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Professional Noticing of Mathematics Teachers: A Comparative Study between Germany and China

Abstract

Research on teacher noticing in the field of mathematics education research has increased in the last decade; however, only a few studies have investigated the social and cultural impact on teachers' noticing. This paper describes results of an international comparative study on (in-service) mathematics teachers' professional noticing in China and Germany, representing Eastern and Western cultures. In this study, 203 teachers in China and 118 teachers in Germany participated in a video-based assessment of their professional noticing, i.e. perception, interpretation, and decision-making competencies. Item response theory (IRT) and differential item functioning (DIF) were employed for data analysis. Findings of the study revealed that German teachers performed significantly better than Chinese teachers on noticing aspects related to general pedagogy; in contrast, Chinese teachers outperformed their German counterparts on noticing aspects connected to mathematics instruction. Further DIF detection analysis found that German mathematics teachers demonstrated particular strengths in "perception" as a sub-facet of noticing; in contrast, Chinese teachers tended to demonstrate strength in "analyzing and decision making" as other sub-facets of noticing. These findings suggest that societal and cultural factors, such as different philosophical paradigms, traditions of teacher education, and teaching and mathematics curriculum traditions are main influencing factors on teachers' professional noticing.

Keywords: comparative study, mathematics teacher competencies, societal and cultural influences, teacher noticing, video-based assessment

Introduction

During the past decade, research on teachers' professional noticing as a major part of teachers' practice in classrooms and as a prerequisite for instructional quality has strongly developed in mathematics education and in other subject areas such as science education (e.g., Authors, 2015; Barnhart & van Es, 2015; Sherin, Jacobs, & Randolph, 2011; van Es, Cashen, Barnhart, & Auger, 2017). Based on the theoretical approach of professional vision by Goodwin (1994), according to which teachers selectively attend to events that take place and then draw on their knowledge to interpret these events, teachers' professional noticing has been described as a key component of teaching expertise (Schoenfeld, 2011). It influences teachers' activities in classrooms and largely determines the overall mathematics instructional quality and consequently, students' mathematics achievement (Kersting, 2008). In psychologically influenced research traditions, the precise observation of a professional situation, in this context a teaching-learning-situation, is called "perception-accuracy", where strong differences between novice and expert teachers exist (Carter, Cushing, Sabers, Stein, & Berliner, 1988). Researchers have also pointed out that teachers' professional noticing is an important construct, which makes explicit the complex work of teaching (Jacobs, Lamb, Philipp, & Schappelle, 2011). Therefore, a deep understanding of the nature of and influences on teachers' professional noticing will contribute to understanding the tacit characteristics of teachers' work and consequently, to finding ways to better support teachers' professional development.

In past years, researchers have explored factors that influence teachers' professional noticing. It has been widely accepted that teachers' beliefs and knowledge (e.g., general pedagogical knowledge and mathematical content knowledge) have a fundamental impact on teachers' noticing during teaching (e.g., Dreher & Kuntze, 2015; Meschede, Fiebranz, Möller, & Steffensky, 2017). In addition, teachers' prior learning and teaching experiences also act as

important factors that influence what teachers notice in teaching (Ding & Dominguez, 2016; Jacobs et al., 2011). Accounting for the situational and contextualized nature of teachers' knowledge, beliefs, and experience, these factors should be shaped and developed in a specific societal and cultural context, and thereby sensitive to cultural contexts. It is, therefore, reasonable to conjecture that social and cultural dimensions of an educational context act as influential factors for the shape and development of teachers' professional noticing. Researchers already argued that "noticing is not purely neutral attention, but culturally shaped perception" (Ball, 2011, p. xxi).

However, in the field of mathematics education research, until recently, there have been only a few studies investigating the cultural impact on teachers' professional noticing (e.g., Ding & Dominguez, 2016; Louie, 2018). Until now, teachers' professional noticing has been mainly studied by qualitative approaches – mainly case studies with small sample sizes - and by descriptive studies of teachers' approaches to noticing and reasoning (Seidel & Stürmer, 2014). A major weakness of case study approaches with small sample sizes is that the sample data is not sufficient for relatively advanced statistical analysis and, therefore, constrains a deep understanding of the inner construct of teachers' professional noticing (Mason, 2016). In addition, a majority of previous studies have focused on the investigation of pre-service mathematics teachers' abilities of noticing. However, the experience of teachers within classrooms acts as a major influence on the development of teachers' professional noticing (Jacobs et al., 2011; Schoenfeld, 2011). Comparative studies have the potential for researchers to serve as windows into their own cultures, because these studies can question the ways we describe and understand education and teaching in our own cultures (Alexander, 1999). Based on this tradition, comparative studies have a long tradition within mathematics education of asking,

among others, the question of how much very distinct cultures, such as East Asian cultures and Western cultures, can learn from each other (Authors, 2013).

The study therefore aims to investigate based on a quantitative study with large samples of in-service teachers the nature of mathematics teachers' professional noticing. It will be especially examined, which influence distinct societal and cultural contexts – in this case Western and East Asian contexts - have on teachers' professional noticing.

Literature Review and Theoretical Framework

The Construct of Teachers' Professional Noticing

In recent years, a growing body of research in mathematics education has focused on the investigation of teachers' professional noticing; however, until now noticing is defined in a variety of ways including different aspects of teachers' thinking and practice (Sherin et al., 2011). In their pioneering work Van Es and Sherin (2002, 2006) conceptualized teachers' noticing as covering the following three aspects: (a) identifying an important or noteworthy event in a classroom situation, (b) making connections between the specific classroom events and broader principles of teaching and learning, and (c) using knowledge about the context to reason about classroom interactions. In their extensive survey on the discussion on noticing, Sherin et al. (2011) pointed out that the conceptualization of noticing is varied, but has two common core facets, namely attending to and making sense of events in an instructional setting, which are described as follows:

Attending to particular events in an instructional setting. To manage the complexity of the classroom, teachers must pay attention to some things and not to others. In other words, they must choose where to focus their attention and for how long and where their attention is not needed Making sense of events in an instructional setting. For those features to

which teachers do attend, they are not simply passive observers. Instead teachers necessarily interpret what they see, relating observed events to abstract categories and characterization what they see in terms of familiar instructional episodes. (p. 5)

Jacobs, Lamb and Philipp (2010) and Erickson (2011) extended this definition and conceptualized the construct of teachers' professional noticing of students' mathematical thinking as a set of three interrelated skills: (a) attending to children's strategies, (b) interpreting children's understandings, and (c) deciding how to respond based on children's understandings.

Overall, although there exist varied conceptualizations of teacher noticing, some common features can be identified in previous studies. First, as described above in the conceptualizations of Sherin and van Es (2002, 2006) and Jacobs et al. (2010), teacher noticing is highly selective and multidimensional (Erickson, 2011). Another common point among the previous definitions of teacher noticing is the description of teachers' professional noticing as a process rather than as static knowledge. Although several aspects of teacher noticing can be clearly identified in the conceptualizations, these aspects are not a distinct static category of knowledge; instead, these aspects are "interrelated and cyclical" (Sherin et al., 2011, p. 5). In addition, it is widely accepted that teacher noticing is a "situated practice" (Thomas, 2011, p. 508). The situational nature of teacher noticing has been recently emphasized in literature as a useful complement to teachers' professional competence and researchers have proposed that teachers' professional competence should reflect the situated and contextualized nature of teaching demands (Authors, 2015; Meschede et al., 2017). For teachers, the transformation of the cognitive dimensions of competence (e.g., knowledge) into classroom performance requires teachers' situation-specific skills (Authors, 2017).

Situated approaches to assess teacher professional competence have led to the development of a series of follow-up studies of the international IEA *Teacher Education and Development Study in Mathematics* (TEDS-M) in Germany, which originally evaluated the efficiency of teacher education internationally, with participating countries coming from the East and the West. TEDS-M assessed teachers' professional knowledge in mathematics, mathematics pedagogy, general pedagogy and their related beliefs. Based on work by authors (2015), the follow-up studies—TEDS-FU and TEDS-Instruct—extended this framework of TEDS-M covering cognition and affect-motivation by situation-specific skills to analyze teachers' professional competencies in a more performance-oriented way (see Figure 1). Teacher noticing was described as an important mediator between teachers' cognitive competence (e.g., knowledge) and teachers' behavior in teaching (Authors, 2015; Authors, 2017).

Fig. 1: Modeling competence as a continuum (Authors, 2015, p. 7)

The essential characteristics of teacher noticing were described via the following model named the “Perception, Interpretation, and Decision-making (PID) Model” of noticing covering the following three sub-facets of noticing: (1) perceiving particular events in an instructional setting; (2) interpreting the perceived activities in the instructional setting; and (3) decision-making, either anticipating a response to students' activities or proposing alternative instructional

strategies. This model was developed specifically to meet the need to investigate teachers' situation-specific skills, which have not yet been fully investigated in literature (Authors, 2017; Depaepe, Verschaffel, & Kelchtermans, 2013).

Like in the conceptualization by Jacobs et al. (2010), three aspects are included in the PID model, which not only require teachers to perceive noteworthy events and interpret them, but also expect teachers to make further decisions and make reasonable responses. However, unlike the conceptualization by Jacobs et al. (2010), which mainly focuses on teachers' professional noticing of students' mathematics thinking, the PID model does not limit teachers' professional noticing to that very special aspect of teachers' work (Authors, 2017). In contrast, the PID model comprises a broad understanding of teachers' noticing, which includes all the aspects that are important to the educational approach of quality-oriented mathematics teaching, such as design of mathematics teaching, potential for cognitive activation of students, and classroom management (Authors, 2015). As the current study has been carried out as an East Asian extension of TEDS-Instruct, the theoretical framework of TEDS-Instruct—especially concerning the PID conceptualization of noticing—is employed in this study.

Influences of Teachers' Professional Noticing

Many factors have been identified as influencing teachers' noticing and its development. One important factor is teachers' knowledge and belief systems. As clearly pointed out by Schoenfeld (2011, p. 232), “what teachers notice, and how they act on it, is a function of the teachers' knowledge and resources, goals, and orientations”. Previous empirical studies provide evidence for this claim. For example, Authors (2014) found out that pre-service teachers' general pedagogical knowledge can predict their performance in interpreting classroom situations.

Similarly, Meschede et al. (2017) identified that teachers' noticing is substantially interrelated with their pedagogical content knowledge and constructivist beliefs about learning and teaching.

In addition, other factors, such as teachers' instruction experience, were also identified as important influences on teacher noticing. For example, Dreher and Kuntze (2015) ascertained a weak but significant relationship between pre-service mathematics teachers' content knowledge and their noticing of multiple representations in mathematics classrooms, whereas no significant correlation was identified among in-service mathematics teachers. This illustrates that due to the differences in length of experience, teachers' knowledge functions differently for teacher noticing. Similarly, Jacobs et al. (2010) reported that teaching experience alone only provides support for teachers at the initial development of expertise in attending to children's strategies and interpreting children's understandings.

Moreover, in terms of the development of teachers' noticing, researchers have also argued that due to the difference of teachers' prior knowledge and experience, teachers develop "their noticing practices in different ways over time" (van Es et al., 2017, p. 174). Teachers' knowledge, orientations, and experience, however, are all essentially societally and culturally developed or accumulated in a specific context. Therefore, it is reasonable to conjecture that teachers from different societal and cultural contexts will develop noticing with different cultural characteristics and, due to their differences of knowledge and orientations, they will notice classrooms differently. The societal and cultural values in a specific context shape teachers' attention to issues they consider important and valuable or to issues they will ignore or consider less relevant. This, to a large degree, further suggests that teachers' professional noticing is "by its very nature, inseparable from a particular context" (Thomas, 2017, p. 510). In fact, when describing professional vision, which is the main theoretical root of teachers' noticing, Goodwin

(1994) argued that “the ability to see a meaningful event is not a transparent, psychological process, but instead a socially situated activity accomplished through the deployment of a range of historically constituted discursive practices” (p. 606).

All these claims suggest that on the one hand, societal and cultural factors existing in a specific context in which the teachers are working act as important influences on teachers’ professional noticing. On the other hand, these claims further imply the cultural nature of teachers’ noticing (Louie, 2018). Previous studies have, indeed, identified some differences in teachers’ noticing between teachers working in two distinct societal and cultural contexts. For example, Miller and Zhou (2007) compared American and Chinese primary school teachers’ noticing of classroom videos and identified striking differences between the teachers from these two cultures. American teachers were more likely to make comments on pedagogical issues and teachers’ personalities, whereas Chinese teachers tended to comment on the mathematical content of the lessons.

Although it is widely accepted that teachers’ professional noticing should be understood as societally and culturally constructed, very few efforts have been made so far to systematically compare how mathematics teachers from different cultures notice classroom events. Furthermore, it has been explored only by a few studies whether differences exist in noticing between the teachers from different cultures and what kinds of societal and cultural factors contribute to these differences. A deep understanding of the societal and cultural influences on teachers’ noticing not only supports the understanding of the nature of teachers’ noticing, but also contributes to the facilitation of its growth. In view of this premise, the present study asks for similarities and differences of professional noticing from mathematics teachers coming from Germany and China,

and whether these similarities and differences can be explained by the different societal and cultural backgrounds of the teachers.

Methodology

Participants

The sample in the present study consisted of 118 and 203 in-service junior secondary school mathematics teachers in Germany and China, respectively. The 203 Chinese teachers were chosen from the 19 junior secondary schools in one district in Chongqing, the biggest metropolitan city in Western China. The German teachers were participants in the TEDS-Instruct study and were in-service teachers in Hamburg, the second biggest city in Germany—comparable to Chongqing due to its metropolitan character. Data collection took place between 2014-2015 in Hamburg and October-December, 2016 in Chongqing. There are differences of schooling culture; for example, junior secondary school mathematics teachers in China only teach one subject—mathematics—to students in Grades 7 to 9 (students' age ranging from 12 to 14), whereas secondary school German teachers teach a second subject in addition to mathematics and teach students from all grades at the secondary level (i.e., Grades 5 to 10 resp. 12/13, students' age ranging from 10 to 16, resp. 18/19). 42 percent of the German teachers were female. By average, they were 39.2 years old ($SD = 10.5$) and had, by average, 10.7 years of work experience ($SD = 9.9$). For the Chinese participants, 31.5 percent of them were female and their average teaching experience was 14.6 years ($SD=9.1$).

Instruments and Data Collection

Brief description of the development of instruments in Germany. The instruments used in the present study to investigate mathematics teachers' noticing were developed in two

large follow-up studies of the TEDS-M study in Germany: TEDS-FU and TEDS-Instruct. The overall aim of TEDS-Instruct was to investigate the relationship between mathematics teachers' professional competence, their quality of instruction, and students' mathematics achievement (Authors, 2017, see Figure 2 for the structure of the framework).

Fig. 2: Impact model of TEDS-Instruct (adapted version from Authors, 2017, p. 179)

To evaluate teachers' professional competence from a cognitive and a situated perspective, TEDS-Instruct used the extended theoretical framework of TEDS-FU and the newly developed instruments as enrichment of the instruments taken from TEDS-M. To examine teachers' domain-specific and general pedagogical perception, interpretation, and decision-making skills, three video vignettes were developed in TEDS-FU. The videos covered a range of mathematical topics (e.g., functions, volumes, and surfaces) usually taught in Germany in Grades 8 to 10. However, the three videos did not consist of episodes taken from real classroom teaching but were scripted plots with different teachers and students considering different types of schools in Germany. In addition, each of the videos focused on different phases of teaching, such as the introduction of a mathematical task followed by work on the task, or work on a task followed by a discussion of the results, to investigate the facets of teachers' professional noticing comprehensively. Each video clip lasted around four minutes with the aim to provide an

overview about the whole lesson. To help participants more completely understand the videotaped teaching, background information of the class and lessons prior to the lesson that was videotaped were provided.

After watching each of the three videos, mathematics teachers were asked to answer closed- and constructed-response items relating to the scenes they had watched. The items distinguished between P_PID (Pedagogy: Perception, Interpretation, and Decision) and M_PID (Mathematics Instruction: Perception, Interpretation, and Decision). The former mainly focused on teachers' general pedagogy-related noticing, such as lesson structure, adaptivity, motivation, classroom management, and assessment. The latter focused on teachers' mathematics instruction-related noticing, such as different ways of explaining a mathematical concept, analyzing mental processes, and identifying task types, mathematical competencies, and mathematics ideas. Generally, the items required teachers to notice mathematics classroom teaching as a whole process; that is, the items were related to almost all aspects of classroom teaching. In total, there were 38 closed items (22 P_PID and 16 M_PID) using Likert-scales and 36 constructed-response items (18 P_PID and 18 M_PID). An expert rating was implemented to develop rating scales for the closed items. The coding manual was developed through a systematic analysis of relevant theories and literature and after wide discussion among experienced mathematics teachers and mathematics education researchers. The coding manual was piloted before it was used in the main study of TEDS-FU to improve its validity. In the following we describe exemplarily one video clip, the "box".

The group of involved students consisted of students 9th year (age 14 to 16) of a German school for higher achieving students (academic track, so-called 'Gymnasium'). The mathematical content focused on the volume of an open box that has to be figured out. The box

is made out of a rectangular sheet with four cut-off square-shaped congruent corners. The volume of the box is influenced by the size of the cut-off corners, thus functional aspects are included.

Three pairs of students are shown in the video, who work on the problem in three different ways. After having worked in pairs the teachers collect the results in a table on the blackboard.

In the following we give two examples of items (see Figure 3 and 4), which were developed in TEDS-FU and used in TEDS-Instruct.

	fully correct	partially correct	partially incorrect	not correct
The task includes some features of an open-ended task.	☐	☐	☐	☐

Fig. 3: Four-point rating scales assessing perceptual skills related to mathematics (from authors, 2015, p. 379)

Many different approaches, like curricular analyses of the mathematical content and comprehensive expert workshops, were employed to ensure the content validity of the instrument. Furthermore, TEDS-FU shortened the original knowledge-based instruments from TEDS-M to complement the evaluation of teachers' professional competencies (for details, see Authors, 2015)

1
2
3

In the video-vignette the working processes of three cooperating pairs have been observed more closely. These working processes are to be examined from two perspectives: (a) mathematics education and (b) pedagogics.

(a) mathematics education perspective

In each of the three approaches the task is represented and solved mathematically in a specific way.

Please describe (in note form) the essential aspects of the approaches in a contrasting mode from a mathematics education view.

Fig. 4: Example of a high-inferential M_PID and P_PID item referring to interpreting a classroom situation (from authors, 2015, p. 380f)

These instruments were used in TEDS-Instruct without further changes, and were complemented by a test on classroom management expertise, which was not applied in this comparative study and is, therefore, not described here.

Adaptation of the instruments and data collection in China. In the present study, the instruments developed in TEDS-FU and TEDS-Instruct in Germany were adapted to the Chinese context to investigate mathematics teachers' professional noticing in China. The video-based instruments were translated from German to English by a native German-speaking mathematics education postgraduate student with a minor degree in English language. The second author checked the accuracy of the translation and made necessary corrections. After an intensive discussion by the first author with the main original developer of the instruments, the first author translated the instruments from English to Chinese. The Chinese translation was checked and corrected by a Chinese professor of mathematics education with many years of international study and research experience. To ensure the content validity, the Chinese version of the instruments was double-checked by a native Chinese PhD student with a major degree in German language. Slight modifications were made to the video script and to some expressions in the questionnaire. Two mathematics education researchers and six junior secondary school

mathematics teachers in China examined the German video clips' issues concerning their adequacy for Chinese mathematics teaching reality. Without changing the original intention of the instruments, modifications were made to some expressions used within the video clips and to some items in the questionnaire. A few items in the German instrument, such as the ones related to German mathematics curriculum and to heterogeneity or multiple cultural backgrounds of students, were deleted since they did not match the situation in Chongqing. One part in a video that showed a teacher checking the attendance of students was deleted since that task is not necessary in Chinese schools. All other parts of the three videos were maintained since they were all similar to Chinese mathematics teaching practice. Three junior secondary school mathematics teachers and their students retook the three videos and performed exactly as their German counterparts.

For data collection, in contrast to the German study, where a web-based survey was used to collect data (i.e., all 118 German teachers of the study finished the survey individually at home), the sampling in China took place during a course on professional development. The 203 Chinese teachers were distributed among five classrooms, watched the video clips together, and answered the questions using a paper-and-pencil test. They finished the test at the same time. The reason for this strong change in the data sampling method was the unfamiliarity of older Chinese teachers, in particular, with this kind of online survey.

Data Analysis

The data were analyzed qualitatively and quantitatively. For the constructed-response items, the coding manual from the TEDS-Instruct project was translated from German into

English by the same German translator, and the accuracy was checked by the second author. Afterwards, 50 randomly chosen Chinese teachers' answers were translated into English by the first author for joint coding sessions with one of the experienced coders from the TEDS-Instruct project. Using the answers of ten Chinese teachers, the first author was trained by the experienced coder; afterwards, the first author and the experienced coder coded the remaining 40 Chinese teachers' answers separately. For all the items, good values of Cohen's Kappa were reached ($k > 0.76$ and $K_{\text{average}} = 0.84$). Differences in the coding were discussed to reach a common understanding of the codes. Some specific answers or expressions only used by Chinese teachers were further examined by the first author and the main developer of the original German coding manual. The coding manual was slightly modified to make it more suitable for the Chinese teaching culture. Based on this final version of the coding manual, the first author coded the other 153 remaining teachers' answers by strictly following the English coding manual and going back to the original developer of the German coding manual in case of uncertainties. For all the constructed-response items, items with no response or an incorrect response were scored 0, and each correct answer was scored 1 (for items with several sub-items, the sum of the correct answers was calculated).

After the completion of coding, the relative item difficulties from a one-parameter (Rasch model) item response theory (IRT) model were calculated separately for the dimensions of P_PID and M_PID. Three items from the aspect on P_PID and two items from the aspect on M_PID with extreme difficulty were removed from the final analysis. The internal consistency of the remaining items on P_PID and M_PID was estimated using the Cronbach alpha reliability coefficient in Germany and China separately (see Table 1). As shown in Table 1, the reliability scores of each aspect in Germany and China were acceptable. Next, a multi-group graded

response model was applied to both P_PID and M_PID aspects. The calibration of the a- (item discriminations) and b-parameters (item difficulties) was conducted on the basis of two teacher groups (concurrent calibration). Thus, item parameters were constrained to be equal in all groups, ensuring the same metric in the groups. After this, the person parameters (maximum likelihood estimates) were computed. Both the P_PID and M_PID person parameters were transformed to a scale with an average score of 500 and standard deviation of 100 within the German teachers (reference group), as in the original tests. A *t*-test was carried out separately to examine the differences between German and Chinese teachers on each of the two aspects.

Table 1

Reliabilities (Cronbach's Alpha) of P_PID and M_PID

	N	P_PID	M_PID
		Cronbach's α	Cronbach's α
German in-service teachers	118	0.82	0.79
Chinese in-service teachers	203	0.84	0.76

Furthermore, differential item functioning (DIF) was conducted with the main aim of identifying which items were typically in favor of German or Chinese teachers. Although DIF has been widely used to judge whether items are functioning in the same manner in various groups of participants in surveys or achievement tests, DIF can also be used for identifying cross-country differences in cultures, as a complement to other methods (Mesic, 2012). Similarly, DIF in the present study was performed with the main aim of identifying societal and cultural factors from the German and Chinese contexts, which would allow reconstructing similarities and differences of teachers' professional noticing at item level. DIF was detected using manifest logistic regressions (Swaminathan & Rogers, 1990), which can identify uniform and non-

uniform DIF (Hambleton, Swaminathan, & Rogers, 1991). According to the difference between uniform and non-uniform DIF, an item showing uniform DIF pointed out that teachers from Germany or China outperformed the counterparts from the other country systematically throughout all the ability levels. This suggests that societal and cultural differences between these two countries might act as main factors that impact teachers' competence on the aspect the item investigated. However, if an item showed non-uniform DIF, the societal and cultural impact on the response behavior was different across teachers' competence levels. That is, societal and cultural factors may only significantly impact some teachers, either with higher or lower competence. The magnitude of DIF in the present study was further classified into three levels according to the thresholds proposed by Jodoin and Gierl (2001), namely: $\Delta R^2 \leq 0.035$ is negligible, $0.035 < \Delta R^2 \leq 0.07$ is moderate, and $0.07 < \Delta R^2$ is large. After the items with DIF had been detected and classified, content analysis of the items with uniform or non-uniform DIF was further carried out to identify possible societal and cultural explanations.

Results

Overall Performance Difference on P_PID and M_PID

The overall performance of German and Chinese mathematics teachers' professional noticing on the aspects P_PID and M_PID is summarized in Table 2. The P_PID and M_PID test scores of German mathematics teachers were transferred and fixed to a mean score of 500 test points with a standard deviation of 100 test points, as was done in the original study. Consequently, the mean score differences on these two aspects of teachers' professional noticing between German and Chinese in-service mathematics teachers can be interpreted in terms of standard deviations of German teachers (i.e., reference group). For P_PID, German teachers showed significantly higher mean scores than the Chinese teachers. As shown in Table 2, the

difference between German and Chinese in-service mathematics teachers on the aspect of P_PID was 64 points and was statistically significant (t -test) difference ($p<0.001$). However, in contrast, for M_PID, Chinese teachers showed significantly higher mean scores than the German teachers; the difference between Chinese and German in-service mathematics teachers was around 63 points and there was a statistically significant (t -test) difference ($p<0.001$) as well. Thus, German in-service mathematics teachers outscored their Chinese counterparts by almost 0.6 standard deviation on the aspect of P-PID. In contrast, Chinese mathematics teachers outscored German teachers by around 0.6 standard deviation on the aspect of M-PID.

Table 2

Mean Scores and Standard Deviations for P_PID and M_PID and Differences

	German Teachers (M $\pm$ SD)	Chinese Teachers (M $\pm$ SD)	T
P_PID	500 $\pm$ 100	436.19 $\pm$ 107.03	5.27***
M_PID	500 $\pm$ 100	567.52 $\pm$ 97.52	-5.93***

Note. *** $p<0.001$

Country-specific Strengths and Weaknesses on P_PID as indicated by DIF

With the main aim of identifying which items on the aspect of P_PID typically favor German or Chinese teachers, DIF was detected with the use of logistic regression identifying both uniform and non-uniform DIF items. The DIF detection results revealed that 15 of the 32 P_PID items show significant uniform DIF, with ten favoring German teachers and five favoring Chinese teachers. In addition, no items showed non-uniform DIF, and the average amount of non-uniform DIF was negligible ($M_R^2_{adj}= 0.005$, $SD_R^2_{adj}= 0.008$), which suggests that the impact of the general competence of teachers' professional noticing on P_PID items did not differ

between the two countries. According to the classification proposed by Jodoin and Gierl (2001), six of the 15 uniform DIF items showed moderate (ΔR^2 ranges from 0.043 to 0.060), and nine showed large DIF (ΔR^2 ranges from 0.075 to 0.234). Table 3 summarizes the distribution of the uniform DIF results considering the two facets of P_PID, perception and analyzing and decision making.

Table 3

Distribution of uniform DIF Items on P_PID

		Perception	Analyzing and Decision	In total
Favor Germany	Class B	3	1	4
	Class C	6	0	6
Favor China	Class B	1	1	2
	Class C	1	2	3

Note. Class B (moderate DIF): $0.035 < \Delta R^2 \leq 0.07$, Class C (large DIF): $0.07 < \Delta R^2$

Most of the items favoring German mathematics teachers were “Perception” related items. Content analysis of the characteristics of these items revealed that the main aim of these items was to investigate teachers’ “perceptions” related to classroom management (e.g., “It takes very long for the student to calm down and for the lesson to start”), students’ behavior (e.g., “Most of the students take an active part in the lesson”), and teachers’ behavior (e.g., “The teacher presents the question visually and acoustically”). In contrast, for the two items with DIF favoring Chinese teachers, one of them related to students’ learning motivation (e.g., “Using the start of her lesson, the teacher tries to evoke an intrinsic motivation in the students”) and the other one related to students’ thinking (e.g., “The teacher provides the students with the opportunity to individually think about the question”).

For the items on the aspect of “Analyzing and Decision Making,” only one item favored German mathematics teachers, and the other three items favored Chinese teachers. This first item asked teachers to judge how one student’s statement is different from the statements made by his classmates, taking a pedagogical perspective (correct answer is the student’s statement relates or refers to his classmates’ statements). Two items, which favored Chinese teachers, were from the same task aiming to investigate teachers’ competencies to modify the lesson they had just watched and identify the phase within the lesson and the activities, where changes would be needed in order to meet the class’s different learning conditions in a better way.

The third item, which favored Chinese teachers, required teachers to evaluate the teaching just watched and to describe two aspects where the teacher’s behavior was not adequate for supporting students’ cognitive activities. A further content analysis on these last three items suggested that teachers need to holistically and deeply evaluate and analyze the teaching they just watched to identify the weaknesses and propose necessary improvements. More importantly, a solid knowledge base on students (e.g., their cognitive development and individual differences) and on alternative ways of acting within a specific classroom situation is needed. In other words, teachers need to have sound knowledge in developing lesson plans flexibly in various ways to meet the needs of students with various backgrounds.

Overall, the DIF results on the P_PID items suggested that German teachers are stronger in professional noticing on the aspect of “Perception,” (i.e., the perception of phenomena related to classroom management and students’ behavior, which were explicitly shown in the videos). In contrast, Chinese teachers tended to show strength on the aspect of “Analyzing and Decision Making,” requiring teachers to use their knowledge to analyze and reason about incidents, which were not explicitly shown in the videos.

Country-specific Strengths and Weaknesses on M_PID as indicated by DIF

Similarly, to identify which items on the aspect of M_PID were typically favoring German or Chinese teachers, DIF was detected, and the DIF results revealed that 19 of the 32 M_PID items showed significant uniform DIF, with 9 favoring German teachers and 10 favoring Chinese teachers. In addition, no items showed non-uniform DIF, and the average amount of non-uniform DIF was negligible ($M_{R^2_{adj}} = 0.006$, $SD_{R^2_{adj}} = 0.007$). That meant the non-uniform results suggested that the impact of the overall competence of teachers' professional noticing on M_PID items also did not differ between the two countries. According to the classification proposed by Jodoin and Gierl (2001), twelve items showed moderate DIF (ΔR^2 ranges from 0.036 to 0.069) and seven items showed large DIF (ΔR^2 ranges from 0.079 to 0.403). Table 4 summarizes the distribution of the uniform DIF results on the aspect of M_PID.

Table 4

Distribution of Uniform DIF Items on M_PID

		Perception	Analyzing and Decision	In Total
Favor	Class B	4	3	7
Germany	Class C	1	1	2
Favor	Class B	2	3	5
China	Class C	1	4	5

Note. Class B (moderate DIF): $0.035 < \Delta R^2 \leq 0.07$; Class C (large DIF): $0.07 < \Delta R^2$.

Again, more items on the aspect of “Perception” were in favor of German teachers, and more items on the aspect of “Analyzing and Decision Making” were favoring Chinese teachers. Content analysis of the characteristics of these items with DIF yielded some further differences

between German and Chinese teachers. The five items on the aspect of “Perception,” favoring German teachers, mainly related to students’ statements in the classroom (e.g., “Jackie’s statement is a helpful contribution to the lesson question”), mathematical modeling (e.g., “There are modeling competencies necessary to work on the central question”) and open-ended characteristics of the task (e.g., “The task shows features of an open question”). Two out of the three items on the aspect of “Perception”, which favored Chinese teachers, required teachers to evaluate which kind of knowledge was necessary to tackle the task successfully (e.g., “The central mathematical idea or functional coherence comes into effect during teaching”), and the third one required teachers to use relevant knowledge to evaluate students’ work (e.g., “When students calculate the volume of the solid, they use the correct measuring unit cm^3 to present their answers”).

For the four items on the aspect of “Analyzing and Decision Making,” which favored German teachers, two items from one problem required teachers to modify the task to make it more realistic or foster modeling competence to solve the task. The two other items required teachers to identify whether a student preferred a formal approach other than a visual approach according to this student’s work. Concerning the seven items, which favored Chinese teachers, three of them required teachers to point out the critical characteristics of students’ group work by using the adequate professional terminology; another two items from one problem required teachers to point out two mistakes of a student’s work. Another item required teachers to identify the difference of a students’ statement from the classmates’ statements from a mathematical point of view (i.e., to answer the item correctly, teachers needed to notice that the student was deducing or making a linear assumption). The last item required teachers to suggest possible

ways to correct a wrong assumption about a linear relation between a ball's diameter and its weight made by a student.

Overall, content analysis of the characteristics of the items with DIF on the aspect of M_PID suggested German teachers' strength concerning stronger professional noticing on aspects like modeling and visual approaches of teaching. In contrast, the Chinese teachers showed strength in professional noticing on aspects like using knowledge to make relevant judgments on students' work or to identify critical characteristics of students' activities, evaluating students' mistakes, and developing alternative ways of teaching.

Discussion

This study aimed to compare mathematics teachers' professional noticing in China and Germany and to identify societal and cultural influences on teachers' noticing. The theoretical framework and the instruments developed and used in follow-up studies of TEDS-M in Germany were adapted in this study.

Differences on Pedagogy-related Professional Noticing (P_PID)

Consistent with known insights from previous studies, the German mathematics teachers performed significantly better than the Chinese teachers on noticing pedagogy-related aspects (P_PID). In the light of the outperformance of Western pre-service teachers at the end of their study by East Asian pre-service teachers in the international comparative study TEDS-M (Authors, 2014), this finding may be astonishing, but it is consistent with findings in previous related international comparative studies. For general pedagogical knowledge, Authors (2011) reported that pre-service German secondary school mathematics teachers performed slightly better than their counterparts from Taiwan. Furthermore, in the corresponding discussion, it was

already pointed out that what teachers notice “is a function of the teachers’ knowledge” (Schoenfeld, 2011, p. 233). Therefore, it is plausible that due to differences of general pedagogical knowledge between teachers from German and Chinese contexts, they notice differently when they watch the same videotaped teaching sequences. Differences in teachers’ opportunities to learn general pedagogy during teacher education may further explain the differences between German and Chinese teachers. As reported in the TEDS-M study, German pre-service secondary mathematics teachers have more opportunities to study educational psychology, philosophy of education, theories of schooling, and knowledge of teaching than their Taiwanese counterparts (Wang & Tang, 2013). Similarly, according to the standard program of teacher education in China, pre-service mathematics teachers in China only have opportunities to attend two courses related to general pedagogical knowledge: namely, general pedagogy and psychology. However, investigations conducted recently found that opportunities provided during teacher education will help pre-service teachers to acquire pedagogical knowledge (Authors, 2013).

Furthermore, DIF detection results reveal more interesting differences between German and Chinese mathematics teachers’ professional noticing on the aspect of P_PID. First, as reported above, most of the items with uniform DIF, which favor German teachers, belong to the category of “Perception” and are mainly related to aspects like classroom management and students’ and teachers’ behavior, such as whether or not students actively participate in the lesson. First, differences as such may be due to differences of teaching culture existing in the two countries. As has been widely discussed in literature, under the influence of Confucian culture, the teacher in China is an authority in teaching, and students need to respect teachers and consequently behave well and participate actively in the teaching-learning process (e.g., Cai &

Wang, 2010). Due to this Confucian culture, to a large degree classroom management is relatively easy for Chinese teachers, especially since many routines are well-established at the beginning of a semester. Therefore, Chinese teachers may not spend much time or attention on classroom management. Teachers' activities, like the presentation of the mathematical task visually and acoustically, are a basic ability for Chinese teachers, who need to explain questions clearly, assisted by the well-organized, clear, and neat writing on the blackboard. Based on this culturally shaped educational experience, Chinese teachers may take a well-organized flow of teaching-and-learning processes for granted. However, in Western teaching culture, effective teaching and learning cannot take place in poorly managed classrooms (Jones & Jones, 2012). Therefore, due to this cultural influence, German teachers must pay attention to classroom management, students' behavior, and student-teacher-interactions.

The two items, which favor Chinese teachers in the category of "Perception", relate to students' learning motivation and thinking, which may be surprising at a first glance. However, because in the most recent mathematics curriculum reform in China, students' interests, motivation, and confidence have been highly emphasized, Chinese mathematics teachers may have already started to emphasize such aspects in their current teaching. In addition, besides the learning of knowledge, the emphasis on the development of students' mathematical thinking has a long tradition in China (Cai & Wang, 2006). Therefore, it is understandable why Chinese mathematics teachers pay particular attention to these aspects while observing the video clips.

For the second category of P_PID, "Analyzing and Decision Making," three items favor Chinese teachers on the aspects of modifying the teaching sequence shown in the video clip and pointing out why the teachers' behavior is not suitable for encouraging students' cognitive activities. As mentioned above, both aspects - making changes to a teaching sequence and

judging the inadequacy of this teaching sequence - require a process of holistic and deep analysis of the teaching just observed by the teachers. Although Chinese teachers have only limited opportunities to acquire such knowledge during teacher education, they have plenty of opportunities to observe and evaluate other teachers' instruction after they start teaching in school (e.g., in activities organized by schools or the districts they are working, such as teaching research group activities, public lessons, and lessons studies) (Han & Paine, 2010). In these activities, besides observing and evaluating colleagues' teaching, Chinese teachers may also be required to develop different lesson plans for the same mathematical topic or to modify lessons repeatedly to improve teaching quality (Huang, Su, & Xu, 2014). These activities may support many Chinese teachers in developing their abilities to make alternative lessons plans to a specific topic or to modify and revise lesson plans flexibly. Moreover, teachers may also have learned how to evaluate a lesson systematically and point out the inadequacy of the teaching. In contrast, German mathematics teachers, due to the difference of working culture (i.e., so-called closed classroom doors), usually do not have such opportunities to develop these skills. Therefore, it is expected that the German teachers may not be able to analyze holistically and deeply the teaching shown in the video clips and develop modifications for teaching.

Differences on Mathematics Instruction-related Professional Noticing (M_PID)

The Chinese mathematics teachers performed significantly better than the German teachers on the mathematics education-related facet of professional noticing. This finding is quite consistent with findings from previous international comparative studies on teachers' knowledge. For example, in TEDS-M, pre-service secondary mathematics teachers from East Asian cultural contexts like Taiwan and Singapore outperformed the German pre-service mathematics teachers concerning the knowledge facets of mathematical content knowledge (MCK) and mathematics

pedagogical content knowledge (MPCK) (Authors, 2014). Similarly, Kleickmann et al. (2015) reported that secondary mathematics teachers from Taiwan outscored secondary mathematics teachers from Germany on the aspects of MCK and PCK as well. As already mentioned above, teachers' knowledge will heavily impact what they notice in teaching (Schoenfeld, 2011). Therefore, the differences in teachers' mathematics content knowledge and pedagogical content knowledge as identified between teachers from German and Confucian cultural contexts may further lead teachers to perceive and reason mathematical teaching differently.

Similarly, as argued earlier, the differences of opportunities to acquire MCK and MPCK provided during teacher education may first contribute to the differences of teachers' mathematics instruction-related professional noticing. Wang and Tang (2013) report that German pre-service secondary mathematics teachers have fewer opportunities to learn advanced mathematical topics and school-level mathematics as pre-service teachers coming from a Confucian culture like Taiwan and Singapore. Overall, pre-service mathematics teachers in Mainland China also need to learn advanced mathematics and attend several courses at school level mathematics. The differences of university learning opportunities may on the one hand equip teachers with different knowledge foundations on MCK and MPCK, which will lead teachers to notice professionally differently. On the other hand, the differences of training experience may already directly make teachers' noticing different, like attending more to the content other than other aspects. Indeed, as reviewed earlier, in a comparative study between Chinese and American mathematics teachers' noticing of videos, Chinese teachers tended to comment more on mathematics content compared to pedagogical issues (Miller & Zhou, 2007).

In addition, DIF detection results revealed further interesting differences between German and Chinese mathematics teachers' professional noticing on the aspect of M_PID. First,

the items on “mathematical modeling” and “open-ended questions” were in favor of the German teachers. This difference may be mainly due to the differences of mathematics curriculum traditions between these two countries. In Germany, there has been a long tradition of the development of students’ mathematics modeling competence and the use of open-ended questions; furthermore, relevant content has already been included in elementary and secondary school mathematics curriculum for more than a decade. However, in China, only in the latest mathematics curriculum standard from 2000 have the first approaches on mathematics modeling and open-ended questions been included; traditionally, mathematics curriculum and mathematics teaching had been dominated by closed routine problems (Cai & Nie, 2007). Therefore, Chinese teachers were likely to have been less familiar with these topics and did not have such rich knowledge and skills.

The other two items in favor of the German teachers were requiring teachers to identify students’ working preferences (i.e., formal versus visual approach). This difference may also be influenced by differences of mathematics teaching traditions between these two countries. Chinese teachers tend to emphasize the purity, generality, and logic of mathematics in teaching; to meet this aim, they tend to use more formal representations of mathematics (e.g., arithmetic, and algebraic symbolic representation) other than visual representations (e.g., diagrams) of mathematics (Cai & Wang, 2006). In contrast, in German mathematics education, the usage of a variety of representations is strongly recommended in the standards for secondary mathematics education and is emphasized in mathematics teacher education.

The items, which were in favor of Chinese teachers, referred to evaluating students’ answers mathematically, judging characteristics of tasks in teaching, and developing teaching sequences to facilitate students’ understanding. First, the differences of content knowledge in

mathematics between German and Chinese teachers may explain these differences. As described above, mathematics teachers from East Asian countries have according to the corresponding studies deeper knowledge in mathematics. Therefore, a deeper understanding of mathematics and its teaching allows Chinese teachers to judge the correctness of students' work and notice students' errors more easily than German teachers. According to Chinese teaching culture, Chinese teachers need to attend activities like Chinese lesson study and have the opportunities to observe and evaluate other teachers' teaching and develop modified lesson plans. Therefore, they may be more skillful in pointing out the critical characteristics of teaching activities and in modifying lesson plans to a certain topic. Furthermore, teaching with variation is an important characteristic of East Asian teaching culture, Chinese mathematics teachers need to design their teaching from various perspectives (Huang, Barlow, & Prince, 2016). Thus, comparatively speaking, Chinese teachers have more knowledge and experience to suggest different possible ways to correct students' misconception.

Conclusions and Limitations

Although it has been argued recently that teachers' professional noticing is a societally and culturally shaped construct (e.g., Louie, 2018), there is little empirical evidence available so far in literature to support this argument. The central goal of the present study was to close this research gap and investigate and compare Chinese and German mathematics teachers' professional noticing and their influencing factors. Therefore, due to the strong differences between East Asian and Western countries, which are currently discussed, we selected Germany as prototype for Western cultures and China as a prototype for East Asian cultures. Our study could provide empirical evidence that mathematics teachers' professional noticing is strongly influenced by these different societal and cultural contexts in which the teachers are working.

As reported above, German teachers performed significantly better than Chinese mathematics teachers in noticing in the general pedagogical domain. In contrast, Chinese teachers outperformed their German counterparts in noticing in the mathematical instruction domain. Further DIF detection analysis found that German mathematics teachers particularly demonstrated strength on the aspect of “Perception,” but in contrast, Chinese mathematics teachers tended to demonstrate strength on the aspect of “Analyzing and Decision Making.” Although the processes of teachers’ professional noticing are described as consisting of the three steps (i.e., attending, interpreting, and responding), which are “interrelated and cyclical” (Sherin et al., 2011, p. 5), the study results suggest that teachers from different contexts may demonstrate a certain pattern of strength or weakness within these processes. More specially, influenced by the societal and cultural differences existing in Germany and China, such as traditions in teacher education, mathematics teaching, mathematical curriculum and working culture, German teachers mainly demonstrated strength on the process of “Perception.” In detail, they were more likely to notice aspects like classroom management and process-oriented mathematical skills like mathematical modeling, which are important or familiar to them. In contrast, Chinese teachers were more likely to show strength on the latter two processes, namely, analyzing and decision making. The Chinese teachers performed better on aspects like modifying teaching sequences or tasks as shown in the video-clips, identifying students’ errors, and suggesting alternative ways to facilitate students’ understanding.

Although the present study is so far one of the very few cross-cultural comparative studies in the field of mathematics teachers’ professional noticing and has identified societal and cultural factors that impact teachers’ professional noticing, limitations of the study should also be discussed. First of all, participants in the study were chosen mainly from one city in each of the

two countries and only at junior or lower secondary school level. Therefore, the samples in the present study may not be typical and representative enough to reflect the general situation or diversity of in-service mathematics teachers' professional noticing in each of the two countries. Future studies need to consider involving more teachers from a wider geographical range and include teachers from other school levels, like primary school or senior school (upper secondary) level. Such an enrichment will provide a deeper understanding of teachers' professional noticing and richer description, on how the specific societal and cultural context influence the shape and development of teachers' professional noticing.

Furthermore, the ways to test teachers in the present study were different in these two countries. In Germany, web-based testing took place, in contrast to China, where a paper-and-pencil test was used due to practical difficulties. The different ways of testing teachers might also impact teachers' noticing to a certain degree. Therefore, generalization of the findings of the present study must be dealt with cautiously. Further studies may consider using the same method of data collection to improve the trustworthiness of the findings.

References

- Authors (2017). *International handbook of research on teacher education*. Thousand Oaks: Sage.
- Authors (2017). *Educational Studies in Mathematics*.
- Authors (2015). *International Journal of Science and Mathematics Education*.
- Authors (2014). *Teaching and Teacher Education*.
- Authors (2014). *International perspectives on teacher knowledge, beliefs and opportunities to learn*. Dordrecht: Springer.
- Authors (2013). *International Journal of Science and Mathematics Education*.
- Authors (2013). *ZDM Mathematics Education*.

- Alexander, R. (1999). Culture in pedagogy, pedagogy across cultures. In R. Alexander, P. Broadfoot, & D. Philips (Eds.), *Learning from comparing. New directions in comparative educational research* (Vol. 1, pp. 149–180). Oxford: Symposium Books
- Ball, D. L. (2011). Foreword. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. xx–xxiv). New York: Routledge.
- Barnhart, T., & van Es, E. (2015). Studying teacher noticing: Examining the relationship among pre-service science teachers' ability to attend, analyze and respond to student thinking. *Teaching and Teacher Education*, 45, 83-93.
- Cai, J., & Nie, B. (2007). Problem solving in Chinese mathematics education: Research and practice. *ZDM Mathematics Education*, 39(5-6), 459-473.
- Cai, J., & Wang, T. (2006). US and Chinese teachers' conceptions and constructions of representations: A case of teaching ratio concept. *International Journal of Science and Mathematics Education*, 4(1), 145-186.
- Cai, J., & Wang, T. (2010). Conceptions of effective mathematics teaching within a cultural context: Perspectives of teachers from China and the United States. *Journal of Mathematics Teacher Education*, 13(3), 265-287.
- Carter, K., Cushing, K., Sabers, D., Stein, P. & Berliner, D. C. (1988). Expert-novice differences in perceiving and processing visual information. *Journal of Teacher Education*, 39, 25-31.
- Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education*, 34, 12-25.
- Ding, L., & Domínguez, H. (2016). Opportunities to notice: Chinese prospective teachers noticing students' ideas in a distance formula lesson. *Journal of Mathematics Teacher Education*, 19(4), 325-347.
- Dreher, A., & Kuntze, S. (2015). Teachers' professional knowledge and noticing: The case of multiple representations in the mathematics classroom. *Educational Studies in Mathematics*, 88(1), 89-114.
- Erickson, F. (2011). On noticing teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 17-34). New York, NY: Routledge.
- Goodwin, C. (1994). Professional Vision. *American Anthropologist*, 96(3), 606-633.
- Hambleton, R. K., Swaminathan, H., & Rogers, H. J. (1991). *Fundamentals of item response theory*. Newbury Park, CA: Sage Publications.
- Han, X., & Paine, L. (2010). Teaching mathematics as deliberate practice through public lessons. *The Elementary School Journal*, 110(4), 519-541.

- Huang, R., Barlow, A. T., & Prince, K. (2016). The same tasks, different learning opportunities: An analysis of two exemplary lessons in China and the US from a perspective of variation. *The Journal of Mathematical Behavior*, 41, 141-158.
- Huang, R., Su, H., & Xu, S. (2014). Developing teachers' and teaching researchers' professional competence in mathematics through Chinese Lesson Study. *ZDM Mathematics Education*, 46(2), 239-251.
- Jacobs, V. R., Lamb, L. L. C. & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41, 169–202.
- Jacobs, V., Lamb, L., Philipp, R. & Schappelle, B. (2011). Deciding how to respond on the basis of children's understandings. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 97–116). New York: Routledge.
- Jodoin, M. G., & Gierl, M. J. (2001). Evaluating type I error and power rates using an effect size measure with the logistic regression procedure for DIF detection. *Applied Measurement in Education*, 14, 329-349.
- Jones, V., & Jones, L. (2012). *Comprehensive classroom management: Creating communities of support and solving problems*. Boston, MA: Pearson.
- Kersting, N. B., Givvin, K. B., Sotelo, F. L., & Stigler, J. W. (2010). Teachers' analyses of classroom video predict student learning of mathematics: Further explorations of a novel measure of teacher knowledge. *Journal of Teacher Education*, 61(1-2), 172-181.
- Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S., & Baumert, J. (2015). Content knowledge and pedagogical content knowledge in Taiwanese and German mathematics teachers. *Teaching and Teacher Education*, 46, 115–126.
- Louie, N. L. (2018). Culture and ideology in mathematics teacher noticing. *Educational Studies in Mathematics*. 97(1), 55-69.
- Mason, J. (2016). Perception, interpretation and decision making: understanding gaps between competence and performance—a commentary. *ZDM Mathematics Education*, 48(1-2), 219-226.
- Meschede, N., Fiebranz, A., Möller, K., & Steffensky, M. (2017). Teachers' professional vision, pedagogical content knowledge and beliefs: On its relation and differences between pre-service and in-service teachers. *Teaching and Teacher Education*, 66, 158-170.
- Mesic, V. (2012). Identifying country-specific cultures of physics education: A differential item functioning approach. *International Journal of Science Education*, 34(16), 2483-2500.
- Miller, K., & Zhou, X. (2007). Learning from classroom video: What makes it compelling and what makes it harder. In R. Goldman-Segal, & R. Pea (Eds.), *Video research in the learning sciences* (pp. 321–334). Mahwah, NJ: Lawrence Erlbaum.

- Schoenfeld, A. H. (2010). Noticing matters. A lot. Now what? In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 223–238). New York: Routledge.
- Seidel, T., & Stürmer, K. (2014). Modeling and measuring the structure of professional vision in preservice teachers. *American Educational Research Journal*, 51(4), 739-771.
- Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (Eds.). (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. New York, NY: Routledge.
- Swaminathan, H., & Rogers, H. J. (1990). Detecting differential item functioning using logistic regression procedures. *Journal of Educational Measurement*, 27, 361-370.
- Thomas, J. N. (2017). The Ascendancy of Noticing: Connections, Challenges, and Questions. In E. O. Schack, H.F. Molly, & J.A. Wilhelm (Eds.) *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 507-514). New York: Springer.
- van Es, E. A., Cashen, M., Barnhart, T., & Auger, A. (2017). Learning to notice mathematics instruction: Using video to develop preservice teachers' vision of ambitious pedagogy. *Cognition and Instruction*, 35(3), 1-23.
- van Es, E. A., Hand, V., & Mercado, J. (2017). Making visible the relationship between teachers' noticing for equity and equitable teaching practice. In E. O. Schack, H.F. Molly, & J.A. Wilhelm (Eds.) *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 251-270). New York: Springer.
- van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, 10(4), 571-596.
- van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education*, 24(2), 244-276.
- Wang, T. Y., & Tang, S. J. (2013). Profiles of opportunities to learn for TEDS-M future secondary mathematics teachers. *International Journal of Science and Mathematics Education*, 11(4), 847–877.