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Citation: [Physics of Fluids](#) **30**, 067102 (2018); doi: 10.1063/1.5034212

View online: <https://doi.org/10.1063/1.5034212>

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Extreme wave statistics of counter-propagating, irregular, long-crested sea states

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(Received 10 April 2018; accepted 31 May 2018; published online 21 June 2018)

Extreme wave statistics of unidirectional and counter-propagating seas are investigated, for the special case of long-crested irregular waves, with laboratory experiments and with numerical simulations using the higher order spectral method. Both the kurtosis of the surface elevation and the exceedance probability of the crest height are larger in unidirectional seas than in counter-propagating seas. Numerical simulations show that even a small amount of wave energy travelling against an essentially unidirectional wave system can significantly reduce the kurtosis of the surface elevation. *Published by AIP Publishing.* <https://doi.org/10.1063/1.5034212>

I. INTRODUCTION

It has been observed that ship accidents often occur in crossing seas; see, for example, the review by Toffoli *et al.*³² Two examples are the Louis Majesty accident which happened in a crossing sea state with crossing angles of about 40° – 60° , investigated by Cavaleri *et al.*,⁶ and the Prestige accident which happened in a crossing sea state with crossing angles of about 90° , investigated by Trulsen *et al.*³⁴

The nonlinear dynamics of two coupled gravity wave systems with arbitrary separation angles, though well separated in wave vector space, can be described by a system of coupled nonlinear Schrödinger (CNLS) equations. This was anticipated by Benney and Newell,² and equations were later presented by Roskes²⁴ for the limiting case that modulations only occur in the mean direction in which the projection of group velocities coincide. CNLS equations were established by Hammack *et al.*,¹⁴ while Bridges and Laine-Pearson⁵ developed a general Hamiltonian framework for their description. CNLS equations were also presented by Onorato *et al.*²¹ for the case that modulations can occur in any direction. For the special limit that the waves are counter-propagating, CNLS equations were presented by Okamura¹⁹ whose results were generalized to gravity–capillary waves by Pierce and Knobloch.²³ Generalisations to coupled higher order or modified nonlinear Schrödinger (CMNLS) equations were presented by Gramstad and Trulsen¹¹ for gravity waves and by Debsarma and Das⁸ for the special case of counter-propagating capillary–gravity waves.

Based on the CNLS equations, Roskes²⁴ found that a system of two interacting Stokes waves is subject to modulational instability due to perturbations in the mean direction in which the projections of group velocities coincide for separation angles less than 70.5° and greater than 136.1° . This result was revisited by Onorato *et al.*,²¹ and generalized to instabilities due to modulations in any direction by Shukla *et al.*²⁶ and Laine-Pearson.¹⁵ For the special limit of

two counter-propagating Stokes waves, Okamura¹⁹ computed unstable growth rates based on the Zakharov equation. For two waves with arbitrary angles of separation, instability analysis based on the CMNLS equations was presented by Gramstad and Trulsen¹¹ and more accurate results based on Zakharov equations were given by Gramstad *et al.*¹² These stability analyses have shown that crossing Stokes waves can have enhanced modulational instability compared to a single Stokes wave.

Due to the possibility of enhanced modulational instability, it was anticipated by Onorato *et al.*²¹ that crossing seas could experience an increased occurrence of extreme waves compared to non-crossing seas. Numerical integration of the coupled NLS equations by Shukla *et al.*²⁶ and by Grönlund *et al.*¹³ demonstrated the occurrence of freak waves for such wave systems. Onorato *et al.*²² showed by means of numerical simulation with a higher order spectral method (HOSM) that rogue waves were more probable in a crossing sea with crossing angles in the range of 40° – 60° . Toffoli *et al.*³³ showed good agreement between experiments and numerical simulations using HOSM, with a maximum of kurtosis for angles between 40° and 60° . Numerical simulations with HOSM for more realistic wave fields by Bitner-Gregersen and Toffoli³ also yielded the same dependence on the crossing angle.

It is known, by the work of Alber¹ and Crawford *et al.*,⁷ that as the bandwidth of a single wave system increases, while the steepness is held constant, the modulational instability will be suppressed. This result was generalized to crossing wave systems by Shukla *et al.*²⁷

In the present paper, we investigate counter-propagating long-crested irregular gravity wave systems with an identical peak period and essentially identical JONSWAP (Joint North Sea Wave Project) spectrum. We consider both the laboratory experiments in a long, narrow flume, as well as numerical simulations with HOSM. Wave modulations in the transversal direction are excluded from our experiments and simulations. In the laboratory, we substituted a damping beach with a reflecting wall to achieve counter-propagating wave systems. In the numerical simulations, we employed a numerical beach that only partially damped the incoming waves, allowing a substantial amount of wave energy to be reflected.

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If the two JONSWAP spectra had been reduced to a pair of Stokes waves, the relevant modulational instabilities would be indicated along the first axis of Fig. 3(a) of Okamura¹⁹ or along the second axis of Fig. 4 of Gramstad *et al.*¹² However, we anticipate that our JONSWAP wave fields are sufficiently wide that the observed wave evolution is not characterized by modulational instabilities.

After describing the experiments in Sec. II and the numerical simulations in Sec. III, we partition the directional spectrum into waves propagating in opposite directions as a necessary prerequisite to discuss the steepness of the wave field in Sec. IV. Analysis of kurtosis and skewness, with particular attention to the degree of wave reflection and the steepness of the two wave systems, is given in Sec. V, and analysis of exceedance probability of crest heights is given in Sec. VI. Our final conclusion is to anticipate fewer extreme waves in counter-propagating seas than in unidirectional seas.

II. SETUP OF THE LABORATORY EXPERIMENTS

The experiments were performed in a wave flume at the Department of Mathematics at the University of Oslo. The dimensions of the tank and the setup are sketched in Fig. 1. The wave tank is 24.6 m long, 0.5 m wide, and was filled to a mean water depth $h = 0.5$ m.

Waves were mechanically generated at one end of the tank by a hydraulic piston, programmed according to a JONSWAP spectrum with peak period $T_p = 0.70$ s and peak enhancement factor 3.3. This corresponds to peak angular frequency $\omega_p = 9.0$ rad/s, and for the given water depth, we anticipate peak wavelength $\lambda_p = 0.76$ m, wavenumber $k_p = 8.2$ rad/m, and nondimensional depth $k_ph = 4.1$.

A 3 m long beach at the opposite end of the tank damped the wave field in the case of a unidirectional wave system. When inserting a reflecting wall just in front of the beach, approximately 60% of the incoming wave energy was reflected to form a counter-propagating wave system.

The surface elevation was measured with an array of 16 ultrasonic probes at an equidistant spacing of 30 cm. The experiments were repeated three times, with the array placed at three different locations, resulting in a uniform grid of 48 equidistant measurement points along the tank. The ultrasonic probes measured the surface elevation at a sampling frequency of 200 Hz, resulting in 140 measurements per peak period in time as opposed to 2.5 measurements per peak wavelength in space.

FIG. 1. Sketch of laboratory experiments in the wave tank.

TABLE I. Key parameters for experiments and simulations.

	Laboratory	Simulation
Number of points along the tank	48	512
Total time (s)	840	200
Start time for analysis (s)	90	90
Analysis time (peak periods)	1071	157
Number of Monte Carlo simulations	1	44 (231)

Key parameters for the experiments are summarized in Table I. Before analyzing the measurements, noise in the data was cleaned and startup effects were removed. Further details of the laboratory experiments and the techniques for filtering the data are given by Rye.²⁵ Visual inspection during the experiments suggested the absence of wave breaking.

The experiments were designed to provide two desired values of wave steepness $\epsilon = k_p a_c$ where $a_c = H_s/\sqrt{8}$ is a characteristic amplitude and where H_s is the significant wave height defined as four times the standard deviation of the surface elevation. The two desired values were $\epsilon = 0.1$ and $\epsilon = 0.06$, denoted in the following as Case 1 and Case 2, respectively. These two values were chosen in order to have a “strong” case close to the maximum steepness observed in reality and a “weaker” case well below this limit. See, for example, Fig. 2 of Socquet-Juglard *et al.*²⁹ and Fig. 7 of Simanese *et al.*²⁸

The specification of steepness in this experiment is complicated, partly due to wave attenuation and partly due to the fact that the concept of the steepness of a crossing sea is not well defined unless we partition the directional spectrum into individual wave systems as discussed below.

III. SETUP OF THE NUMERICAL WAVE TANK

Numerical simulations were carried out with a higher order spectral method, the HOS-NWT numerical wave tank described by Bonnefoy *et al.*⁴ and Ducroz *et al.*⁹ We applied a 1D wave tank with the same length and depth as in the laboratory. A numerical wave paddle at one end of the tank was programmed to generate a JONSWAP spectrum with the same peak period and peak enhancement factor as described above. The performance characteristics of the paddle were set to the default values. The damping beach was simulated by the inherent numerical beach with an absorption coefficient of unity, while the reflecting wall was modeled by a short beach with an absorption coefficient of 0.06 in order to limit the effect of energy accumulation in the wave tank. The standard higher order spectral method does not include dissipation, and so operating the wave paddle will cause a constant increase of energy in the tank unless the waves are damped at the beach. We did not achieve both wave reflection and constant energy in the tank over time; however, we did achieve energy growing slowly within the time frame of the simulation. As a result, the reflected energy was much lower in the numerical simulations (approximately 25%) than in the laboratory experiments (approximately 60%).

Key parameters for the numerical simulations are listed in Table I. The number of Monte Carlo simulations was 44

for all cases, except for Case 1 for which we carried out 231 simulations.

IV. PARTITION OF THE DIRECTIONAL SPECTRUM

Following the approach of Trulsen *et al.*,³⁴ we partition the spectrum into wave fields propagating out from the wave maker (to the right) and back from the reflecting wall or beach (to the left). This can be done by means of the two-dimensional spectrum $S(k, \omega)$, where the wavenumber k corresponds to the x -axis oriented from the wave maker toward the reflecting wall or beach. In the following, we present the one-sided (k, ω)-spectrum, for $k \geq 0$, such that the first quadrant of the (k, ω)-plane corresponds to waves going to the right and the fourth quadrant corresponds to waves going to the left.

The wavenumber-frequency spectra for Case 1 of the laboratory experiments and the simulations are given in Figs. 2 and 3, respectively. We have deliberately focused the domain around the peaks of the spectrum ($k_p, \pm \omega_p$), disregarding non-linear contributions at higher harmonics. The laboratory data suffer from coarse resolution in space, which leads to strong aliasing for values slightly larger than the peak. Following

Nieto Borge *et al.*,¹⁸ the de-aliased spectrum has been retrieved by rearranging tiles of the computed aliased spectrum.

The directional spectrum is partitioned by distinguishing waves going to the right ($\omega > 0$) and to the left ($\omega < 0$). Integrating over only one of the quadrants at a time, we find the global significant wave heights of the wave systems going to the right and to the left, $\bar{H}_{sr}$ and $\bar{H}_{sl}$, as four times the standard deviation of the surface elevation corresponding to either the first or the fourth quadrant, respectively. Assuming that the two wave fields, going to the right and to the left, are uncorrelated, we obtain the total global significant wave height as

$$\bar{H}_s = \sqrt{\bar{H}_{sr}^2 + \bar{H}_{sl}^2}. \quad (1)$$

On the other hand, we also compute the local significant wave height $H_s(x)$, for each fixed position x along the flume, as four times the standard deviation of the surface elevation by time averaging at each fixed position.

The significant wave heights are presented as dimensionless steepnesses by the scaling

$$\{\epsilon(x), \bar{\epsilon}_r, \bar{\epsilon}_l\} = \frac{k_p}{\sqrt{8}} \{H_s(x), \bar{H}_s, \bar{H}_{sl}\}. \quad (2)$$

FIG. 2. One-sided (k, ω)-spectrum for laboratory experiment Case 1 and reconstruction of the de-aliased spectrum: the aliased spectrum for damping beach (left), de-aliased spectrum for damping beach (middle), and de-aliased spectrum for reflecting wall (right).

FIG. 3. One-sided (k, ω)-spectrum for numerical simulation Case 1: Unidirectional waves (left) and counter-propagating waves (right).

FIG. 4. Local steepness $\epsilon(x)$ (solid line with markers) and total global steepness $\bar{\epsilon}$ (dashed line) for laboratory experiments and numerical simulations.

We define the reflection coefficient of the wall or beach as the proportion of right-going energy that is reflected to the left, and for this computation, we use the global significant wave heights or steepnesses,

$$R = \frac{\bar{H}_{sl}^2}{\bar{H}_{sr}^2} = \frac{\bar{\epsilon}_l^2}{\bar{\epsilon}_r^2}. \quad (3)$$

TABLE II. Global steepnesses and reflection coefficients for experiments and simulations.

Case	Case 1				Case 2			
Parameter	$\bar{\epsilon}$	$\bar{\epsilon}_r$	$\bar{\epsilon}_l$	R	$\bar{\epsilon}$	$\bar{\epsilon}_r$	$\bar{\epsilon}_l$	R
Lab unidirectional	0.089	0.087	0.017	0.04	0.064	0.063	0.012	0.04
Lab crossing	0.100	0.080	0.063	0.62	0.068	0.054	0.041	0.59
Sim unidirectional	0.092	0.091	0.013	0.02	0.059	0.059	0.005	0.01
Sim weak crossing	0.092	0.090	0.016	0.03	0.058	0.057	0.007	0.02
Sim moderate crossing	0.092	0.088	0.027	0.10	0.058	0.056	0.015	0.07
Sim crossing	0.094	0.084	0.043	0.26	0.058	0.052	0.026	0.25

FIG. 5. Two-sided frequency spectra for Case 1. All spectra are scaled by the peak value of the spectrum of the unidirectional case. Left: Laboratory experiments with “unidirectional” (red) and “crossing” (blue). Right: Simulations with “unidirectional” (red), “weak crossing” (purple), “moderate crossing” (green), and “crossing” (blue).

The local steepness $\epsilon(x)$ and the total global steepness $\bar{\epsilon}$ are shown in Fig. 4. Table II gives a summary of the global steepnesses and the reflection coefficients for all experiments and simulations.

The two-sided ω -spectra in Fig. 5 have been obtained by integrating the one-sided (k, ω) -spectra over $k > 0$.

V. KURTOSIS AND SKEWNESS

The local kurtosis κ and the local skewness γ were computed for each fixed position along the tank. For Case 1 and Case 2, for laboratory and simulation, and for unidirectional and crossing wave systems (see Table II), the kurtosis is shown in Fig. 6 and the skewness is shown in Fig. 7. In all cases, 80% confidence intervals are indicated.

For the laboratory measurements, the time series from each probe was divided into 10 subintervals. Skewness and kurtosis were calculated for each subinterval and the highest and lowest values were discarded, resulting in the desired confidence interval. The mean over those values is indicated inside the confidence interval.

For the simulations, we have employed a lower time resolution of $\Delta t = 0.2$ s and the part of the simulation time subject to analysis is limited to 110 s. To ensure sufficient surface elevation measurements for adequate estimates of kurtosis and skewness, 16 neighboring grid points were considered as one evaluation point in space, and four (24 in Case 1) simulations were merged; thus, the mean values and the confidence intervals are based on 44 (216) simulations arranged into 11 (9) groups.

FIG. 6. Local kurtosis along the tank for Case 1 (upper row) and Case 2 (lower row) for laboratory (left column) and simulation (right column), with 80% confidence intervals.

Both the experiments and the simulations exhibit higher local kurtosis for unidirectional seas than for crossing seas for both steepness cases. The difference in local kurtosis is more pronounced in the simulations.

While the experiments do not reveal a trend in local skewness, the simulations show increased local skewness for the unidirectional sea.

In order to further investigate the dependence of kurtosis and skewness on wave steepness, we carried out several additional simulations at other values of steepnesses. The resulting estimates of global kurtosis and global skewness are shown in Fig. 8. The 80% confidence intervals (dotted lines) and

FIG. 7. Local skewness along the tank for Case 1 (upper row) and Case 2 (lower row) for laboratory (left column) and simulation (right column), with 80% confidence intervals.

the means (solid lines) were calculated by averaging over the spatial interval $[5\lambda_p, 23\lambda_p]$ along the tank. The laboratory results have been included as two pairs of data points. Theoretical results are also indicated for unidirectional sea states considered by Mori and Janssen¹⁶ and Srokosz and Longuet-Higgins.³⁰

Figure 8 also shows the difference in dependence of global kurtosis and global skewness on total global steepness $\bar{\epsilon}$ and global steepness of the waves going to the right $\bar{\epsilon}_r$. For unidirectional waves, we have $\bar{\epsilon} = \bar{\epsilon}_r$. From this comparison, we anticipate that the total global steepness $\bar{\epsilon}$ is the correct parameter for this analysis, at least for a proper characterization of the laboratory experiments.

FIG. 8. Global kurtosis (left column) and global skewness (right column) against total global steepness (top row) and global steepness of the wave system going to the right (bottom row). The mean (solid line) and 80% confidence interval (dotted line) of the simulations, as well as the mean (point) and 80% confidence interval (vertical bar) of the experiments. The black dashed lines are the theoretical results for unidirectional seas of Mori and Janssen¹⁶ and Srokosz and Longuet-Higgins.³⁰

FIG. 9. Global kurtosis for different steepnesses $\bar{\epsilon}$ or $\bar{\epsilon}_r$ and different reflection coefficients: $R = 1.4\%$ “unidirectional” (red), $R = 2.6\%$ “weak crossing” (purple), $R = 8.9\%$ “moderate crossing” (green), and $R = 25.7\%$ “crossing” (blue).

The global kurtosis and global skewness are not only higher for unidirectional waves, they also grow faster with the steepness than for crossing waves.

We also carried out simulations with other damping beaches with different reflection coefficients corresponding to the “weak crossing” and “moderate crossing” rows in Table II. The global kurtosis is shown in Fig. 9 based on averages of 12 simulations for each configuration. For these simulations, we reach the same conclusions regardless of using $\bar{\epsilon}$ or $\bar{\epsilon}_r$.

It is clear from Fig. 9 that even a small reflection coefficient of $R = 2.6\%$ is enough to reduce the global kurtosis significantly.

Our unidirectional Case 1 is comparable to the laboratory experiment of Mori *et al.*¹⁷ whose setup is similar, except that the wave tank and the peak wavelength are longer so that

dissipation is less important. Mori *et al.*¹⁷ used a JONSWAP spectrum with peak enhancement factor 6. The maximum kurtosis values are similar but they occur a few peak wavelengths later than in our simulations.

Gramstad *et al.*¹² have also considered counter-propagating wave fields and observe that the kurtosis of unidirectional waves is higher than equivalent counter-propagating waves, albeit with lower values of kurtosis than obtained herein. Referring to their Fig. 7(a), for a JONSWAP spectrum with peak enhancement factor 6 and rather narrow directional spread, they obtained kurtosis 3.13 for counter-propagating waves and kurtosis 3.18 for unidirectional waves. For a JONSWAP spectrum with peak enhancement factor 3.3 and wider directional spread, they obtained kurtosis 3.10 in both configurations. Their kurtosis values are clearly lower than ours. We believe that the difference in magnitude may be due to the fact that their wave fields are weakly directionally spread, whereas our wave fields are perfectly long-crested. Similar influence of the crest length on kurtosis was previously reported by Onorato *et al.*²⁰ and by Gramstad and Trulsen.¹⁰

VI. EXCEEDANCE PROBABILITY OF CREST HEIGHT

Finally, we show the local exceedance probability of the crest height in Fig. 10. The crest height is defined as the maximum surface elevation between two zero-down crossings. The simulated data were interpolated by the truncated Fourier series in order to determine precisely the maximum crest height; this was not needed for the laboratory data due to the high temporal resolution of the measurements. The horizontal dotted lines mark the limit for the 100 largest observations in the dataset.

FIG. 10. Exceedance probability of the crest height, Case 1: Probe at $7\lambda_p$ (upper row), probe at $15\lambda_p$ (lower row), experiments (left column), and simulations (right column).

The exceedance probability is characterized by local variation, similar to what we previously found for the significant wave height, skewness, and kurtosis. To illustrate the amount of local variation, we show in Fig. 10 the exceedance probability at two locations, at the probe located at $7\lambda_p$ and the probe located at $15\lambda_p$; see Fig. 1 for the locations of these probes.

As approximate reference distributions, we show the Rayleigh distribution corresponding to the limit of a unidirectional narrowband Gaussian wave field and the Tayfun³¹ distribution corresponding to the limit of a unidirectional narrowband wave field with second order bound waves. We have employed the local significant wave height in the corresponding local Tayfun distribution. These are only approximate reference distributions since our wave fields are not narrowband and not all are unidirectional.

We see that the unidirectional wave fields systematically have greater exceedance probability than the counter-propagating wave fields and that the Tayfun distribution better approximates the exceedance probability for the counter-propagating wave fields than for the unidirectional wave fields. This conclusion applies to both experiments and simulations. Thus the exceedance probability for counter-propagating long-crested waves appears to better resemble field observations of ocean waves than what the exceedance probability for unidirectional long-crested waves appears to do.

VII. CONCLUSION

We conducted laboratory experiments and numerical simulations of unidirectional and counter-propagating, irregular, long-crested sea states with a JONSWAP frequency spectrum. We found that both local kurtosis of surface elevation and local exceedance probability of crest height are larger in unidirectional seas than in counter-propagating seas. Even a small amount of waves propagating against an essentially unidirectional wave field is enough to notably reduce the kurtosis of the surface elevation. The wave fields considered are sufficiently wide-banded that we do not expect Benjamin–Feir instability to occur; therefore, we anticipate that the observed behavior is due to nonlinear interactions different from the modulational instability. Based on our observations, we anticipate fewer extreme waves in counter-propagating than in unidirectional, irregular, long-crested sea states.

ACKNOWLEDGMENTS

We are grateful for the constructive remarks from three anonymous referees. This research has been funded by the Research Council of Norway (RCN) through Project Nos. RCN 214556 and RCN 256466.

- ¹Alber, I. E., “The effects of randomness on the stability of two-dimensional surface wavetrains,” *Proc. R. Soc. A* **363**, 525–546 (1978).
- ²Benney, D. J. and Newell, A. C., “The propagation of nonlinear wave envelopes,” *J. Math. Phys.* **46**, 133–139 (1967).
- ³Bitner-Gregersen, E. M. and Toffoli, A., “Occurrence of rogue wave states and consequences for marine structures,” *Ocean Dyn.* **64**, 1457–1468 (2014).
- ⁴Bonnefoy, F., Ducroz, G., Le Touzé, D., and Ferrant, P., “Time-domain simulation of nonlinear water waves using spectral methods,” in *Advances in Numerical Simulation of Nonlinear Water Waves* 2009.

- ⁵Bridges, T. J. and Laine-Pearson, F. E., “The long-wave instability of short-crested waves, via embedding in the oblique two-wave interaction,” *J. Fluid Mech.* **543**, 147–182 (2005).
- ⁶Cavaleri, L., Bertotti, L., Torrisi, L., Bitner-Gregersen, E., Serio, M., and Onorato, M., “Rogue waves in crossing seas: The Louis Majesty accident,” *J. Geophys. Res.* **117**, C00J10, <https://doi.org/10.1029/2012jc007923> (2012).
- ⁷Crawford, D. R., Saffman, P. G., and Yuen, H. C., “Evolution of a random inhomogeneous field of nonlinear deep-water gravity waves,” *Wave Motion* **2**, 1–16 (1980).
- ⁸Debsarma, S. and Das, K. P., “Fourth-order nonlinear evolution equations for counterpropagating capillary-gravity wave packets on the surface of water of infinite depth,” *Phys. Fluids* **14**, 2225–2234 (2002).
- ⁹Ducroz, G., Bonnefoy, F., Le Touzé, D., and Ferrant, P., “A modified high-order spectral method for wavemaker modeling in a numerical wave tank,” *Eur. J. Mech. B/Fluids* **34**, 19–34 (2012).
- ¹⁰Gramstad, O. and Trulsen, K., “Influence of crest and group length on the occurrence of freak waves,” *J. Fluid Mech.* **582**, 463–472 (2007).
- ¹¹Gramstad, O. and Trulsen, K., “Fourth-order coupled nonlinear Schrödinger equations for gravity waves on deep water,” *Phys. Fluids* **23**, 062102 (2011).
- ¹²Gramstad, O., Bitner-Gregersen, E., Trulsen, K., and Nieto Borge, J. C., “Modulational instability and rogue waves in crossing sea states,” *J. Phys. Oceanogr.* **48**, 1317–1331 (2018).
- ¹³Grönlund, A., Eliasson, B., and Marklund, M., “Evolution of rogue waves in interacting wave systems,” *Eur. Phys. Lett.* **86**, 24001 (2009).
- ¹⁴Hammack, J. L., Henderson, D. M., and Segur, H., “Progressive waves with persistent two-dimensional surface patterns in deep water,” *J. Fluid Mech.* **532**, 1–52 (2005).
- ¹⁵Laine-Pearson, F. E., “Instability growth rates of crossing sea states,” *Phys. Rev. E* **81**, 036316 (2010).
- ¹⁶Mori, N. and Janssen, P. A. E. M., “On kurtosis and occurrence probability of freak waves,” *J. Phys. Oceanogr.* **36**, 1471–1483 (2006).
- ¹⁷Mori, N., Onorato, M., Janssen, P. A. E. M., Osborne, A. R., and Serio, M., “On the extreme statistics of long-crested deep water waves: Theory and experiments,” *J. Geophys. Res.* **112**, C09011, <https://doi.org/10.1029/2006jc004024> (2006).
- ¹⁸Nieto Borge, J., Ziemer, F., Seemann, J., and Senet, C., “Overcome the Nyquist limit in frequency in nautical radar measurement of wave fields,” in *17th International Conference on Offshore Mechanics and Arctic Engineering*, Lisbon, Portugal, 1998.
- ¹⁹Okamura, M., “Instabilities of weakly nonlinear standing gravity waves,” *J. Phys. Soc. Jpn.* **53**, 3788–3796 (1984).
- ²⁰Onorato, M., Osborne, A. R., and Serio, M., “Extreme wave events in directional, random oceanic sea states,” *Phys. Fluids* **14**, L25–L28 (2002).
- ²¹Onorato, M., Osborne, A. R., and Serio, M., “Modulational instability in crossing sea states: A possible mechanism for the formation of freak waves,” *Phys. Rev. Lett.* **96**, 014503 (2006).
- ²²Onorato, M., Proment, D., and Toffoli, A., “Freak waves in crossing seas,” *Eur. Phys. J. Spec. Top.* **185**, 45–55 (2010).
- ²³Pierce, R. D. and Knobloch, E., “On the modulational stability of traveling and standing water waves,” *Phys. Fluids* **6**, 1177–1190 (1994).
- ²⁴Roskes, G. J., “Nonlinear multiphase deep-water wavetrains,” *Phys. Fluids* **19**, 1253–1254 (1976).
- ²⁵Rye, L. B., “Freak waves in crossing sea with counterpropagating wave systems,” M.S. thesis, University of Oslo, 2014.
- ²⁶Shukla, P. K., Kourakis, I., Eliasson, B., Marklund, M., and Stenflo, L., “Instability and evolution of nonlinearly interacting water waves,” *Phys. Rev. Lett.* **97**, 094501 (2006a).
- ²⁷Shukla, P. K., Marklund, M., and Stenflo, L., “Modulational instability of nonlinearly interacting incoherent sea states,” *JETP Lett.* **84**, 645–649 (2006b).
- ²⁸Simanesew, A., Krogstad, H. E., Trulsen, K., and Nieto Borge, J. C., “Bimodality of directional distributions in ocean wave spectra: A comparison of data-adaptive estimation techniques,” *J. Atmos. Oceanic Technol.* **35**, 365–384 (2018).
- ²⁹Socquet-Juglard, H., Dysthe, K., Trulsen, K., Krogstad, H. E., and Liu, J., “Probability distributions of surface gravity waves during spectral changes,” *J. Fluid Mech.* **542**, 195–216 (2005).
- ³⁰Srokosz, M. A. and Longuet-Higgins, M. S., “On the skewness of sea-surface elevation,” *J. Fluid Mech.* **164**, 487–497 (1986).
- ³¹Tayfun, M. A., “Narrow-band nonlinear sea waves,” *J. Geophys. Res.* **85**, 1548–1552, <https://doi.org/10.1029/jc085ic03p01548> (1980).
- ³²Toffoli, A., Lefevre, J. M., Bitner-Gregersen, E., and Monbaliu, J., “Towards the identification of warning criteria: Analysis of a ship accident database,” *Appl. Ocean Res.* **27**, 281–291 (2005).

³³Toffoli, A., Bitner-Gregersen, E. M., Osborne, A. R., Serio, M., Monbaliu, J., and Onorato, M., “Extreme waves in random crossing seas: Laboratory experiments and numerical simulations,” *Geophys. Res. Lett.* **38**, L06605, <https://doi.org/10.1029/2011gl046827> (2011).

³⁴Trulsen, K., Nieto Borge, J. C., Gramstad, O., Aouf, L., and Lefèvre, J.-M., “Crossing sea state and rogue wave probability during the Prestige accident,” *J. Geophys. Res.: Oceans* **120**, 7113–7136, <https://doi.org/10.1002/2015jc011161> (2015).