thus we might reduce the uncertainty at the fundamental level of regional understanding. Thus, the proposed method aligns with the typical processing applied to the final stack of a processing project, but formulated more ambitiously as an inverse problem, and arguably filling an often empty gap between a conservative final stack and typical QI/AVO inversion workflows.

CONCLUSIONS

We have defined a general framework for a genuine 3D inversion of seismic amplitude data, utilizing a stationary point spread function and a weak 3D regularization. The synthetic example included show that using a total variation norm rather than the weighted L2 improves the fit in the case of discontinuities in the earth parameter. The approach is in particular useful in the exploration setting where little is known about rock properties. Joint inversion of the point spread function and the parameter is considered to be an important topic for future research.

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Figure 4: Field data and inversion product. The seismic data are displayed in top. The inversion product being a mix of the inversion and the reflection coefficients in bottom.

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