Wave analysis of porous geometry with linear resistance law

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Abstract: The wave diffraction-radiation problem of a porous geometry of arbitrary shape located in the free surface of a fluid is formulated by a set of integral equations, assuming a linear resistance law at the geometry. The linear forces, the energy relation and the mean horizontal drift force are evaluated for non-porous and porous geometries. A geometry of large porosity has an almost vanishing added mass. The exciting forces are a factor of 5-20 smaller compared to a solid geometry. In the long wave regime, the porous geometry significantly enhances both the damping and the mean drift force, where the latter grows linearly with the wavenumber. The calculated mean drift force on a porous hemisphere and a vertical truncated cylinder, relevant to the construction of fish cages, is compared to available published results.

Keywords: Wave analysis, fish cages, mean drift force, wave exciting force, added mass, damping

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1 Introduction¹

Analysis of the loads on floating bodies is an established research discipline (see Faltinsen, 1990) including wave-current-body interaction (see Grue and Palm, 1996) and computational advances of complex geometries (see Newman, 2014). Besides the more common wave analysis of solid floating bodies, more general geometries with slots or pores have caught industrial interest, where one of the applications has been to enhance the damping and reduce the wave loads on offshore structures (see Molin, 2011). Another application is tuned liquid damping of the lateral motions of

land buildings (see Molin and Remy, 2013). The aerodynamic effect of a porous geometry and flow through screens have been analyzed (see Taylor, 1956; Laws and Livesey, 1978). It is commonly assumed that the geometry is represented by a thin wall with the effect of the screens or fine pores modelled by a relation between the pressure jump (Δp) and normal velocity (w_n), where a linear or quadratic resistance law is assumed.

Wave interaction with porous geometries assuming a pressure jump that is linear in w_n has been investigated by Chwang (1983), Chwang and Wu (1994), Chwang and Chan (1998) and Williams *et al.* (2000). In relation to coastal engineering purposes and the practical construction of breakwaters Yu (1995) demonstrated that the neglect of the inertia effect underestimates the functional performance of such porous structures. Huang *et al.* (2011) reviewed the transmission and reflection characteristics and the hydraulic performance of perforated coastal breakwaters including Jarlan (1961) type geometries with or without a back-wall. Wave reflection and transmission from breakwaters were further studied by Liu and Li (2013) while Koley *et al.* (2015a, 2015b) modelled oblique wave scattering and wave trapping at a vertical flexible porous plate, including also a two-layer fluid, see Behera *et al.* (2015).

In relation to marine and offshore hydrodynamics, Zhao *et al.* (2011a) developed generalizations of the common expressions of the linear radiation and diffraction forces as well as obtaining the mean drift force, where calculations were obtained for a vertical truncated cylinder assuming normal mode expansions of the external and internal wave potentials. In Zhao *et al.* (2011b) they obtained experimental relations between the porosity factor and the opening fraction of the geometry, in combination with the wave slope of the incoming waves. A quadratic relation between the pressure drop and normal velocity, i.e., $\Delta p = \text{const.} \times w_n^2$, has been pursued by Molin (2001,

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2011), An and Falinsen (2012) and Molin and Remy (2013), in wave radiation and diffraction studies of porous or slotted geometries.

In the present paper, the main novelties, for a porous geometry located in the free surface, assuming a linear resistance law, include: a) a wave analysis of porous geometries of arbitrary shape; b) a formulation based on integral equations valid for bodies in three dimensions; c) numerical representation and calculation of the various force coefficients of the general body including the energy balance; d) particular efforts obtaining the linear exciting force and mean drift force where the latter is second order in the wave amplitude; e) practical evaluation of these quantities, where the exciting force and mean drift force are important quantities in a wave analysis of floating fish farms, for example. The evaluation of the mean drift force on the porous geometry – of arbitrary shape – is a particular novel contribution, since none of the above mentioned publications have considered this force, or its numerical values have been obtained erroneously as in Zhao *et al.* (2011a). Thus, particularly, section 5 of the paper is devoted to the evaluation of the mean drift force. Computations for porous and solid geometries including a hemisphere and a truncated vertical cylinder – where both geometries are relevant for the case of fish cages – are carried out with comparison to existing publications of the solid geometry case published by Grue and Biberg (1993) and Molin (1994).

The paper is organized as follows: Section 2 describes the mathematical representation of the flow exterior and interior to the geometry exposed to the incoming waves as well as the radiation problem. In section 3 we develop the kinematic-dynamic boundary condition at the porous geometry. Section 4 describes the linear forces including added mass, damping and the exciting forces, as well as the energy relation, and their numerical evaluation. In section 5 we rederive the mathematical expression for the mean horizontal drift force and perform numerical calculations for solid and porous geometries with comparison to existing publications for the solid body case (Grue and Biberg, 1993; Molin, 1994) as well as for the porous body case (Zhao *et al.*, 2011a) correcting the numerical calculations in the latter for an obvious error. Finally section 6 is a conclusion.

2 Mathematical formulation

A coordinate system is introduced with the x and y axis in the horizontal plane and z the vertical axis, with the water surface at rest at $z = 0$ (Fig. 1). The incoming waves interact with a floating geometry which is assumed to have a porous boundary. The geometry is

floating because of a buoyant element at the intersection between the geometry and the free surface, but this part of the geometry is disregarded in the present analysis where only the effect of the main porous geometry is considered. The fluid motion exterior and interior to the geometry is assumed to be inviscid and irrotational, and obtained by linear potential theory. The analysis is carried out in the frequency domain where the (periodic) incoming waves are characterized by the frequency ω , wavenumber K , wave angle β and wave amplitude A .

2.1 Representation of the exterior and interior wave fields

We assume in this application that the motion of the porous geometry may be modelled as a rigid body motion. The more general case where the geometry is flexible may be modelled within linear theory, assuming an additional set of motion modes corresponding to the flexible modes of the geometry. The flexible case is left for future study.

A periodic motion in time of frequency ω is assumed. Assuming a rigid body motion, the velocity at a position P on the geometry, with coordinates (x_P, y_P, z_P) , is given by $\mathbf{v}_P = (v_1, v_2, v_3)_P = \text{Re}\{e^{i\omega t} i\omega [(\xi_1, \xi_2, \xi_3) + (\xi_4, \xi_5, \xi_6) \times (x_P, y_P, z_P)]\}$, where the motion amplitudes ξ_j of the six degrees of freedom correspond to the translatory modes in surge, sway and heave ($j = 1, 2, 3$), respectively, and the rotational modes in roll, pitch and heave, about the x , y and z axes ($j = 4, 5, 6$), respectively.

The wave potentials describing the fluid motion in the exterior and interior domains (see Fig. 1), $\tilde{\phi}^E$ and $\tilde{\phi}^I$, respectively, may be expressed by

$$\begin{aligned} \tilde{\phi}^{E,I}(x, y, z, t) &= \tilde{\phi}_D^{E,I}(x, y, z, t) + \tilde{\phi}_{rad}^{E,I}(x, y, z, t) \\ &= \text{Re}\left(e^{i\omega t} \left[\frac{iAg}{\omega} \phi_D^{E,I}(x, y, z) + i\omega \sum_{j=1}^6 \xi_j \phi_j^{E,I}(x, y, z) \right] \right), \end{aligned} \quad (1)$$

where $\phi_D^{E,I}$ and $\phi_j^{E,I}$ denote the diffraction potential and radiation potential of motion mode number j , respectively, in the exterior and interior domains, and g denotes the acceleration of gravity (and A is the wave amplitude). The diffraction potentials $\phi_D^{E,I}$ are composed by the velocity potential ϕ_0 due to the incoming waves plus the remainders $\phi_7^{E,I} = \phi_D^{E,I} - \phi_0$.

Assuming deep water, the incoming wave potential is given by

$$\phi_0 = e^{Kz - iK(x \cos \beta + y \sin \beta)}, \quad (2)$$

where β denotes the wave angle, with $\beta = 0$ corresponding to incoming waves propagating along the positive x -axis.

The velocity potentials $\phi_j^{E,I}$ ($j = 1, \dots, D$) satisfy the Laplace equation in the respective fluid domains. The linear free surface boundary condition applies at $z = 0$:

$$-K\phi_j^{E,I} + \frac{\partial\phi_j^{E,I}}{\partial z} = 0, \quad j = 1, \dots, D, \quad (3)$$

where $K = \omega^2/g$. Further, the potentials ϕ_j^E in the exterior domain satisfy a radiation condition in the far field:

$$\frac{\partial\phi_j^E}{\partial R} = -iK\phi_j^E + O\left(\frac{1}{R}\right), \quad R \rightarrow \infty, \quad j = 1, \dots, 7, \quad (4)$$

where $R = \sqrt{x^2 + y^2}$. The velocities $|\nabla\phi_j^E| \rightarrow 0$ for $z \rightarrow -\infty$. A remaining boundary condition relating the normal velocity to the pressure drop at the geometry, assuming a linear relationship is obtained in section 3 below.

The potentials $\phi_j^{E,I}$ ($j = 1, \dots, D$) are obtained by a set of integral equations. The exterior potentials ϕ_j^E are obtained by, for a point (x', y', z') on the porous geometry:

$$2\pi\phi_j^{E'} + \int_{S_B} \left(\phi_j^E \frac{\partial G}{\partial n} - G \frac{\partial\phi_j^E}{\partial n} \right) dS = 4\pi\phi_0'\delta_{D,j}, \quad (5)$$

($j = 1, \dots, D$), and for a point (x', y', z') in the exterior fluid:

$$4\pi\phi_j^{E'} + \int_{S_B} \left(\phi_j^E \frac{\partial G}{\partial n} - G \frac{\partial\phi_j^E}{\partial n} \right) dS = 4\pi\phi_0'\delta_{D,j}, \quad (6)$$

($j = 1, \dots, D$.) Here, G denotes the wave Green function corresponding to a rankine source in the point (x, y, z) , satisfying the linear free surface boundary condition at $z = 0$, the radiation condition in the far field as well as the bottom boundary condition for $z \rightarrow -\infty$. The function G is specified in section 2.2 below. Further, $\delta_{D,j}$ on the r.h.s. of (5) denotes the Kroenecker-delta, a prime means evaluation at the point (x', y', z') and the normal n points out of the exterior fluid. The internal potentials are obtained similarly, for a point (x', y', z') on the porous geometry,

$$-2\pi\phi_j^{I'} + \int_{S_B} \left(\phi_j^I \frac{\partial G}{\partial n} - G \frac{\partial\phi_j^I}{\partial n} \right) dS = 0, \quad (7)$$

($j = 1, \dots, D$), where a minus appears in front of the first term on the l.h.s. of (7) since the normal n is pointing into the interior domain.

Sum and difference potentials along the geometry are introduced by $\Phi_j = \phi_j^E + \phi_j^I$ and $\Psi_j = \phi_j^E - \phi_j^I$ ($j = 1, \dots, D$). Combination of (5) and (7) gives:

$$2\pi\Phi_j' + \int_{S_B} \Psi_j \frac{\partial G}{\partial n} dS = 4\pi\phi_0'\delta_{D,j}, \quad (8)$$

$$2\pi\Psi_j' + \int_{S_B} \Phi_j \frac{\partial G}{\partial n} dS = \int_{S_B} G \frac{\partial\Phi_j}{\partial n} dS + 4\pi\phi_0'\delta_{D,j}, \quad (9)$$

($j = 1, \dots, D$), where in (8) we have used that the normal velocity is continuous at the boundary, see also (19) below.

Fig. 1 Coordinate system and sketch of (a) porous hemisphere and (b) truncated vertical cylinder.

2.2 Far field behavior of the Green function and the potentials

The Green function is given by

$$G = \frac{1}{r} + \int_0^\infty \frac{k + K}{k - K} e^{k(z+z')} J_0(kr_0) dk \quad (10)$$

where $r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$, $r_0 = \sqrt{(x-x')^2 + (y-y')^2}$, J_0 denotes the Bessel function of first kind and order zero, and the path of integration is above the pole at $k = K$. For $R' = \sqrt{x'^2 + y'^2} \rightarrow \infty$, G takes the form, see Grue and Palm (1993, eq. (4.1)),

$$G = R'^{-1/2} h(\theta') e^{Kz' - iKR'} (1 + O((KR')^{-1})) \quad (11)$$

where the angle θ' is related to the far field coordinates by $(x', y') = R'(\cos \theta', \sin \theta')$ and

$$h(\theta') = (8\pi K)^{1/2} e^{K(z + ix \cos \theta' + iy \sin \theta') - i\pi/4}. \quad (12)$$

The potentials are obtained in the far field by (6)

$$\begin{aligned} \phi_j^{E'}(x', y', z') &= -\frac{1}{4\pi} \int_{S_B} \left(\phi_j^E \frac{\partial G}{\partial n} - G \frac{\partial\phi_j^E}{\partial n} \right) dS \\ &= \left(\frac{K}{2\pi R} \right)^{1/2} H_j(\theta') e^{K(z' - iR')} + O\left(\frac{1}{R}\right), \end{aligned} \quad (13)$$

($j = 1, \dots, 7$), where

$$H_j = \int_{S_B} \left(\frac{\partial\phi_j^E}{\partial n} - \phi_j^E \frac{\partial}{\partial n} \right) e^{K(z + ix \cos \theta' + iy \sin \theta') - i\pi/4} dS, \quad (14)$$

($j = 1, \dots, 6$),

$$H_7 = \int_{S_B} \left(\frac{\partial \phi_D^E}{\partial n} - \phi_D^E \frac{\partial}{\partial n} \right) e^{K(z + ix \cos \theta' + iy \cos \theta') - i\pi/4} dS. \quad (15)$$

3 Kinematic-dynamic boundary condition at the geometry

The wave induced motion sets up a local flow through the porous geometry. Let $w_n = (\mathbf{v}^{E,I} - \mathbf{v}_P) \cdot \mathbf{n}$ denote the velocity normal to the geometry where $\mathbf{v}^E = \nabla \phi^E$ and $\mathbf{v}^I = \nabla \phi^I$ denote the velocity of the exterior and interior fluid, respectively, and $\mathbf{v}_P$ is defined in the text above (1). A resistance law at the geometry is generally of the form $w_n = F(\Delta p)$ where Δp denotes the pressure drop between the exterior and interior fluid. Following Taylor (1956) studying aerodynamical flow applications $\Delta p = (\rho C_0/2)w_n^2$ models a quadratic resistance law and $\Delta p = (\mu/b)w_n$ a linear law where ρ denotes density, C_0 a dimensionless coefficient, μ viscosity and b a coefficient with dimension. Here we pursue the linear pressure drop assumption which is a valid approach for geometries with fine pores, where

$$w_n = \frac{b}{\mu} \Delta p \quad (16)$$

at S_B . The ratio $\rho b/\mu$ has dimension of time over length.

Wave generation by a moving porous plate assuming a linear pressure drop was studied by Chwang (1983) and Chwang and Li (1983), expressing the porosity effect by a parameter $G_0 = \rho b \omega / K \mu$ where $K = \omega^2/g$. Zhao *et al.* (2011a) studied the linear and mean drift forces on a vertical porous circular cylinder as function of the parameter ' b ' = $B_0 = 2\pi G_0$ and the wavenumber. By a set of experiments Zhao *et al.* (2011b) connected the porosity parameter B_0 to the opening rate τ of the material as well as the waveslope $\epsilon = KA$, see particularly their equation (29), obtaining

$$B_0 = \frac{(17.8/\epsilon + 143.2)\tau^2}{1 + 1.06\tau}. \quad (17)$$

Table 1 illustrates practical values of τ as function of $B_0 = 2\pi G_0$ obtained from (17) where a waveslope of $KA = 0.05$ is assumed.

Returning to the derivation of a boundary condition at the geometry we obtain the pressure drop by $\Delta p = p^E - p^I$, where p^E, p^I denote the pressure of the exterior and interior fluid, respectively. The Bernoulli equation gives, assuming a linear motion: $\Delta p = -\rho(\partial \phi^E / \partial t - \partial \phi^I / \partial t)$. Using $w_n = (b/\mu)\Delta p$

Table 1 Porosity parameter B_0 and the corresponding opening rate τ of the geometry for $KA = 0.05$ using (17).

B_0	2π	8.5	5π	10π	20π
τ	0.12	0.14	0.19	0.29	0.43

we obtain

$$\begin{aligned} & -\frac{i\rho b\omega}{\mu} \left(\frac{iAg}{\omega} \Psi_D + i\omega \sum_{j=1}^6 \xi_j \Psi_j \right) \\ & = \frac{iAg}{\omega} \frac{\partial \phi_D^{E,I}}{\partial n} + i\omega \sum_{j=1}^6 \xi_j \left(\frac{\partial \phi_j^{E,I}}{\partial n} - n_j \right), \end{aligned} \quad (18)$$

giving

$$\frac{\partial \phi_j^E}{\partial n} = \frac{\partial \phi_j^I}{\partial n} = n_j \delta_{jJ} - \frac{i\rho b\omega}{\mu} (\phi^E - \phi^I)_j, \quad (19)$$

$$\frac{\partial \Phi_j}{\partial n} = 2n_j \delta_{jJ} - \frac{2i\rho b\omega}{\mu} \Psi_j, \quad (20)$$

($j = 1, \dots, D, J = 1, \dots, 6$), in both equations, where the relations close the set of equations (8-9) for the potentials Φ_j, Ψ_j as well as ϕ_j^E, ϕ_j^I .

Numerical solution of the set of integral equations (8-9,19-20) is obtained by two different codes. One is a Python-script derived specifically for the unbounded fluid case where the Green function is replaced by the Rankine source ($G = 1/|(x, y, z) - (x', y', z')|$) and the fluxes are evaluated by high-order Gauss quadrature. The other is a diffraction-radiation panel code which is a generalization of the low-order version of the WAMIT-code (wamit.inc), see also Nossen *et al.* (1991), Grue and Palm (1996) and Finne and Grue (1997). Geometries explored in the present paper include a sphere in unbounded fluid and a hemisphere as well as a truncated vertical cylinder in the free surface. The number of panels is $N = 1024$ or 2048 for the sphere, $N = 1024$ for the hemisphere and $N = 1131$ for the truncated cylinder. Running two different codes for the case of the submerged sphere makes a double check of the computations of the effect of porosity at the geometry, without the wave effects. The calculations involving the wave effects with the geometries in the free surface are qualified by evaluation of energy checks and obtaining the linear exciting force in two different ways. In the calculations of the mean drift force in section 5 we compare with published results for the case of a solid hemisphere and a truncated cylinder at finite and infinite water depth published in Grue and Biberg (1993) as well as in Molin (1994).

Fig. 2 Added mass and damping coefficients a_{11} and b_{11} . (a) Porous sphere of radius R_0 in unbounded fluid.

(b) Floating hemisphere of radius R_0 for $\rho b \sqrt{g R_0} / \mu = 0.157$. (c) Same as (b) but a_{11} for $\rho b \sqrt{g R_0} / \mu = 1.25, 3.13, 6.26$. (d) Same as (c) but a_{33} .

4 Linear forces

A set of relations of the linear force coefficients have by Zhao *et al.* (2011a) been obtained by use of Green's theorem to the set of wave potentials. The relations are generalizations of the usual added mass and damping symmetry and the Haskind-relation in the diffraction problem for solid geometries, where a linear resistance law is assumed for the porous case. We here evaluate the added mass and damping force which in unbounded fluid are functions of the parameter $\rho\omega bR_0/\mu$. A main effect of the porous geometry is to introduce a significant damping. The coupling between the exterior and interior motion further modifies the forces. In the free surface case the contributions to the damping coefficients from the dissipation at the geometry and the radiated waves are obtained. The exciting force is calculated by both pressure integration and the generalized Haskind-relations which are rederived here. In the diffraction problem the incoming and scattered energy flux balances exactly the porous energy loss.

4.1 Added mass and damping

The forces and moments in the radiation problem are obtained by integrating the linear pressure over the body surface including the outer and inner motion, giving, $F_i(t) = \text{Re}(\xi_j(\omega^2 a_{ij} - i\omega b_{ij})e^{i\omega t})$, where a_{ij} and b_{ij} denote the added mass and damping coefficients, respectively, obtained by:

$$a_{ij} - ib_{ij}/\omega = \rho \int_{S_B} \Psi_j n_i dS, \quad i, j = 1, 2, \dots, 6. \quad (21)$$

The symmetry of a_{ij} and b_{ij} was demonstrated by Zhao *et al.* (2011a).

The work per unit time performed by the force opposing the pressure force on the body, in the radiation mode J , averaged in time, is expressed by

$$\overline{W_J} = - \overline{\int_{S_B} (p_{rad,J}^E - p_{rad,J}^I) \mathbf{n} \cdot \mathbf{v}_{P,J} dS}, \quad (22)$$

where $p_{rad,J}^{E,I}$ denotes the external/internal pressure in the radiation mode number J , $\mathbf{v}_{P,J}$ the velocity at the point P on the geometry S_B for mode J and an overline time average. Carrying out the time average in (22) we obtain $\overline{W_J} = -(1/2)\rho\omega^3 |\xi_J|^2 \text{Im} \int_{S_B} \Psi_J n_J dS = \omega^2 |\xi_J|^2 b_{JJ}/2$.

The work in (22) contributes to a) an energy loss due to the pressure drop across the porous geometry, given by $\int_{S_B} \Delta p v_n dS$, and b) an energy flux at a control surface S_∞ in the far field given by $\int_{S_\infty} -\rho(\partial\phi_{rad,J}^E/\partial t)(\partial\phi_{rad,J}^E/\partial n)dS$ where $\mathbf{n}$ is point-

ing out of the external fluid. By carrying out the respective integrals we obtain

$$\frac{b_{JJ}}{\rho\omega} = \frac{b\omega}{\mu} \int_{S_B} |\Psi_J|^2 dS + \frac{K}{4\pi} \int_0^{2\pi} |H_J(\theta)|^2 d\theta, \quad (23)$$

where the first term expresses the dissipation along the porous geometry and in the latter integral the far field form (13) has been used with H_J obtained in (14). We note that (23) is here derived directly from the energy balance equation, a superior principle. Using a different method and Green's theorem, Zhao *et al.* (2011a, eq. (17)) have derived a set of relations for the damping coefficients b_{ij} but these are not used in the present calculations.

Fig. 2(a) illustrates the added mass and damping coefficients a_{11} and b_{11}/ω for a sphere of radius R_0 in unbounded fluid as function of the parameter $\rho\omega bR_0/\mu$. In the case when $\rho\omega bR_0/\mu \rightarrow 0$ the added mass coefficient tends to $a_{11} \rightarrow 2\pi\rho R_0^3$ where a contribution of $(2/3)\pi\rho R_0^3$ corresponds to the added mass due to the external flow and another contribution of $(4/3)\pi\rho R_0^3$ is due to the internal flow, where the latter is just the displaced volume of the sphere. The two different codes produce the same results (results not shown).

Fig. 2(b) shows the added mass and damping coefficients of a floating hemisphere of radius R_0 , of small porosity with $\rho b\sqrt{gR_0}/\mu = 0.094$. The forces, function of the wavenumber KR_0 , exhibit a high, finite damping coefficient close to the resonance at $KR_0 = \pi/2$. A finite added mass is observed close to the resonance frequency as well. The added mass is negative for the computed frequencies with $KR_0 > \pi/2$. The added mass coefficients a_{11} and a_{33} are approximately zero for the higher porosity factors with $\rho b\sqrt{gR_0}/\mu$ up to 6.26 for all KR_0 (Figs. 3(c,d)).

The floating hemisphere with a small porosity factor of $\rho b\sqrt{gR_0}/\mu = 0.157$ exhibits a large porous damping in vicinity of the resonance frequency, in the lateral mode of motion (b_{11}). The wave radiation contribution is comparatively small (Fig. 3(a)). The dissipation of the porous geometry is high in the long wave range in the heave mode of motion (b_{33}) (Fig. 3(b)). The hemisphere with larger porosity factors $\rho b\sqrt{gR_0}/\mu$ up to 12.53 exhibits a dissipation that is always exceeding the damping of the solid geometry, in the long wave range, in both sway and heave (Figs. 3(c,d)).

Note that the calculations of b_{jj} using the pressure integration (21) and the energy balance (23) produce the same damping coefficients illustrating the accuracy of the calculations. Note that the pressure integration of b_{11} for the solid geometry in Fig. 3(a) is not accurate close to resonance.

4.2 The wave exciting force and moment

The wave exciting force and moment are obtained by $F_j^{ex} = \text{Re}(\rho g A X_j e^{i\omega t})$ where $X_j = \int_{S_B} (\phi_D^E - \phi_D^I) n_j dS$ ($j = 1, \dots, 6$). The latter expression, as also analyzed by Zhao *et al.* (2011a), may be rewritten by use of the body boundary conditions (19) obtaining $n_j = \partial \phi_j^E / \partial n + (i\rho b\omega/\mu)(\phi_j^E - \phi_j^I)$ and $\phi_D^E - \phi_D^I = (\mu/(i\rho b\omega))\partial \phi_D^E / \partial n = (\mu/(i\rho b\omega))\partial \phi_D^I / \partial n$. This gives

$$\begin{aligned} X_j &= \int_{S_B} (\phi_D^E - \phi_D^I) \left(\frac{\partial \phi_j^E}{\partial n} + (i\rho b\omega/\mu)(\phi_j^E - \phi_j^I) \right) dS \\ &= \int_{S_B} \left(\phi_D^E \frac{\partial \phi_j^E}{\partial n} - \phi_j^E \frac{\partial \phi_D^E}{\partial n} \right) dS \\ &\quad + \int_{S_B} \left(\phi_D^I \frac{\partial \phi_j^I}{\partial n} - \phi_j^I \frac{\partial \phi_D^I}{\partial n} \right) dS. \end{aligned} \quad (24)$$

The latter integral in (24) is zero. The integral $\int_{S_B} \left(\phi_D^E \partial \phi_j^E / \partial n - \phi_j^E \partial \phi_D^E / \partial n \right) dS$ is equal to $-\int_{S_\infty} \left(\phi_D^E \partial \phi_j^E / \partial n - \phi_j^E \partial \phi_D^E / \partial n \right) dS$ where the free surface boundary condition of the potentials has been used. Noting that ϕ_j^E ($j = 1, \dots, 7$) satisfy the same radiation condition at infinity we obtain

$$X_j = - \int_{S_\infty} \left(\phi_0^E \frac{\partial \phi_j^E}{\partial n} - \phi_j^E \frac{\partial \phi_0^E}{\partial n} \right) dS. \quad (25)$$

The exciting force acting on the porous geometry is thus obtained by a generalized Haskind relation, formally of the same form as for a solid body, see e.g. Newman (1977, p. 303). By carrying out the integration in the far field we obtain the exciting force by the far field amplitude of the radiation potential obtained in (14), i.e., $X_j = H_j(\beta + \pi)$ (β the incoming wave angle).

The exciting forces X_1 and X_3 are visualized in Figs. 4(a,b), respectively. The forces are up to $X_{1,3}/R_0^2 \sim 0.3$ for $\rho b\sqrt{gR_0}/\mu = 3.13$ and approximately the half for $\rho b\sqrt{gR_0}/\mu = 6.26$ where calculations are performed for $0 < KR_0 < 2.5$. Note that $X_1 = 0$ at the resonance at $KR_0 = \pi/2$ corresponding to a wave length of $\lambda/R_0 = 4$. Pressure integration and the Haskind relation give the same force and is another check of the computations. Depending on the wavenumber KR_0 the exciting forces on the solid geometry are 5-20 times greater than those on a porous geometry with $\rho b\sqrt{gR_0}/\mu = 6.26$.

In the diffraction problem the energy flux of the incoming waves contributes to a) a time averaged energy loss at the porous geometry obtained by $\int_{S_B} \Delta p w_n dS$, and b) an energy flux due to the scattered waves which

couple to the incoming waves. The time averaged energy flux of the waves evaluated at the far field becomes $\int_{S_\infty} -\rho(\partial \tilde{\phi}_D^E / \partial t)(\partial \tilde{\phi}_D^E / \partial n) dS$. By carrying out the integration we obtain

$$\begin{aligned} 0 &= \frac{\rho b\omega}{\mu} \int_{S_B} |\Psi_D|^2 dS \\ &\quad + \frac{K}{4\pi} \int_0^{2\pi} |H_7(\theta)|^2 d\theta - \frac{1}{2} \text{Im}(H_7(\beta)). \end{aligned} \quad (26)$$

The net far field energy flux corresponding to the negative of terms two and three in (26) is dissipated in the porous geometry at exactly the same rate, corresponding to the first term in (26), see Fig. 4(c).

5 Mean drift force

The expression for the mean drift force on a porous geometry is a generalization of the similar analysis for a solid geometry. This has been obtained by Zhao *et al.* (2011a). They also obtained calculations of the force on a truncated cylinder geometry assuming an eigenfunction expansion. Our comparison with their calculations, particularly the case with a solid geometry, which they also include in their paper, shows that there is an error in their calculations in the sense that their force is a factor of 2.5 too large. The expression of the drift force is here rederived. New calculations are presented for both porous and solid geometries. The calculations of the solid body are compared to published calculations for the hemisphere and truncated cylinder (Grue and Biberg, 1993) as well as asymptotics in the short wave range (Molin, 1994). By dividing the results of Zhao *et al.* (2011a) of a factor $\pi^2/4$ we obtain a correspondence between our and their results, both in the porous and solid geometry case.

Conservation of momentum of the external fluid gives

$$\begin{aligned} \frac{d}{dt} \int_{V^E} \rho \mathbf{v}^E dV &= \\ -\rho \int_{S_\infty} \left[\left(\frac{p^E}{\rho} + gz \right) \mathbf{n} + \mathbf{v}^E \mathbf{v}^E \cdot \mathbf{n} \right] dS \\ -\rho \int_{S_B} \left[\left(\frac{p^E}{\rho} + gz \right) \mathbf{n} + \mathbf{v}^E (\mathbf{v}^E - \mathbf{v}_P) \cdot \mathbf{n} \right] dS. \end{aligned} \quad (27)$$

In (27) V^E denotes the fluid volume of the external fluid, bounded by S_∞ (a geometrical surface with zero velocity), S_B , the free surface and the bottom (for $z \rightarrow -\infty$). Eq. (27) is obtained by use of the equation of motion as well as the continuity equation. There is no momentum flux through the free surface. Similarly we obtain for the internal fluid

Fig. 3 Damping of hemisphere of radius R_0 at the free surface. (a) b_{11} by pressure integration (21) (—), porous contribution (---), wave radiation contribution (- · - ·) and total (—) using (23) for $\rho b\sqrt{gR_0}/\mu = 0.157$. b_{11} for solid geometry by far field and near field (pressure integration) (••••). (b) Same as a) but b_{33} . (c) b_{11} by (23) (lines) for $\rho b\sqrt{gR_0}/\mu = 1.25, 3.13, 12.53$. Solid geometry, far field (••••). (d) Same as (c) but b_{33} .

$$\begin{aligned} \frac{d}{dt} \int_{V^I} \rho \mathbf{v}^I dV = \\ \rho \int_{S_B} \left[\left(\frac{p^I}{\rho} + gz \right) \mathbf{n} + \mathbf{v}^I (\mathbf{v}^I - \mathbf{v}_P) \cdot \mathbf{n} \right] dS, \end{aligned} \quad (28)$$

where V^I denotes the volume at the interior of the geometry, bounded by S_B and the free surface, and where $\mathbf{v}^I = \nabla \tilde{\phi}^I$. (The normal $\mathbf{n}$ points into V^I .) Evaluating the time average of (27-28), assuming that there is no net total mass flux, the mean drift force, $\overline{\mathbf{F}} = \int_{S_B} (p^E - p^I) \mathbf{n} dS$, is obtained by

$$\begin{aligned} \overline{\mathbf{F}} = -\rho \overline{\int_{S_\infty} \left[\left(\frac{p^E}{\rho} + gz \right) \mathbf{n} + \mathbf{v}^E \mathbf{v}^E \cdot \mathbf{n} \right] dS} \\ - \rho \int_{S_B} (\mathbf{v}^E - \mathbf{v}^I) (\mathbf{v}^E - \mathbf{v}_P) \cdot \mathbf{n} dS, \end{aligned} \quad (29)$$

where an overline means time average. For the horizontal force along the x -direction the former term in (29) attains the usual far field contribution, see e.g. Grue and Biberg (1993, eq. (48)):

$$\begin{aligned} \frac{\overline{F}_x^{(i)}}{\rho g A^2} = -\frac{K}{8\pi} \int_0^{2\pi} |H_7(\theta)|^2 \cos \theta d\theta \\ - \frac{1}{2} \text{Im}(H_7(\beta)) \cos \beta, \end{aligned} \quad (30)$$

with H_7 given by (15), where the effect of porosity appears in the boundary condition for $\partial \phi_D^{E,I} / \partial n$ on S_B , see (19). The second integral in (29) becomes, in the diffraction problem, with $\mathbf{v}_P \cdot \mathbf{n} = \text{Re}(i\omega \xi_j n_j e^{i\omega t}) = 0$,

$$\frac{\overline{F}_x^{(ii)}}{\rho g A^2} = -\frac{1}{2K} \text{Re} \left[\int_{S_B} \left(\frac{\partial \phi_D^E}{\partial x} - \frac{\partial \phi_D^I}{\partial x} \right) \left(\frac{\partial \phi_D^E}{\partial n} \right)^* dS \right], \quad (31)$$

where a star denotes complex conjugate. The total horizontal drift force becomes $\overline{F}_x = \overline{F}_x^{(i)} + \overline{F}_x^{(ii)}$.

Calculations illustrate the horizontal drift force on the fixed hemisphere (in the diffraction problem) where the contribution from the porous term (31) is seen to be the larger, while the contribution (30) from the scattered waves in the far field is small to moderate, where the porosity factor is $\rho b \sqrt{g R_0} / \mu = 2.19$ in Fig. 5(a). Results for a truncated cylinder of draught $d/R_0 = 1$ are included as well. The considerable contribution to the drift force in the long wave regime for the porous geometry differs fundamentally from that on the solid body. This is further illustrated in Fig. 5(b) for $\rho b \sqrt{g R_0} / \mu$ in the range $\sim 0.31 - 6.26$. The drift force on the porous geometry grows almost linearly with the wavenumber when $K R_0 \ll 1$ and means that a body of even moderate radius is exposed

to a severe drift force in the long wave regime. This effect of the porous geometry is also observed in the calculations by Zhao et al. (2011a). Note also that the viscous effect is known to enhance the mean drift force on semi-submersibles in the long wave regime, where the contributions from a pure potential formulation underpredict the forces. This is well known to specialists in ocean engineering (Nestegård, 2017, personal communication).

Fig. 5(c) obtains the drift force as function of the parameter $G_0 = \rho b \omega / (\mu K)$ for wavenumbers in the range $K R_0 \sim 0.25 - 2.5$ where the value of the ' b ' = B_0 -parameter of 8.5 in Zhao et al. (2011a) corresponds to $G_0 = B_0 / (2\pi) = 1.35$ in our notation, see (17). Note from Fig. 5(c) (for the hemisphere), for $G_0 = 10$ ($B_0 = 20\pi$), $\overline{F}_x \simeq 0.1 \rho g A^2 R_0$ for $K R_0 = 0.25, 0.45, 1.01$ and $\overline{F}_x \simeq 0.07 \rho g A^2 R_0$ for $K R_0 = 2.5$. For illustrative purposes we estimate from table 1 an opening rate of the geometry of $\tau \sim 0.14$ for $G_0 = 1.35$ ($B_0 = 8.5$) and of $\tau \sim 0.43$ for $G_0 = 10$ ($B_0 = 20\pi$), both for $K A = 0.05$.

Table 2 compares the present drift force calculations on the porous hemisphere and the truncated cylinder. Note that the drift force on the two different geometries are rather close. We also compare to the mean drift force obtained by Zhao et al. (2011a, Fig. 5) for a truncated porous cylinder, as reproduced here in Fig. 5(d). The results of Zhao et al. are approximately a factor of 2.5 larger than our. Such a big discrepancy is unphysical. Note that it is unclear from the series representation of the diffraction wave potential in Zhao et al. (2011a, (5a)) how the flow below the truncated cylinder is represented. It is also unclear if the cylinder bottom is porous or not in Zhao et al.

In order to qualify our calculations we consider the mean drift force on the solid geometry where results presented in existing publications are summarized in table 3. It is observed that the present calculations of the drift force on the non-porous hemisphere agree with those presented in Grue and Biberg (1993, Fig. 6a). Note a small discrepancy at $K R_0 = 1.6$ due to the two different discretizations of the hemisphere. Results for the truncated cylinder with draught $d/R_0 = 1$ at infinite water depth are also included. The results by Zhao et al. (2011a, Fig. 5) for the drift force on the non-porous solid cylinder geometry are approximately a factor of 2.5 larger compared to the published calculations by Grue and Biberg (1993) as well as the present calculations. Another reference is the asymptotic result of Molin (1994, eq. (6)) for the drift force on a fixed cylinder or hemisphere of radius R_0 in the short wavelength regime ($K R_0 = \infty$) obtaining $\overline{F}_x = (2/3) \rho g A^2 R_0$, also given in table 3. We may speculate if Zhao et al. (2011a) have multiplied their force by $\pi^2/2^2$. If their published results for $\overline{F}_x$ are

Fig. 4 Exciting force on a hemisphere by pressure integration (symbols) and Haskind relation (lines): (a) X_1 and (b) X_3 . (c) The energy relation (26) with dissipation loss at S_B (lines) and energy flux in the far field (symbols). $\rho b \sqrt{g R_0} / \mu = 1.57, 3.13, 6.26$, and solid case. Wave angle $\beta = 0$.

divided by a factor of $\pi^2/2^2$, their drift force calculations fit quite well with the present calculations for the non-porous case, see the last row in table 3. In the case of the porous truncated cylinder case with $B_0 = 8.5$ their calculations underpredict our calculations by 20 % (at $KR = 1.01$) and 30 % (at $KR = 2.5$), see the last row in table 2.

Table 2 Mean drift force $\bar{F}_x/(\rho g A^2 R_0)$ on porous fixed geometry with porosity factor ' b' ' = $B_0 = 2\pi G_0 = 2\pi\rho b\omega/K\mu = 8.5$ and wavenumber $KR_0 = 0.25, 0.45, 1.01, 2.50$. Hemisphere (HS) and truncated cylinder (TC) of draught $d/R_0 = 1$, both of radius R_0 . N number of panels. PR (present). Z et al. (Zhao et al. (2011, Fig. 5))

KR_0	0.25	0.45	1.01	2.50
HS, $h = \infty$, $N = 1024$, PR	0.23	0.43	0.43	0.35
TC, $h = \infty$, $N = 1311$, PR	0.25	0.44	0.43	0.37
TC, Z et al.	0.55	1.02	0.84	0.63
TC, Z et al./ $(\pi^2/2^2)$	0.22	0.41	0.34	0.26

Table 3 Mean drift force $\bar{F}_x/(\rho g A^2 R_0)$ on fixed solid (non-porous) hemisphere (HS) and truncated cylinder (TC) of draught $d/R_0 = 1$, both of radius R_0 . N number of panels. $h = \infty$. PR means present. GB (Grue & Biberg (1993, Fig. 6a)), M (asymptotic value from Molin (1994, eq. (6)), Z et al. (Zhao et al. (2011, Fig. 5))

KR_0	0.25	0.45	1.01	1.6	2.50	∞
HS $N = 1024$ PR	0.03	0.13	0.48	0.53	0.57	-
HS $N = 784$ GB	0.03	0.13	0.48	0.52	-	-
TC $N = 1311$ PR	0.03	0.13	0.58	0.59	0.63	-
M	-	-	-	-	-	0.67
Z et al./ $(\pi^2/2^2)$	0.02	0.13	0.60	0.61	0.62	-

6 Conclusion

We perform a mathematical analysis of the linear wave diffraction-radiation problem of a porous geometry localized in the surface of a inviscid fluid. The geometry

may also be submerged. A linear resistance law is assumed at the geometry where the outer and inner flows are connected. The formulation valid for geometries of general shape obtains the field variables by a set of integral equations. Calculations using a panel method obtain the added mass, damping and exciting forces as well as the mean drift force on porous geometries including a submerged sphere and a hemisphere as well as a truncated cylinder located in the free surface. The energy loss at the porous geometry and the far field energy flux are evaluated providing global checks on the computations. The generalized Haskind relations are evaluated as well, with excellent agreement to the local pressure integration of the exciting forces.

The added mass and damping of a body in unbounded fluid are function of the porosity parameter $\rho\omega b R_0/\mu$. In the more general case of a porous geometry interacting with a free surface suitable proxies include the wavenumber $KR_0 = \omega^2 R_0/g$ and a nondimensional porosity parameter $\rho\omega b\sqrt{gR_0}/\mu$. Other publications have used $G_0 = \rho\omega b/(K\mu)$ (see Chwang, 1983; Chwang and Li, 1983) and ' b' ' = $B_0 = 2\pi G_0 = 2\pi\rho\omega b/(K\mu)$ (see Zhao *et al.*, 2011a) as dimensionless porosity parameter, in addition to the wavenumber. Experiments by Zhao *et al.* (2011b) provide a relation between the porosity parameter and the opening rate of the material, τ , see eq. (17) in section 3. Their and our porosity parameters give an opening rate of the material of $\tau = 0.12$ for $\rho\omega b\sqrt{gR_0}/\mu = 1$, $KR_0 = 1$, and $\tau = 0.43$ for $\rho\omega b\sqrt{gR_0}/\mu = 10$, $KR_0 = 1$.

The added mass of a porous geometry located in the free surface is close to zero for a porosity parameter $\rho\omega b\sqrt{gR_0}/\mu > 3$. The porous geometry significantly enhances the damping in the long wave regime. The exciting force $X_{1,3}/R_0^2$ is up to ~ 0.3 for all KR_0 and $\rho\omega b\sqrt{gR_0}/\mu = 3.13$, and up to about the half for $\rho\omega b\sqrt{gR_0}/\mu = 6.26$ and is a factor of 5-20 smaller than the exciting force on a solid hemisphere.

The mean horizontal drift force $\bar{F}_x$ on the porous geometry is significantly larger in the long wave regime compared to that on a non-porous geometry. The force $\bar{F}_x$ is found to grow almost linearly with the wavenumber and means that a body of even moderate radius is exposed to a severe drift force in the long wave regime. This effect of the porous geometry is also found in the calculations by Zhao *et al.* (2011a). Note that a viscous effect is known to enhance the mean drift force on semi-submersibles in the long wave regime, where the contributions from a pure potential formulation underpredict the forces, according to specialists in ocean engineering (Nestegård, 2017, personal communication).

The present evaluation of the mean drift force for both porous and non-porous geometries, with comparison to existing publications regarding the mean

drift force on a solid hemisphere and truncated vertical cylinder, shows that the numerical values of $\bar{F}_x$ obtained by Zhao et al. (2011a) are a factor $\pi^2/2^2$ too large, both for the porous and non-porous case. Dividing by this factor, the results by Zhao *et al.* still underpredict the drift force on a porous body by a factor of 20-30 per cent for $KR_0 > 1$. The present analysis and calculations provide important references to a similar wave analysis assuming a quadratic resistance law at the porous geometry.

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Fig. 5 Mean drift force. Fixed hemisphere (HS) and truncated cylinder (TC). (a) $\bar{F}_x$ vs. $K R_0$, for $\rho b \sqrt{g R_0} / \mu = 2.19$, for HS (thick dots) and TC (long dashes), porous part for HS $\bar{F}_x^{ii}$ (thin dashed) and far field for HS $\bar{F}_x^i$ (thin dash-dot). Solid HS and TC (—). (b) $\bar{F}_x$ vs. $K R_0$ for $\rho b \sqrt{g R_0} / \mu = 0.31, 1.57, 3.13, 6.26$. (c) $\bar{F}_x$ vs. $G_0 = \rho b \omega / (K \mu)$ for $K R_0 = 0.25, 0.45, 1.01, 2.50$. (d) Zhao et al. (2011a, Fig. 5) for $\bar{F}_x$ for $b' = B_0 = 8.5$ (porous) and $b' = B_0 = 0$ (solid) TC, draught $d/a = 1$, water depth $h/a = 3$, cylinder radius a .

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