Analytical and numerical modeling of cavity closure in rock salt

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Thesis submitted for the degree of Philosophiae Doctor

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November, 2017

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*Series of dissertations submitted to the
Faculty of Mathematics and Natural Sciences, University of Oslo
No. 1966*

ISSN 1501-7710

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Cover: Hanne Baadsgaard Utigard.
Print production: Reprosentralen, University of Oslo.

If you quit once it becomes a habit. Never quit!!!

Michael Jordan (1963-)

Acknowledgements

When I started my PhD four years ago on analytical and numerical modeling of rock flow, I had no idea what I was getting into. I knew that I liked coding and working on rocks so it seemed like a reasonable choice. I did meet a couple of PhD students at the time who strongly advised me not to do one but I thought I knew better, so I went for it. In the beginning, the title of my PhD alone scared me and once I got to know my supervisors, I became terrified. My knowledge on numerical modeling and rock mechanics did not reassure me, to say the least, and it became clear that the road ahead would be long. During the past four years there have been many times when I got stuck and considered quitting, but every time I decided to rely instead on sheer determination to go on and get myself out of these tricky situations. Now that I am finally about to finish this scientific endeavor, I would like to thank all the people who helped me in these challenging times.

Marcin, I would like to thank you for the countless times you came to my rescue when I was stuck or went out of track. Your extensive knowledge of so many different fields felt extremely intimidating (it still does but a bit less ;)) but it allowed me to get a glimpse at some of the connections that exist between different communities. Your ability to break down complex problems into simpler ones taught me a lot about facing difficult questions and eventually led me to where I am now. Your positive attitude towards science has been, and still is, very inspiring and it has been a privilege to learn from and work with you during the past four years.

Dani, I would like to thank you for being my supervisor and for sharing with me your graphical view of the world. We realized quite early on that we are wired differently but I did nonetheless try to introduce some of your Swiss efficiency in my working habits. The lessons you gave me on how to write articles and organize my research have especially been invaluable. I would also like to thank you for challenging the assumptions that I made and for reminding me to link back the models to reality.

I would also like to thank all the academic staff from the PGP group and from the University of Oslo for providing a vibrant environment to work in. I express my deep gratitude to Statoil for funding my PhD and to Anders Malthe-Sørenssen for keeping the link between industry and academia alive.

I would also like to thank all the people who helped me keep some mental sanity when things were crumbling down. Piyoosh, Saturday nights were quite something and Kjøkken was not the same once you left. Daniel, our talks about everything and nothing were extremely enjoyable and the few travels we did together are some of the highpoints of my time in Oslo. Alban, it was nice to have someone with whom I could share the details of the problems I was facing. Seeing that you are not the only one struggling always helps, and being able to complain in French was priceless.

My office mates have been very important to me during the past years, even though I was living in a different time zone, and I would like to give a big up to you. Kristina, I did try several times to get into a German time zone but my body couldn't take the sauerkraut diet. Shagha, I enjoyed sharing my office with you even though you are such a nerd ;) Aylin, you go girl!

I would also like to thank the Pluymakers and Jørgen T for the social life of PGP. Anne, I am so grateful that you have organized so many dinners and parties that were always fun. I almost feel like your place is funded by PGP. For making my time at PGP unforgettable and without who PGP would not be itself I would like to thank you: Jesus, Marcel, Anne, Sigve, Arianne, Franck, Øystein, Lisa, Tobi, Ben, Ole Ivar, Håvard, Kjetil, Amelie, Felix and all the others. I am also not going to forget you, the group (which in my head is) of the girls from pharmacy Julia, Lilia and Benjy.

I would also like to use this opportunity to thank the people from Wroclaw for welcoming me in their group: Marta, Marcin, Maciej, Lukasz and the others. Wroclaw has become my new home and I am very excited about it.

Finally, my family and friends from my previous life have always been there for me and for that I am very grateful. These past few years have not been easy and it has been very helpful to know you by my side, even though we were not geographically close, so for that thank you.

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1 Introduction

1.1 Abstract

This thesis investigates how cavities close in rock salt using analytical and numerical methods. Time dependent hole convergence is an on-going problem in the industry and a thorough understanding of how it occurs is needed. The constitutive law for salt is best described based on the deformation mechanisms that are active at depth. The viscous component of the constitutive law used here relies on both pressure solution and dislocation creep which are combined using either a Carreau or an Ellis model. The steady state closure of pressurized cylindrical holes in incompressible Carreau materials subjected to various far-field loads is first considered. New analytical solutions are presented for the maximum closure velocity at the rim for far-field pressure and pure shear loads. A proxy is also proposed based on the two previous end-members for the maximum closure velocity at the rim when a general load is applied at infinity. These analytical solutions are directly applied to rock salt to evaluate the magnitude of the closure velocities under various conditions. Rock salt behavior is strongly dependent on grain size, via pressure solution, so the impact of varying the grain size on closure velocity is especially investigated. The deformation mechanism governing hole closure is also analyzed as a function of the in-situ conditions and grain size. Pressure solution is found to be governing closure in fine-grain salts under low loads and temperatures while dislocation is dominant in coarse-grain salts under high loads and temperatures. Finally, the effect of the transient behavior of salt on pressure driven hole closure is investigated by considering an incompressible non-linear viscoelastic constitutive law. More precisely, a Maxwell body is used where the viscous component follows an Ellis model. The closure is fully characterized initially and its variation with time is described by both a characteristic relaxation time and a pseudo steady state time. Taking pressure solution into account is important because closure approaches pseudo steady state instantaneously where it is the dominant deformation mechanism, e.g. at shallow depths. This underlines the

relevance of using a model for the viscosity of salt that combines dislocation creep and pressure solution and shows that the analytical solutions derived for steady-state conditions can be used close to the surface where pressure solution is likely to be the dominant viscous deformation mechanism.

1.2 Motivation and scope of the thesis

1.2.1 Why is hole closure in salt an issue?

Closure of holes in salt is caused by the creeping property of rock salt. When a hole is created in the underground, it perturbs the local in-situ stress and creates shear stresses that lead to its time dependent closure. It is a problem encountered across many industries that can result in dangerous and expensive scenarios.

In the oil industry, a major challenge is to drill safely through thick sequences of salt and to assure the integrity of the well during its lifetime (Dusseault et al., 2004a; Dusseault et al., 2004b; Mackay et al., 2008; Poiate et al., 2006; Wang et al., 2013; Willson et al., 2003; Xie and Tao, 2013; Zhang et al., 2008). Many of the salt related oil reservoirs around the world are located in ultra-deep water as is the case in the Gulf of Mexico, offshore Brazil or Angola and the time and money costs related with creating a well are huge. Ensuring the safety and the success of the well is especially capital in these scenarios. The problems encountered during and after drilling through salt are associated with salt creep leading to hole closure and casing collapse (Hackney, 1985; Infante and Chenevert, 1989; Kim, 1988; Senseny, 1990). To achieve success, engineers have adopted measures like increasing mud weight, decreasing temperature, under-reaming, decreasing mud salinity to dissolve creeping salt, and decreasing open hole time (Dusseault et al., 2004a; Holt and Johnson, 1986; Yearwood et al., 1988). The key is to evaluate the downhole conditions required to ensure that enough time is available to properly complete the well before the diameter of the hole becomes too small. Ensuring that cement is set properly is the only way to guarantee the long-term safety of the well (Cheatham and McEver, 1964). An analytical expression has been developed by Barker et al. (1994) which allows to quickly evaluate the instantaneous closure velocity and the time available before the well is too narrow to proceed securely. Since then, increasingly complex models have been used to forecast hole closure in the high pressure, high temperature environment encountered during drilling

(Carcione et al., 2006; Fossum and Fredrich, 2007; Hou et al., 2012; Liu et al., 2011; Orlic and Buijze, 2014; Paraschiv-Munteanu and Cristescu, 2001; Weijermars et al., 2013). The challenge, however, of understanding the underlying mechanisms of hole closure and providing the best model for it has remained the same.

In the mining industry, mines are abandoned after extraction has ended and the question arises of what happens after (Bérest et al., 2001; Bérest and Brouard, 2014; Berest et al., 2011; Brouard et al., 2013; Huang and Xiong, 2011; Karimi-Jafari et al., 2006; van Heekeren et al., 2009; Wang et al., 2011; Wang et al., 2015). Mine closure does not stop after the production ceased as can be seen from direct measurements (Frayne and Mraz, 1991). Figure 1 shows pictures of tunnel closure from the Wieliczka salt mine in Poland. Mine closure can potentially have disastrous consequences due to the subsidence it can cause (Fokker et al., 1995; Mancini et al., 2009; Raucoules et al., 2003) therefore it must be monitored for a long time after the end of the extraction. It is obviously important to ensure that the subsidence remains below a level that does not cause damage to the buildings on the surface. Subsidence is an ever more critical issue in countries like the Netherlands where large land surfaces are situated close to sea level and where subsidence could cause these areas to be covered by water. Properly forecasting the long-term closure of salt cavities in this context is therefore critical.

Rock salt also has very good sealing properties (low permeability, low porosity and no fractures at depth) so it is used as a storage medium for chemicals like oil and gas, pressurized air, hydrogen, and nuclear wastes. Creating large underground salt caverns by solution mining being cheap and easy to do, it makes salt even more attractive for large scale storage. Salt caverns are typically planned to be used during tens of years and it is important to understand their behavior to increase their lifetime (Guillhaus, 2007; Heusermann et al., 2003). In the case of oil and gas, seasonal trends in production and injection have to be especially taken into account because they translate in changes in cavern pressure which affects cavern closure and the storage volume available. For compressed air, the production-injection cycles happen more often and faster (Djizanne et al., 2014) and it is critical to model the loss of cavern volume associated with these extreme pressure variations. Nuclear waste is another material that is considered for storage in salt (Serata and Gloyna, 1959; Westbrook, 2016). The

time scale involved in this application is much longer than in the previous ones and the long-term closure of the storing cavities is of interest. In this hundred to millions of years context, it is important to make sure that all the deformation mechanisms governing salt have been taken into account when modeling hole closure, even the ones active at very low deformation rates.

Figure 1 Photos from the Wieliczka and Kłodawa salt mines in Poland showing convergence with time. In a), the timber battens on the floor have been broken by the progressive closure of the mine (Wieliczka mine). In b), a metallic beam has been dramatically deformed by the downward movement of the ceiling (Kłodawa mine). In c), the convergence of a drift is directly observed from its shape (Wieliczka mine).

1.2.2 How to model salt?

‘Salt’ usually refers to deposits dominated by large proportions of sodium chloride NaCl. Many salts are very homogeneous with less than 5% impurities (Fredrich et al., 2007). Other salt minerals also exist like carnallite, bischofite or taquihydryte (Frayne and Mraz, 1991; Poiate et al., 2006) and the basic mechanisms governing their

behavior are the same as the ones for sodium chloride so the results presented here can be extended to these salts. The elastic properties of different salts are very similar and they can be assumed to be the same (Fredrich et al., 2007). This, however, is not true for their viscous properties because the emplacement and the patterns made by heterogeneities and dislocations can have a big influence on the flow of salt (Hunsche and Hampel, 1999; Renard et al., 2001).

The choice of a constitutive law for salt is a critical and complicated one as there are many different available models. To name a few there is: the Multimechanism Deformation (MD) model (Munson, 1997; Munson and Dawson, 1981; Weatherby et al., 1996) and its improvement the Multimechanism Deformation Coupled Fracture (MDCF) model (Fossum and Fredrich, 2002), the model developed by Cristescu (1993) based on a viscoplastic potential and its improvements (Hunsche and Hampel, 1999; Jin and Cristescu, 1998), the LUBBY2 creep model (Heusermann et al., 2003), and the Hou-Lux model (Hou, 2003; Hou et al., 2012). All these viscoelastoplastic models aim at providing a complete description of salt behavior covering the range of conditions encountered in the underground. They are all based on the identification and the description of the individual mechanisms governing salt deformation. This approach allows to extrapolate the models to times for which they cannot be tested (like millions of years in the case of nuclear waste storage). The deformations taken into account by the aforementioned models can be divided into three categories: elastic, plastic, and viscous. The elastic behavior of salt is well constrained and can be considered as independent of its type, as mentioned above. The plastic behavior of salt is best described by a dilatancy band in stress space which is strongly influenced by the confining pressure (Cristescu, 1993; Hunsche and Hampel, 1999; Jin and Cristescu, 1998). At 14 MPa confining pressure, the shear has to be bigger than 14 MPa to notice dilatancy. At 35 MPa confining pressure, the shear limit is increased to 22 MPa (Jin and Cristescu, 1998). Plasticity is therefore mainly a problem for holes subjected to large under-pressures. Finally, the time dependent behavior of salt is characterized by three different types of creep (Figure 2). Primary - or transient - creep is associated with strain hardening and a decreasing strain rate, secondary – or steady state – creep is characterized by a constant strain rate and tertiary creep is defined by an increasing strain rate leading to failure. The steady state viscous behavior of salt is governed by

different mechanisms depending on the temperature and stress which can be represented in a diagram (Figure 3).

Figure 2 Typical axial strain versus time curve measured in confined uniaxial compression tests (Khaledi, 2017).

Figure 3 Steady state deformation map of rock salt from Khaledi (2017) summarizing the stress and temperature conditions under which the different deformation mechanisms are dominant. Grain boundary migration is not indicated as it is a recovery process associated with dislocation creep. Grain boundary healing is also not indicated as a macroscopical stress threshold has not yet been determined for salt.

1.2.2.1 Dislocation creep and grain boundary migration

Under high shear stress the creep of rock salt is governed by the different types of dislocation creep inside salt grains (Carter and Hansen, 1983; Carter et al., 1993; Senseny et al., 1992; Spiers and Carter, 1996; Wawersik and Zeuch, 1986). Under medium shear stresses the climb of edge dislocations is the limiting mechanism while at higher stresses it is the cross slip of screw dislocations that is rate limiting. Both processes are usually presented in the context of confined uniaxial compression so the temperature activated power law looks like:

$$\dot{D} = C \exp\left(-\frac{Q}{RT}\right) \tilde{\tau}^n \quad (1)$$

where $\dot{D}$ is the uniaxial strain rate, C is a constant depending on the type of dislocation creep, Q is the activation energy for the specific type of dislocation creep, R is the universal gas constant, T is temperature, $\tilde{\tau}$ is the stress difference between the largest principal stress and the confining pressure, and n is the stress exponent. n is typically around 3 when the climb of edge dislocation is the limiting mechanism and around 5 when cross slip of screw dislocations is the limiting factor.

Figure 4 Left: Scanning Electron Microscope image showing that dislocations are present either as free dislocations (black dots) or accumulate along sub-grain boundaries (black lines) (Senseny et al., 1992) Right: Photomicrograph of a gamma-irradiated salt showing a strain free grain growing at the expense of the old neighboring deformed grain. Grain boundaries are indicated in black while sub-grains boundaries appear in white (Schléder and Urai, 2005).

Dislocation creep is characterized by the generation and displacement of dislocations inside salt grains to form sub-grains (Figure 4, left). The amount of dislocations is

strongly dependent on the applied shear stress and the size of the sub-grains can be estimated from the applied stress. The sub-grains are progressively rotated by the accumulation of dislocations which eventually leads to the creation of small new grains. This is a grain size reduction process which is referred to as rotation recrystallization and is one of the end members of dynamic recrystallization (Drury and Urai, 1990). A recovery process is associated with this mechanism: grain boundary migration. This process, which is the other end member of dynamic recrystallization, is driven by differences in chemical potentials across grains. Grains with higher dislocation densities have higher chemical potentials which creates a flux of ions to the less deformed grains. Small strain free grains grow at the expense of old deformed grains in this scenario (Figure 4, right) and this process is strongly enhanced by the presence of water at the grain boundaries, as water facilitates the movement of ions. In the absence of boundary water, impurity drag effects considerably slow down or even prevents migration and strain hardening prevails. In nature, grain boundary migration is widespread in salt and it effectively removes the evidences of past deformations. Peach et al. (2001) provided a mathematical description of grain boundary migration which gives an expression for the uniaxial rate of deformation combining dislocation creep and grain boundary migration:

$$D = \frac{V}{C_{GBM}} \exp\left(\frac{E-Q}{RT}\right) \tilde{\epsilon}^{n-m} \quad (2)$$

where V is the velocity of grain boundary migration, C_{GBM} is a constant, E is the activation energy for grain boundary diffusion which can be taken from Spiers et al. (1990) even though it was established for pressure solution and m is equal to 1 or 2 depending on whether subgrains have developed or not, respectively. From Eq.(2) we see that the effect of grain boundary migration is to reduce the degree of nonlinearity and the activation energy. Experiments (Peach et al., 2001) confirm that the flow stress is decreased by grain boundary migration as forecasted by Eq.(2).

Creep tests conducted under laboratory conditions usually use large stresses to reduce observation time and the behavior of salt falls thus in the dislocation creep regime. If steady state is reached (strain has to be large enough to reach secondary creep) and the salt samples are wet, it is very likely that the stress-strain curves are influenced by both end members of the dynamic recrystallization such that the parameters of Eq.(2) are

measured instead of those of Eq.(1). This means that the effect of grain boundary migration on salt behavior is likely to be already included in the parameters obtained from the data fit.

Steady state in wet salt is associated with dynamic recrystallization so the mean grain size remains constant due to a balance between grain growth and reduction. Based on this assumption referred to as the Field Boundary Hypothesis by De Bresser et al. (2001), Ter Heege et al. (2005b) established a piezometric relation relating grain size and stress for rock salt:

$$\frac{d}{b} = 10^{-1.55} \left(\frac{\tilde{\tau}}{G} \right)^{-1.85} \exp \left(\frac{14.2 \times 10^3}{RT} \right) \quad (3)$$

where d is the mean recrystallized grain size in mm , b is the Burgers vector equal to $3.99 \times 10^{-7} mm$ and G is the shear modulus equal to $15 GPa$.

1.2.2.2 Pressure solution

Pressure solution – or solution precipitation creep – is a solution-precipitation process where salt is dissolved at grain contacts and diffuses through wet grain boundaries to precipitate on low stressed surfaces (Rutter, 1983). The limiting mechanism controlling deformation is the diffusion of ions through water (de Meer et al., 2002) so pressure solution can be described macroscopically by a linear law. Pressure solution is dominant at low stresses and temperatures in wet salts (Figure 3) but it is seldom considered as a deformation mechanism in applications like hole closure. This lack of reconnaissance is due to the high strain rates usually used in creep tests which leads dislocation creep to be the investigated deformation mechanism and not pressure solution.

Pressure solution is associated microscopically with grain boundary sliding, dissolution and precipitation structures (Urai and Spiers, 2007). Figure 5 shows typical microstructures like truncations (dissolution) at grain contacts and overgrowths (precipitation) in the pore space.

Figure 5 Optical micrograph showing truncation (t) and overgrowth (o) structures typical of pressure solution in a wet synthetic salt aggregate (Spiers et al., 1990).

A mathematical description of pressure solution was given by Spiers et al. (1990) in the framework of confined uniaxial compression:

$$D = 5V_m \frac{Z^*}{T} \frac{\tilde{\tau}}{d^3} \quad (4)$$

where D is the uniaxial strain rate, V_m is the molar volume of rock salt, Z^* is the effective grain boundary diffusivity and $\tilde{\tau}$, T and d are, as above, the differential stress, the temperature and the grain size. Expressing that Z^* is temperature activated, the classical equation describing pressure solution is obtained:

$$D = B_{PS} \frac{\exp(-\frac{Q_{PS}}{RT})}{T} \frac{\tilde{\tau}}{d^3} \quad (5)$$

where Q_{PS} is the activation energy for boundary diffusion and B_{PS} is a material constant:

$$B_{PS} = 5V_m D_0 C_0 S \quad (6)$$

where D_0 is the diffusivity of dissolved salt in grain boundary fluid at $T = \infty$, C_0 is the average solubility of salt in grain boundary fluid at $T = \infty$ and S is the average thickness of grain boundary fluid. The value of the parameters in Eq.(5) were evaluated by Spiers et al. (1990) from a synthetic salt aggregate and then applied to dense rock salt. There is a consensus in the scientific community to use the values established by Spiers et al. (1990) to model pressure solution in salt but there seems to be a transcription mistake in the B_{PS} value. The reported value by Spiers et al. (1990) is

$B_{PS} = 5V_m (2.79 \pm 1.40) \times 10^{-15} \text{ K.m}^3.\text{Pa}^{-1}.\text{s}^{-1}$ which is equal to $B_{PS} = 3.7 \times 10^{-4} \text{ K.mm}^3.\text{MPa}^{-1}.\text{s}^{-1}$ and not $B_{PS} = 4.7 \times 10^{-4} \text{ K.mm}^3.\text{MPa}^{-1}.\text{s}^{-1}$ as found when quoted in other publications. The latter value however remains in the uncertainty bounds so it is valid but arbitrary.

Figure 6 Optical micrographs of grain contacts in wet salt showing channel like inclusions. Left: Scanning Electron Microscope image (Spiers et al., 1990). Right: (Schoenherr et al., 2010).

Although pressure solution is supposed to be governing deformation at low stresses, a macroscopic threshold stress exists below which it is cancelled (van Noort et al., 2008). Observation of salt grains contacts under very low applied stresses shows grain boundary healing driven by the reduction of interfacial energy (Hickman and Evans, 1992; Hickman and Evans, 1991). If grain boundaries initially display an island-channel structure (Figure 6), the neck (or grain contact) growth occurring during healing progressively isolates grain boundary water in disconnected inclusions which effectively prevents pressure solution (Figure 7). This leads to immobile grain boundaries.

Figure 7 Rock salt grain boundaries will tend to heal with a decrease in differential stress (from Li and Urai (2016) quoting Drury and Urai (1990)).

1.2.2.3 Chosen constitutive law

Creep laboratory measurements are carried out at deformation rates that are much higher than the ones encountered in nature or in long term applications and the question of the determination of a representative constitutive law for the long term arises. An extrapolation of laboratory data to underground conditions has to be done based on the deformation mechanisms identified above. Figure 8 shows how laboratory measurements are fit by the functions corresponding to pressure solution and dislocation creep and what the extrapolated constitutive law for salt at small strain rates looks like. In long term applications, both pressure solution and dislocation creep can be the dominating deformation mechanism, depending on the load. A constitutive law for salt should therefore at least take both deformation mechanisms into account to reproduce the flow of salt encountered at depth (Li and Urai, 2016; Spiers and Carter, 1996; Urai and Spiers, 2007; van Keken et al., 1993).

Figure 8 Deviatoric stress - strain rate diagram summarizing laboratory data for a wide range of salts collected at different temperatures but corrected to 20°C here. The broken line represents the best fit to the temperature corrected dislocation creep data and the solid lines are the pressure solution creep law for different grain sizes (modified from Urai and Spiers (2007)).

In this work, we focus on long term hole closure so we rely primarily on secondary creep. Simplified Carreau (Carreau, 1972) or Ellis (Bird et al., 1960) models for salt viscosity are used which are linear at low deformation rates and non-linear otherwise to fit the observations from Figure 9. Pressure solution dominates in the linear regime, while dislocation creep dominates in the non-linear one. We dismiss the plastic behavior of salt because we are interested in confined salts subjected to low to medium shear stresses that never reach tertiary creep. Incompressibility, i.e. volume constant deformation, is assumed (Fossum and Fredrich, 2002; Senseny et al., 1992). Elastic compressibility is also considered, but only when short term or transient answers to loading are investigated.

Figure 9 Same data as in Figure 8 but the apparent viscosity is represented as a function of the deviatoric strain rate instead. The dislocation creep data obtained at different temperatures has been corrected to 20°C .

1.3 Summary of the thesis

The thesis is a compilation of three papers presenting analytical solutions and numerical results investigating the driving mechanisms of hole closure in salt at different times. The numerical results were obtained using modified versions of in-house numerical codes. Several one-dimensional codes based on radial coordinates have been developed to model pressure driven hole closure. A two-dimensional numerical code based on MILAMIN (Dabrowski et al., 2008), a fast finite element code solving Stokes problem on large unstructured meshes has been used for the more general cases. The resolution of the non-linear system of equations relied on the solver implemented in FOLDER (Adamuszek et al., 2016).

1.3.1 Paper 1: Long-term cavity closure in non-linear rocks

Published in *Geophysical Journal International*.

The paper investigates the steady state closure of a pressurized cylindrical cavity occurring in a Carreau viscous body under different far field loads. An analytical solution describing the one-dimensional closure under far field pressure is given and benchmarked numerically. The solution shows that the regime in which the material behaves depends on the applied differential pressure. Using the three characteristic numbers defining a simplified Carreau model, namely the linear viscosity threshold, the transition rate and the stress exponent, we establish the load limit under which the material behavior is dominated by its linear regime. Above this limit, a zone where the non-linear behavior of the material dominates appears in the neighborhood of the hole. This illustrates the impact of the viscosity threshold from the Carreau viscous model. Another analytical solution is provided for the maximum closure velocity at the hole rim when the material is subjected to far field shear. The solution is based on a linearization in the far field which transforms the non-linear isotropic material into a linear anisotropic one. The solution is benchmarked numerically and shows again that, depending on the shear load at infinity, the closure velocity is determined by either the linear or non-linear properties of the material. Using the two end member solutions, we propose a new proxy for the maximum closure velocity at the rim under general far field loads in a Carreau viscous body. Compared with numerical results, the misfit of the predictions made by this new solution is always less than 50%, which is acceptable compared to the orders of magnitude error potentially done if the speed ups due to the linear viscosity threshold from the Carreau model or due to the far field shear are not taken into account.

1.3.2 Paper 2: Long-term creep closure of salt cavities

Submitted to *International Journal of Rock Mechanics and Mining Sciences*.

The paper quantifies the steady state closure of salt cavities using a Carreau model for viscosity. We apply the analytical solutions and the numerical approaches developed in the first paper to rock salt under typical conditions encountered in the underground. The viscosity threshold in the Carreau model describes pressure solution which is expressed at the macroscopic level by a grain size dependent linear viscosity defined by Spiers et al. (1990). The non-linear part of the Carreau model corresponds to dislocation creep and three different salts are considered to capture the variety of

answers due to this end member. An Avery salt with a grain size of 7.5 mm (Carter et al., 1993) is taken as a reference and two other salts are chosen which display extreme dislocation creep behaviors. The behavior of salt is strongly dependent on the applied load and grain size while salt type and temperature only have a limited impact. Pressure solution is the dominant deformation mechanism in cold fine-grained salts under small shear loads while dislocation creep is dominant in warm coarse-grained salts under large loads. The dichotomy in material behavior is observed during hole closure because of the spatial variations of the stress around the hole: the zones subjected to a high shear load behave in the non-linear regime while the others are linear. Investigating the closure rate at the hole rim as a function of loads shows that it is very sensitive to grain size. The type of far-field load also matters: if the material behaves non-linearly, shear stresses are n (the stress exponent) times more important than pressure difference. Even though far field shear stress in salt is usually considered to be negligible, this new result may change the definition of what a negligible shear stress is. Under the proper combination of loads, grain size and temperature a 10% closure can be recorded in less than one day. Although this is an extreme case, it stresses the importance of taking pressure solution into account because it can explain fast closure velocities without referring to transient creep. The determination of the dominant deformation mechanism governing closure is also investigated. Under typical in-situ conditions, the closure is governed by both deformation mechanisms which confirms our choice of constitutive law. Determining the dominant deformation mechanism of a particular salt based solely on its grain size is not possible and a more thorough study including load, temperature and dislocation properties should always be done. Finally, the influence of dynamic recrystallization on hole closure is investigated. A stress dependent grain size is implemented to model grain boundary migration which leads to the cancellation of pressure solution when the applied pressure difference is smaller than 32 MPa. This result, which is the exact opposite of the one obtained without using a stress dependent grain size, is interesting but it is only preliminary as it is based on an oversimplification of grain size distributions. Further studies are necessary which take into account the complete grain size distribution of rock salt before a definitive conclusion can be reached.

1.3.3 Paper 3: Non-linear viscoelastic closure of salt cavities.

Submitted to *Rock Mechanics and Rock Engineering*.

The paper investigates pressure driven hole closure in salt using a non-linear Maxwell viscoelastic constitutive law. The viscous component follows an Ellis model for which the deformation is the sum of the deformations of the individual mechanisms: pressure solution and dislocation creep. It is very similar to the Carreau model used previously but analytical solutions at $t = 0+$ can be derived for Maxwell bodies based on this model. In the study, pressure loads are applied at infinity and the problem reduces to one dimension so we use one-dimensional numerical codes to solve it.

We start by considering a Maxwell body in which the viscous component is purely non-linear as was done by Karimi-Jafari et al. (2006); Wang et al. (2015). We rederive the expressions obtained by Wang et al. (2015) for the initial closure velocity at the rim which allows us to fully characterize the instantaneous hole closure. A corrected version of the characteristic relaxation time proposed by Wang et al. (2015) is also presented. For power law based Maxwell bodies, the uncorrected and corrected versions of the characteristic time are almost equal. Using the numerical codes, the characteristic time is shown to accurately describe the moment at which the closure velocity evolves from its initial value to its long-term one, irrespective of the salt type or load. Having analytical solutions for the initial and final closure velocities as well as an expression for the transition time, we propose an empirical proxy for the time dependent closure velocity at the rim. Numerical simulations show that the proxy has less than 3% inaccuracy for all types of salts and loads and for times smaller than three times the characteristic one. At larger times, the salts behaving highly non-linearly depart from the proxy. The time evolution of the stress and rate of deformation distributions are also investigated and show the growth of a non-linear viscously dominated zone in the neighborhood of the hole.

We then consider a Maxwell body in which the viscous component follows an Ellis model. The initial closure for this case is also fully characterized analytically. The same characteristic load as for a Carreau fluid is used to evaluate whether the viscous behavior is dominated by the linear or non-linear regime. Defining pseudo steady-state as the moment when the viscous rate of deformation is larger than the elastic one in the entire domain, we show that this state can be achieved initially for small loads

compared with the characteristic one. Investigating further the impact of the dominant viscous deformation mechanism on closure we characterize the ratio of the initial to the steady state closure velocities as a function of the load and salt type. When pressure solution is dominant, pseudo steady state is reached instantaneously and initial and long-term closure velocities are very close. When dislocation creep is dominant, the ratio of the velocities is n^{n-1} with n the stress exponent. Investigating the evolution with time of closure, we show that our version of the characteristic time also works for Ellis based Maxwell materials. Knowing the initial and long-term closure velocities as well as the transition time, we attempt to describe the time dependent drop in closure velocities by the master curve obtained for a power law based Maxwell material. The master curve is approached when the viscous deformation is dominated by dislocation creep but not otherwise when pressure solution is dominant. However, when pressure solution is dominant, the variation with time of the closure velocity is so small that pseudo steady state is always reached.

We finish by studying the time evolution of closure velocities in an application. The case of a cylindrical hole extending in depth is considered. Based on typical temperature and pressure gradients we compute the profiles for the characteristic relaxation time and for the pseudo steady state time. It shows that pseudo steady state is reached almost instantaneously at shallow depths, where pressure solution is the governing viscous deformation mechanism. At greater depths, the viscous deformation is initially governed by dislocation creep and the characteristic relaxation time should be used to interpret closure. Pseudo steady-state is reached in less than a year along the whole profile indicating that elastic deformation rates are only dominant over viscous deformation rates at short time scales away from the hole. This example also illustrates the importance of taking pressure solution into account when studying the viscoelastic closure of cavities in rock salt.

1.4 Discussion and outlook

This work is based on several assumptions which limit the scale of the applicability of the solutions and conclusions presented above. Of all the assumptions, homogeneity has one of the largest uncertainties. Although many salts are very homogeneous with little impurities (Fredrich et al., 2007), other salts, like bedded salts, cannot be

considered as homogeneous and their creeping behavior have to be studied separately (Liang et al., 2007; Zhang et al., 2012). Renard et al. (2001); Zubtsov et al. (2004) also reported that the addition of hard particles into homogeneous salts can greatly enhance deformation by pressure solution. The law established by Spiers et al. (1990) relies on a completely homogeneous synthetic rock salt which does not take this effect into account. The effect of pressure solution on closure is therefore probably underestimated from this perspective.

The other major uncertainty concerns pressure solution and whether it is active at depth. Having noticed the presence of stringers in a Zechstein salt pillow, Li et al. (2012) concluded that the stringers did not sink through salt with time because salt deformation was governed only by dislocation creep. Summarizing their opinion on whether pressure solution is active at depth, Li and Urai (2016) stated that: “Combined with experimental results in laboratories and microstructure research as the reliable exploration of experimental results, we conclude that rock salt rheology can be simplified to dislocation creep corresponding to power law creep ($n = 5$) with the appropriate material parameters in the salt tectonic modeling.” Direct observations of the microstructure of buried salts have indeed never confirmed that pressure solution is active at depth. It seems like pressure solution is only observed in extruding salt

Figure 10 Picture from the Wieliczka salt mine in Poland showing patterns of heterogeneities on the wall of one of the drifts.

bodies and associated with rainwater (Desbois et al., 2010; Schlöder and Urai, 2007; Schoenherr et al., 2010). In buried salts, only microstructural evidences of dislocation creep and grain boundary migration have been found. Creep experiments done on synthetic salts show that pressure solution stops being active after some threshold strain has been reached (Pennock et al., 2006) which suggests that pressure solution is only active at the beginning of halokinesis in buried salts. The small shear stresses found in nature can also lead to grain boundary healing which could prevent pressure solution (Li and Urai, 2016). The water at grain boundaries is progressively isolated in unconnected inclusions during healing due to neck growth and this effectively stops pressure solution. Another possible explanation for the deactivation of pressure solution is the progressively smoother grain contacts created by this mechanism (Spiers and Schutjens, 1999). Diffusivity of grain boundary water is very small (Nakashima, 1995; Rutter, 1983; Watanabe and Peach, 2002) so if water becomes distributed as thin fluid films between grain contacts, pressure solution can be slowed down by several orders of magnitude.

Other processes however exist that enhance the contribution from pressure solution in several applications. First, many cavities have been created by solution mining i.e. by injecting fresh water to dissolve salt. The impact of this water on the behavior of the salt body has not yet been investigated but it could possibly enhance pressure solution due to the progressive permeation of water in the formation. Second, under large under-pressures, plastic deformation in the vicinity of the hole can occur and increase permeability by several orders of magnitude (Alkan, 2009; Hou, 2003; Peach and Spiers, 1996; Schulze et al., 2001) which can also greatly enhance the importance of pressure solution. Finally, as rainwater has been associated with pressure solution at the surface, the salinity of the water in rock salt seems to be an important factor controlling the governing deformation mechanism. An analysis of fluid pathways in rock salt should be done to identify water movements in and around salt bodies to better assess the salinity of the water present in salt.

Further developments in the study of hole closure should include the modeling of the excavation disturbed zone to better assess the role played by pressure solution. In addition, estimating the time dependent grain growth in the salt body due to dynamic recrystallization could be useful to understand the grain size distribution evolution

away from the hole. A new approach for the steady state constitutive law of salt could be used that rely on the local grain size distribution encountered at any point away from the hole (Ter Heege et al., 2005a). Including transient creep in the constitutive law when studying viscoelastic hole closure would also be interesting to better describe the short term answer. Finally, a new estimation of the parameters describing pressure solution macroscopically should be done to confirm the ones established by Spiers et al. (1990) in synthetic salt.

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Paper I: Long-term cavity closure in non-linear rocks

Published in *Geophysical Journal International*, Vol. **210**, 2017, pp. 1231-1243.

doi: 10.1093/gji/ggx227

2 Long-term cavity closure in non-linear rocks

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2.1 Abstract

The time dependent closure of pressurized cavities in viscous rocks due to far field loads is a problem encountered in many applications like drilling, cavity abandonment and porosity closure. The non-linear nature of the flow of rocks prevents the use of simple solutions for hole closure and calls for the development of appropriate expressions reproducing all the dependencies observed in nature.

An approximate solution is presented for the closure velocity of a pressurized cylindrical cavity in a non-linear viscous medium subjected to a combined pressure and shear stress load in the far field. The embedding medium is treated as homogeneous, isotropic, and incompressible and follows a Carreau viscosity model. We derive analytical solutions for the end-member cases of the pressure and shear loads. The exact analytical solution for pressure loads shows that the closure velocity

v_R is given by the implicit expression $\frac{\Delta p}{2\mu_0 D_{II}^*} = -\frac{1}{2} B\left(\frac{v_R^2}{RD_{II}^* + v_R^2}; \frac{1}{2}, -\frac{1}{2n}\right)$, where Δp

is the pressure load, R is the hole radius, B is the incomplete beta function, and μ_0 , D_{II}^* , n are respectively the threshold viscosity, transition rate and stress exponent of the Carreau model. The closure velocity is dominated by the linear mechanism under pressure loads smaller than $1.8\mu_0 D_{II}^*$ and by the non-linear one under large pressure loads. In the non-linear regime, pressure variations support an increasing part of the load with increasing degree of non-linearity. The decay of the stress perturbation in the non-linear zone varies as $r^{-2/n}$ where r is the radial distance to the hole.

A solution for the maximum closure velocity at the cavity rim $v_{R\max}$ under far field

shear is given: $v_{R\max} = \left(1 + \overline{M}_s^{-1/2}\right) R \overline{D}_{II}$, where

$\overline{M}_s = \left(1 + \overline{D}_{II}^2 / n D_{II}^{*2}\right) / \left(1 + \overline{D}_{II}^2 / D_{II}^{*2}\right)$ and $\overline{D}_{II}$ is the second invariant of the far field

deformation rate. The solution remains valid in the limit of ideal power law fluid when

$\overline{M}_s = 1/n$. The solution is based on an approximation which transforms the non-linear

isotropic constitutive law into a linear anisotropic one in the far field.

The proposed approximate solution for closure velocity for general far field load is

based on the two end member analytical solutions. They are additively combined after

replacing the material threshold viscosity μ_0 by the apparent background viscosity due

to the far field shear. Benchmarking the solution shows that there is a 50% misfit at

most between the analytical and the numerical solution for closure rate. Comparing

the closure velocities obtained with a Carreau viscosity model with the ones from a

power law material shows an increase of several orders of magnitude for pressure loads

smaller than $2\mu_0 D_{II}^*$. Far field shear can also increase hole closure rate by several

orders of magnitude. Compared with other available solutions, the approximate

solution presented here ensures that both speed ups are taken into account making it

well suited for actual underground conditions where both diffusion creep and shear

stresses occur. The additional closure mechanisms considered here can potentially

explain the fast closure rates observed underground without referring to transient

mechanisms.

2.2 Introduction

Viscous materials flow under stress and cavities within them either close or expand

depending on their internal pressure. Rocks like ice and salt are especially of interest

but minerals like quartz and calcite can also exhibit significant creep if the temperature

is high enough. Applications for the problem of a hole closing in a viscous material

include the long-term creep of salt caverns used as storage facilities (Bérest et al.,

2001; Brouard et al., 2013; Heusermann et al., 2003), the establishment of rules for

drilling through salt or ice (Barker et al., 1994; Dusseault et al., 2004a; Talalay et al.,

2014), porosity closure in shear enhanced compacted materials (McKenzie, 1984;

Xiao and Evans, 2003), the estimation of material parameters (Naruse et al., 1988; Talalay and Hooke, 2007) and more (Mackerle, 2000).

The steady state flow of a material can be described using a deformation map showing the dominant deformation mechanisms as a function of strain rate, temperature and grain size (Fossum and Fredrich, 2002; Mackerle, 2000; Munson and Dawson, 1981). Rock viscosity can typically be described as being the combination of a linear and non-linear solid-state deformation mechanism. The non-linear mechanism describes the motion of dislocations in the material (Carter and Hansen, 1983; Glen, 1955; Kirby and Kronenberg, 1987; Ranalli, 1995) while the linear one refers to a diffusion process which is either pressure-solution (Spiers et al., 1990; Urai and Spiers, 2007), Coble (Coble, 1963) or Nabarro-Herring (Herring, 1950) creep depending on the temperature and water saturation. We use a Carreau viscosity model (Carreau, 1972) to combine both linear and non-linear steady state processes into a single smooth constitutive law. The purely viscous rock behavior approach is valid for long-term flow where the initial elastic contribution to the total strain is negligible. It relies on the assumption that transient phenomena like time dependent stress redistribution in the rock mass are no longer active (Wang et al., 2015).

Analytical solutions have been derived for the problem of hole closure in a creeping host under far field loads using different constitutive laws and hole shapes. Power law viscosities are a common choice to model steady state creep. Nye (1953) derived a solution using this rheology for a circular opening when pressure boundary conditions are applied at the rim and at infinity. Bogobowicz et al. (1991) provided a more general answer in the case where the opening is elliptical. Van Sambeek (1990) used a circular opening but gave the solution in terms of volume loss rather than velocity at the rim. Using Perzyna viscoplastic theory, Liu et al. (2011) rederived the same solution as Nye (1953) when the chosen yield function was a Von Mises one. Finally, an approximate solution using a power law viscosity has been provided by Barker et al. (1994) and it has been used in the oil industry even though it overestimated the closure velocity (Willson et al., 2003). All the mentioned analytical solutions have been derived for pressure loads but in reality, the stress state is not isotropic. Using power law materials, Birnbaum et al. (1968); Cornet et al. (2016); Lee and Smith (1987); Sofronis and McMeeking (1992) provided approximate solutions under general loads. Birnbaum et

al. (1968); Lee and Smith (1987) used an approach based on complex potentials and gave an approximation for the Airy stress function. Cornet et al. (2016); Sofronis and McMeeking (1992), on the other hand, relied on solutions for the pressure and shear end members to interpolate intermediate cases. The constitutive law used in this study relies on a combination of power law and constant viscosity which has not been extensively dealt with in relation to cavity closure. Dismissing the diffusion creep under low stress can have a big impact on the predicted closure velocity of a hole (Orlic and Buijze, 2014). Brouard et al. (2009) provided a solution when pressure loads are applied and a sharp transition from linear to non-linear behavior is used in the constitutive law.

We study the problem of hole closure in a Carreau material under general far field stress. We adopt an approach involving both analytical and numerical modelling and remain as general as possible to fit all previously mentioned applications. First, we derive the analytical solution for cavity closure under far field pressure loads. Then we investigate the shear end member. Based on the solutions for the two end members we propose a proxy for the maximum closure velocity at the rim under general far field loads.

2.3 Model

2.3.1 Setup

We study the velocity field around a circular hole of radius R embedded in a Carreau fluid under general far-field loads. We introduce a Cartesian reference frame $(0, x, y, z)$ centered on the hole (Figure 11). We assume a plane strain condition perpendicularly to the z axis, making the problem two dimensional, because we consider highly elongated cylinders are representative of boreholes and caverns. We assume the host body is homogeneous and isotropic. Exploiting symmetry, we simplify the problem to study only one quadrant (Figure 11).

We set all boundaries to free slip. The normal components of the velocity are set to zero along the left and bottom boundaries. At the hole rim, we apply normal tractions corresponding to the well pressure $-p_w$. At the exterior far field boundaries, we consider a general stress state that is the sum of a pressure $-\bar{p}$ and deviatoric stress

$\bar{\tau}$. Note that $\bar{p}$ is always greater or equal to p_w so that $\Delta p = \bar{p} - p_w$ is always positive leading to borehole closure. The external boundaries are set at a finite distance but far enough to approach the infinite body condition of the analytical models.

The problem studied hereafter is considered to be purely mechanical except for the indirect dependency on temperature via the material coefficient. We neglect any sort of chemical interactions or phase transitions. Porous fluid flow is also dismissed because of the low permeabilities of the studied materials.

Figure 11 Sketch of the geometry, mesh, and boundary conditions v_x and v_y are respectively the horizontal and vertical components of the velocity t_x and t_y are the components of the traction. In the actual models the far field boundaries are much farther away from the hole and the mesh adjacent to the hole substantially finer.

2.3.2 Constitutive model

The constitutive model is incompressible viscous and is made of a linear and a non-linear branch. The linear behavior refers to diffusion creep and is described by:

$$\tau_{kl} = A^L D_{kl}^L \quad (7)$$

where k, l are spatial indices going from 1 to 2, τ is the deviatoric stress, A^L is a material parameter, and D^L is the deformation rate due to linear behavior.

The non-linear part of the rheology corresponds to dislocation creep and accounted for by the following power law relationship:

$$\tau_{kl} = A^{NL} \tau_{II}^{1-n} D_{kl}^{NL} \quad (8)$$

where A^{NL} is a material parameter, τ_{II} is the second deviatoric stress invariant, n is the stress exponent ($n > 1$ for shear thinning media) and D^{NL} is the deformation rate due to non-linear behavior. Since dislocation creep is a thermally activated process, A^{NL} is usually taken to follow an Arrhenius law.

We use a Carreau viscosity model (Carreau, 1972), with zero viscosity for infinite shear rates, to smoothly combine linear and non-linear behavior in a single constitutive law:

$$\tau_{kl} = 2\mu_{app} D_{kl} \quad (9)$$

where the apparent viscosity μ_{app} is given by:

$$\mu_{app} = \mu_0 \left(1 + \left(\frac{D_{II}}{D_{II}^*} \right)^2 \right)^{\frac{1-n}{2}} \quad (10)$$

with

$$D_{II} = \sqrt{\frac{1}{2} D_{kl} D_{kl}} \quad (11)$$

μ_0 is the threshold viscosity under zero deformation rate equal to $A^L/2$, n is the same stress exponent as the one in the power law Eq.(8), D_{kl} is the total deviatoric strain rate, D_{II} is the second invariant of the rate of deformation tensor, D_{II}^* is the deformation rate at the transition from linear to non-linear behavior. D_{II}^* is derived from Eq.(8) and (7) by stating that the deformation rates due to the two mechanisms are equal to D_{II}^* at the transition:

$$D_{II}^* = \frac{1}{A^L} \left(\frac{A^{NL}}{A^L} \right)^{\frac{1}{n-1}} \quad (12)$$

The constitutive law from Eq.(10) is fully characterized by three parameters: μ_0 the Newtonian viscosity, D_{II}^* the transition deformation rate and n the stress exponent. Once normalized using these quantities, all the curves representing a Carreau viscosity as a function of deformation rate have the same shape: constant at low strain rates and non-linear at higher ones (i.e. Figure 12). The three characteristic values are readily extracted from such a plot: the viscosity for low deformation rates is the threshold viscosity μ_0 , the transition from linear to non-linear regime occurs at the transition rate D_{II}^* and the slope of the non-linear branch in a logarithmic plot is equal to the stress exponent n . These parameters, however, do not control the exact transition from one branch to another. In Figure 12 we compare the transitions in apparent viscosities using three different approaches. The first one consists in putting a sharp transition from linear to non-linear behavior so that only one mechanism is active for any deformation rate (Brouard et al., 2009). Another way of modeling both deformation mechanisms is to consider the total deformation rate to be the sum of the contributions from the two mechanisms. This results in an Ellis model for the apparent viscosities where the effective viscosity is the geometric mean of the two separate viscosities (van Keken et al., 1993). Finally, the Carreau viscosity considered in this study is plotted. Figure 12 shows the normalized apparent viscosities at the transition. The difference in behavior between the different approaches is small. We therefore dismiss any parameter referring to the smoothness of the transition in the following.

Figure 12 Comparison of the Carreau and Ellis models to fit Newtonian viscosity at low deformation rates and power law at high ones.

2.3.3 Numerical model

The numerical code used to study the effect of far field differential stresses on borehole closure builds up on MILAMIN (Dabrowski et al., 2008), a fast 2D finite element code solving linear viscous Stokes flow. It uses triangular element meshes to accurately resolve interfaces. To take into account our non-linear rheological model, we use FOLDER (Adamuszek et al., 2016), a modified version of MILAMIN. FOLDER uses a combination of Picard and Newton-Raphson iterations to efficiently reach a converged solution.

Mesh refinement is introduced close to the hole where large variations in velocity and pressure are expected to occur. The size of the studied domain is big compared to the hole radius to facilitate comparisons to the analytic solution (five thousand times borehole radius). A coarse mesh is used far away from the hole to decrease computing time without significant loss in the accuracy of the solution. We keep the ratio of the area of the largest to the smallest element limited to avoid numerical issues during pressure iterations for large domains and large stress exponents. In our case, the maximum value for this ratio is equal to ten orders of magnitude.

2.4 Analytical solution for pressure loads

2.4.1 Derivation

We consider the two-dimensional plane strain setup from Figure 11 where an infinite homogeneous isotropic medium has a circular hole of radius R and pressures $-p_w$ and $-\bar{p}$ are prescribed in the hole and at infinity, respectively. In this case, there is an axial symmetry making the problem one dimensional along the radial coordinate and the radial velocity v_r of any point in the material is only a function of its distance r to the center of the hole. All the other velocity components are zero. We use polar coordinates with r the radial distance, θ the polar angle and z the depth.

We establish the relationship between the velocity at any point and the velocity v_R at the rim. The non-zero components of the rate of deformation tensor in polar coordinates are:

$$D_{rr} = \frac{\partial v_r}{\partial r}, \quad D_{\theta\theta} = \frac{v_r}{r} \quad (13)$$

where D_{rr} and $D_{\theta\theta}$ are respectively the radial and hoop components of the rate of deformation tensor. Using the plane strain condition, $D_{zz} = 0$, the incompressibility condition reads $D_{rr} + D_{\theta\theta} = 0$. Using it with Eq.(13), and multiplying by r leads to:

$$\frac{\partial r v_r}{\partial r} = 0 \quad (14)$$

Integrating with respect to r gives:

$$v_r = v_R \frac{R}{r} \quad (15)$$

with v_R the velocity at $r = R$. When r tends to infinity we see that the velocity converges to zero, as expected. We can use Eq.(15) to find the radial deformation rate:

$$D_{rr} = -v_R \frac{R}{r^2} \quad (16)$$

To fully determine the velocity field, an expression for v_R has to be found. Using incompressibility, $D_{\theta\theta} = -D_{rr}$, which, in turn, gives:

$$D_{II} = |D_{rr}| \quad (17)$$

Moving on to force balance when gravity and inertia terms are neglected, we have:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r} = 0 \quad (18)$$

where σ_{rr} is the radial component of stress, and τ_{rr} and $\tau_{\theta\theta}$ are respectively the radial and hoop deviatoric stresses. Due to plane strain and to the definition of deviatoric stresses the force balance becomes:

$$\frac{\partial \sigma_{rr}}{\partial r} + 2 \frac{\tau_{rr}}{r} = 0 \quad (19)$$

Integrating from r to ∞ and using the boundary conditions at infinity:

$$[\sigma_{rr}]_r^\infty + \int_r^\infty 2 \frac{\tau_{rr}}{r} dr = 0 \quad (20)$$

Since

$$\tau_{rr} = 2\mu_0 \left(1 + \frac{D_{rr}^2}{D_{II}^{*2}} \right)^{\frac{1/n-1}{2}} D_{rr} \quad (21)$$

we have

$$\int_r^\infty 2 \frac{\tau_{rr}}{r} dr = 4\mu_0 \int_r^\infty \left(1 + \frac{D_{rr}^2}{D_{II}^{*2}} \right)^{\frac{1/n-1}{2}} \frac{D_{rr}}{r} dr \quad (22)$$

and using Eq.(16), we have

$$dD_{rr} = \frac{-2\nu_R R}{r^3} dr = \frac{-2D_{rr}}{r} dr \quad (23)$$

so

$$\int_r^\infty 2 \frac{\tau_{rr}}{r} dr = 2\mu_0 \int_0^{\beta/r^2} \left(1 + \frac{D_{rr}^2}{D_{II}^{*2}} \right)^{\frac{1/n-1}{2}} dD_{rr} \quad (24)$$

which is an integral which can be evaluated using the incomplete beta function defined by

$$B(z; a, b) = \int_0^z u^{a-1} (1-u)^{b-1} du \quad (25)$$

Making the change of variable $u = D_{rr}^2 / D_{II}^{*2} / (1 + D_{rr}^2 / D_{II}^{*2})$ in Eq.(24) gives:

$$\int_r^\infty 2 \frac{\tau_{rr}}{r} dr = \mu_0 D_{II}^* B \left(\frac{\tilde{v}_R^2 \frac{R^4}{r^4}}{1 + \tilde{v}_R^2 \frac{R^4}{r^4}}; \frac{1}{2}, -\frac{1}{2n} \right) \quad (26)$$

with

$$\tilde{v}_R = v_R / R D_{II}^* \quad (27)$$

the normalized velocity. Using the boundary condition at infinity $\sigma_{rr} = -\bar{p}$, we get:

$$\sigma_{rr} = -\bar{p} - \mu_0 D_{II}^* B \left(\frac{\tilde{v}_R^2 \frac{R^4}{r^4}}{1 + \tilde{v}_R^2 \frac{R^4}{r^4}}; \frac{1}{2}, -\frac{1}{2n} \right) \quad (28)$$

Finally, using the boundary condition at the rim $\sigma_{rr} = -p_w$, we obtain:

$$-\Delta p - \mu_0 D_{II}^* B \left(\frac{\tilde{v}_R^2 \frac{R^4}{r^4}}{1 + \tilde{v}_R^2 \frac{R^4}{r^4}}; \frac{1}{2}, -\frac{1}{2n} \right) = 0 \quad (29)$$

This solution is only valid for materials that are strictly non-linear ($n > 1$). It breaks in the limit $n \rightarrow 1$ because the transition rate D_{II}^* is not defined anymore. We present in Appendix 1 the derivation for the power law end member. The result for this case is:

$$v_R = -\frac{(\Delta p / n)^n}{A_{NL}} R \quad (30)$$

This result is the same as the one obtained by Bogobowicz et al. (1991); Cornet et al. (2016); Liu et al. (2011); Nye (1953).

2.4.2 Solution and analysis

The complete solution for the problem of a hole closing under pressure load in a Carreau fluid is:

$$\frac{\Delta p}{\tau_{II}^*} = -\frac{1}{2} B \left(\frac{\tilde{v}_R^2}{1 + \tilde{v}_R^2}; \frac{1}{2}, -\frac{1}{2n} \right) \quad (31)$$

where $\tau_{II}^* = 2\mu_0 D_{II}^*$. This equation is solved for $\tilde{v}_R$ and once this quantity is known the velocity and stress fields are entirely determined:

$$v_r = v_R \frac{R}{r} \quad (32)$$

$$\tau_{rr} = \tau_{II}^* \tilde{v}_R \left(\frac{R}{r} \right)^2 \left(1 + \tilde{v}_R^2 \left(\frac{R}{r} \right)^4 \right)^{\frac{1/n-1}{2}} \quad (33)$$

$$\sigma_{rr} + \bar{p} = -\frac{\tau_{II}^*}{2} B \left(\frac{\tilde{v}_R^2 \left(\frac{R}{r} \right)^4}{1 + \tilde{v}_R^2 \left(\frac{R}{r} \right)^4}; \frac{1}{2}, -\frac{1}{2n} \right) \quad (34)$$

$$p - \bar{p} = \tau_{rr} - (\sigma_{rr} + \bar{p}) \quad (35)$$

We plot Eq.(31) in Figure 13 for different stress exponents. The horizontal axis represents the value of the load level $\Delta P / \tau_{II}^*$, which is a measure of the degree of non-linearity of the problem and which is used to estimate the dominant deformation mechanism: values much smaller than 1 mean that the material responds linearly while values much bigger than one indicate non-linear behavior. For rock salt for example, if we use the material parameters established by Carter et al. (1993); Spiers et al. (1990) at a temperature of 60°C and a grain size of 7.5 mm, then $\tau_{II}^* = 1.22$ MPa. In the underground, pressure differences of a couple of MPa are common which justifies the relevance of this characteristic stress. The vertical axis represents the normalized velocity $\tilde{v}_R$ (Eq.(27)) which is equal to D_{II} / D_{II}^* at the rim (Figure 12) and which is used to determine the deformation mechanism. For $\Delta P / \tau_{II}^* < 1$, the reduced velocity is smaller than one and the three materials behave linearly. For load levels larger or equal to one, the reduced velocity indicates that the non-linear regime is reached so the three materials have distinct closure velocities depending on their stress exponents. The higher the stress exponent, the lower the apparent viscosity (Eq.(10)) and hence the larger the reduced closure velocity. Finally, Figure 13 confirms that the ratio $\Delta P / \tau_{II}^*$ provides a measure of the dominant deformation regime as the ratio D_{II} / D_{II}^* does.

Figure 13 Graphical representation of Eq.(31). It displays the reduced rim velocity $\tilde{v}_R$ as a function of the load level $\Delta p / \tau_{II}^*$ for different stress exponents n .

In Figure 14 we plot the stress distributions around the hole for a stress exponent of 3 for different values of the load level $\Delta p / \tau_{II}^*$. The case $\Delta p / \tau_{II}^* = 0.1$ approximates the linear end member: the pressure is almost equal to the pressure load, the radial deviatoric stress is equal to the radial stress and they converge to zero as a function of normalized distance with a power of -2 . The case $\Delta p / \tau_{II}^* = 1$ is very similar to the linear end member with some differences close to the rim $r = R$. When $\Delta p = \tau_{II}^*$ we are at the transition between the two deformation mechanisms, which explains that discrepancies from the linear case start to appear. Going directly to the non-linear end member $\Delta p / \tau_{II}^* = 100$, the role played by the pressure in the pressure-deviatoric stress split is very different. The particularity of the non-linear regime is precisely that pressure does support more stress than in the linear regime. At the rim for example, the radial deviatoric stress is equal to $\tau_{rr}(R) = -\Delta p / n$ while the pressure is $p(R) = -\bar{p} + \Delta p(n-1)/n$ (Appendix 1). This shows that the pressure is not uniformly equal to the far field pressure and that the deviatoric stress is smaller than in the linear case ($\tau_{rr}(R) = -\Delta p$). Increasing the stress exponent increases the contribution of

pressure while reducing the one of the deviatoric stress in the pressure-deviatoric stress split. Another particularity of the non-linear end member is that the stress components decay away from the hole as $(r/R)^{-\frac{2}{n}}$ (Appendix 1). The size of the zone affected by the cavity is thus increased due to a slower decay of the stresses with distance. The case $\Delta P/\tau_{II}^* = 10$ finally is an intermediate case between the linear and non-linear end members. A transition occurs from non-linear behavior close to the rim to linear in the far field. This is illustrated both by the pressure-deviatoric stress split at the rim and by the change in slope of the stress decay. An error is obviously introduced if one uses the solution derived for purely non-linear behavior and it is especially important to have a two mechanisms constitutive law for this case.

Figure 14 Normalized stress distributions as a function of the distance to the hole for different values of the load level $\Delta P/\tau_{II}^*$ for $n = 3$. The normalization is based on the linear case where pressure is equal to the far field pressure everywhere but at the rim and far field σ_{rr} is equal to the opposite of far field pressure.

Figure 14 also demonstrates the ability of our two-dimensional numerical model to reproduce the analytical solutions. It can hence be extended to study more general configurations like far field shear loads.

From Figure 14 we see that it is possible to define zones where the linear and non-linear behaviors are dominating. Away from the hole, the material is governed by its Newtonian part. Close to the hole, shear rate is elevated and a non-linear zone can exist if the applied load is high enough. The transition from linear to non-linear regime is reached when $|D_{rr}|$ is equal to D_{II}^* . Substituting D_{rr} with its expression from Eq.(16) gives an expression for the transition radius R^* :

$$R^* = \left(\frac{|V_R| R}{D_{II}^*} \right)^{\frac{1}{2}} \quad (36)$$

In Figure 15 we plot R^* as a function of the load level $\Delta P / \tau_{II}^*$ for different stress exponents. When the pressure load is lower than the characteristic stress τ_{II}^* the material behaves only linearly which means that the transition radius is equal to the hole radius. When the characteristic stress τ_{II}^* is approached or passed, a zone of limited size appears where non-linear behavior is dominant. The size of this zone increases with increasing pressure load and increasing stress exponent. This fits our previous observation on the effect of the stress exponent on the zone of influence of the hole on the stress distributions. Finally, the pressure load at which non-linear creep starts to influence the material behavior is estimated to $0.9\tau_{II}^*$. This value depends however on the stress exponent, with higher stress exponents giving slightly lower thresholds.

Figure 15 Normalized size of the domain where non-linear material behavior is dominant as a function of the load level $\Delta P/\tau_{II}^*$ for different stress exponents n .

2.5 Approximate solution for shear loads

2.5.1 Derivation

We consider the same two dimensional plane strain setup as above but instead of loading it with pressure we apply a far field pure shear $\bar{\tau}$ where the direction of compression is in the horizontal direction. There is no exact analytical solution to this problem but several authors (Birnbaum et al., 1968; Cornet et al., 2016; Lee and Smith, 1987; Sofronis and McMeeking, 1992) have provided approximate solutions for the power law end member. Following the method used by Fletcher (1974), we linearize the constitutive law using the stress state at infinity as a background state. The total and deviatoric deformation rates are equal because the body is incompressible so we write everything using the total deformation rates for clarity. The total deformation rate is equal to:

$$D_{kl} = \overline{D}_{kl} + D'_{kl} \quad (37)$$

where the overbar quantities refer to the ones at infinity and the primed are the perturbing quantities due to the hole. Inserting Eq.(37) into Eq.(10) and neglecting second and higher order terms we get:

$$\mu_{app} = \overline{\mu_{app}} \left(1 + \frac{1-n}{2n} \frac{\overline{D_{kl}} D'_{kl}}{D_{II}^{*2} + \overline{D_{II}}^2} \right) \quad (38)$$

where $\overline{\mu_{app}}$ is the apparent viscosity at infinity. We write similarly for the deviatoric stress:

$$\tau_{kl} = \overline{\tau_{kl}} + \tau'_{kl} \quad (39)$$

which gives:

$$\tau'_{ij} = 2\overline{\mu_{app}} \left(D'_{ij} + \frac{1-n}{2n} \frac{\overline{D_{kl}} D'_{kl}}{D_{II}^{*2} + \overline{D_{II}}^2} \overline{D_{ij}} \right) \quad (40)$$

If we consider the x and y axes to be in the same direction as the principal directions of $\overline{D_{ij}}$ then $\overline{D_{ij}} = 0$ when $i \neq j$ and:

$$\overline{D_{kl}} D'_{kl} = \overline{D_{xx}} D'_{xx} + \overline{D_{yy}} D'_{yy} \quad (41)$$

The incompressibility and plane strain conditions further give:

$$\overline{D_{kl}} D'_{kl} = 2\overline{D_{xx}} D'_{xx} \quad (42)$$

and

$$\overline{D_{II}} = \overline{D_{xx}} \quad (43)$$

which in turn yields:

$$\begin{aligned} \tau'_{xx} &= 2\overline{\mu_{app}} \overline{M_s} D'_{xx} \\ \tau'_{yy} &= 2\overline{\mu_{app}} \overline{M_s} D'_{yy} \\ \tau'_{xy} &= 2\overline{\mu_{app}} D'_{xy} \end{aligned} \quad (44)$$

where

$$\overline{M_s} = \frac{1 + \overline{D_{II}}^2 / n D_{II}^{*2}}{1 + \overline{D_{II}}^2 / D_{II}^{*2}} \quad (45)$$

At this stage, we have transformed the non-linear and isotropic material into a linear and anisotropic one where the principal direction of anisotropy is at 45 degrees from our system of reference. The convention that normal viscosity is larger than shear viscosity explains this rotation. Using this linearization, the problem of a hole loaded under far field pure shear is decomposed in the sum of two intermediate problems:

1. One setup where no hole is present and far field shear loads are applied
2. Another setup where a hole is loaded internally by tractions $-t_w$. These tractions correspond to the opposite of the ones obtained along the virtual rim from problem 1. They insure that the sum of the setups 1 and 2 does not produce any traction along the rim.

Figure 16 Linear approximation of the original setup. Two sub problems 1 and 2 are addressed instead of the initial one. In problem 1 far field shear is applied at infinity on the isotropic Carreau material with no hole. In problem 2 the perturbation due to the hole is added. It occurs in the anisotropic medium defined by Eq.(44) and the applied traction $-t_w$ is introduced to correct for the traction created by sub problem 1 along the virtual emplacement of the rim.

Following this decomposition, the maximum closure velocity (point indicated by a cross on Figure 16), can be rewritten as:

$$v_{R\max} = \overline{v_{R\max}} + v'_{R\max} \quad (46)$$

Where $\overline{v_{R\max}} = \overline{DR}$. $v'_{R\max}$ is determined by solving problem 2 in the anisotropic body stemming from the linearization of Eq.(44). In this anisotropic body, a similar decomposition as the one presented above can be done and is presented in Figure 17.

Figure 17 Problem 2 is decomposed into the sum of two sub problems: one where far field shear is applied and a hole is present and one where the same far field load is applied but without the hole in the body. The material properties are the same in the three setups and only the boundary conditions are modified such that the two sub problems have known solutions.

The decomposition can be written as:

$$v'_{R\max} = v_{R\max}^{aniso} - \overline{\overline{RD_{II}}} \quad (47)$$

where $\overline{\overline{D_{II}}} = \overline{D_{II}} / \overline{M_s}$ from Eq.(44). Our focus is still the velocity in the point indicated by a cross on Figure 17. Fletcher (2009) derived the solution for an inviscid and incompressible inclusion in a linear incompressible anisotropic viscous body. Under shear, an inviscid and incompressible inclusion is equivalent to a hole and we use this solution to calculate $v_{R\max}^{aniso}$. Because the direction of anisotropy is at 45 degrees to our system of reference, we use an auxiliary coordinate system aligned with the direction of anisotropy and we use the expression for the shear deformation rate given by Fletcher (2009) in Eq.(21). In this new system, the angle of anisotropy is zero and the system is loaded in simple shear. Rotating back to the initial coordinate system transforms the shear deformation rate into the horizontal one and we can directly integrate it over the inclusion to get:

$$v_{R\max}^{aniso} = \left(1 + \frac{1}{\sqrt{\delta}}\right) \overline{\overline{RD_{II}}} \quad (48)$$

with the anisotropy factor $\delta = 1/\overline{M_s} \geq 1$. This leads to:

$$v'_{R\max} = \overline{\overline{RD_{II}}} \overline{M_s}^{-1/2} \quad (49)$$

The maximum closure velocity at the rim under pure shear is:

$$\frac{v_{R\max}}{RD_{II}} = 1 + \overline{M}_s^{-1/2} \quad (50)$$

where $\overline{M}_s$ is a function of the deformation rate ratio $\overline{D}_{II}/D_{II}^*$ and of the stress exponent n . For the power law end-member we have $D_{II}^* = 0$, so $\overline{M}_s$ reduces to $1/n$ and Eq.(50) to:

$$\frac{v_{R\max}}{RD_{II}} = 1 + \sqrt{n} \quad (51)$$

which only depends on the stress exponent n .

2.5.2 Solution analysis

We benchmark the expression for maximum closure velocity under far field shear presented in Eq.(50) using our numerical code. We present the results in Figure 18. The left plot shows the comparison over a wide range of normalized background deformation rates $\overline{D}_{II}/D_{II}^*$ for different stress exponents. This deformation rates ratio provides again a measure of the degree of non-linearity of the material: if a shear much smaller than $2\mu_0 D_{II}^*$ is applied at infinity, only the linear part of the constitutive law plays a role in the material response. If on the other hand a load much larger than $2\mu_0 D_{II}^*$ is applied, then the answer is mostly non-linear. We estimate the deformation rate limit from which effects due to non-linearity start to appear to be approximately $0.1D_{II}^*$ for all stress exponents. The misfit between the numerical solution and the analytical linearization is largest for the power law end member, which is investigated in the right plot of Figure 18. The misfit increases for increasing stress exponent but remains below or equal to 1% which proves the relevance of this approach. The resolution of the mesh used in the numerical code is high enough so that no more changes are observed if the mesh is refined. This confirms that the misfit is related to the linearization.

Figure 18 Comparison of the normalized closure velocities under pure shear for the isotropic Carreau fluid and for its anisotropic linearization. Left: comparison over a wide range of normalized far field loads for different stress exponents (Eq.(50)). Right: comparison for the power law end member as a function of stress exponent (Eq.(51)).

2.6 Approximate solution for combined loads

2.6.1 Apparent viscosities

In this section, we focus on the intermediate far field loads between the pressure and shear end members. The effective far field load $\bar{\sigma}$ is:

$$\bar{\sigma} = -\Delta p + \bar{\tau} \quad (52)$$

where Δp is the pressure difference and $\bar{\tau}$ is the pure shear load. We introduce the β coefficient to describe the magnitude of the shear compared to the one of the pressure load. We define it as: $\beta = |\bar{\tau}| / |\Delta p|$. The pressure end member corresponds

to $\beta = 0$ and the shear end member to $\beta \rightarrow \infty$. We define a characteristic deformation rate for the intermediate loads as:

$$D_{II_Char} = D_{II_p} + D_{II_s} \quad (53)$$

where D_{II_p} and D_{II_s} are, respectively, the maximum deformation rates for the pressure and shear end members along the rim. They are calculated thanks to the expressions established above. Using this characteristic deformation rate and the constitutive law Eq.(10), a characteristic viscosity μ_{Char} is computed.

In Figure 19, the distributions of normalized apparent viscosities around the hole are plotted for different combinations of stress exponents and β values. The viscosities are plotted using a load level $\Delta p / \tau_{II}^* = 10$ that ensures that some non-linear behavior is apparent. The viscosities are evaluated numerically using the finite element code and normalized by the characteristic viscosity μ_{Char} . The plots show the changes in the distributions with increasing shear for different stress exponents. The pressure end member ($\beta = 0$) gives the same results as the ones obtained previously, with distributions that depend only on the radial distance to the rim. For $n = 1.1$ the material is close to being linear which explains the uniform distribution for all the β coefficients. Applying a small shear at infinity, $\beta = 0.1$, breaks the symmetry and introduces an angular dependency. The viscosities along the rim are lower than the predicted one μ_{Char} showing that the behavior around the hole is more complicated than suggested by the linear approach used in Eq.(53). The distributions for $\beta = 0.4$ show a zone of high viscosities along the x axis, the direction of contraction, and a zone of low viscosities in the perpendicular direction. The zone of high viscosities is referred to as a stagnant point which is characterized by low deformation rates. It is again clear that the characteristic viscosity can overestimate the viscosities at the rim. The pattern for $\beta = 1$ is a bit different from the case $\beta = 0.4$ with the stagnant point from the x-axis getting closer to the rim and interacting with the stagnant point from the shear end member located at $\theta = \pi/4$. This gives rise to a pattern illustrating complex interactions. Finally, the case $\beta = 100$, approaches the shear end-member. The viscosity distributions are symmetric with respect to the line $\theta = \pi/4$ and a

stagnant point is located along this line at the rim. From this collection of plots, we can conclude that the intermediate loads between pressure and shear give rise to complex viscosity distributions with interactions occurring close to the rim. They cannot be reproduced based on any linear combination of the two end members. We can also conclude that the stress exponent increases the contrast of the viscosity distributions, even though the actual viscosity is in general within one order of magnitude of the proposed viscosity μ_{Char} .

Figure 19 Apparent viscosity distributions normalized by μ_{Char} in the vicinity of the hole for different stress exponents and β values. The log10 values of the normalized viscosities are indicated by the right-hand color bars. These figures have been obtained using $\Delta p/\tau_{II}^* = 10$.

Figure 19 focuses on the near cavity viscosity distributions and resolves the large variations occurring there. In Figure 20, we plot the apparent viscosities computed numerically along the far field outer boundary of the domain as a function of the load ratio β for the same stress exponents as in Figure 19. The viscosities have been computed for the same load level $\Delta p / \tau_{II}^* = 10$ but they are normalized by the material property μ_0 . This figure shows that the far field viscosity is equal to μ_0 for the pressure end member ($\beta = 0$) and it is decreased by several orders of magnitude for increasing shear. The larger the stress exponent, the smaller the viscosity. This figure illustrates the transition that occurs in the far field viscosity: from being a material property under pressure loads it becomes governed by stress under shear. The transition occurs around a load ratio β of 0.1.

Figure 20 Far field normalized apparent viscosities as a function of the load ratio β for different stress exponents. The load level $\Delta p / \tau_{II}^*$ is set to 10.

2.6.2 Closure velocity approximation

Although we have an exact analytical solution for the closure velocity under pressure and an approximate one under shear, a solution for general loads is missing. In a similar way as done above for deformation rates, we write that the maximum closure velocity at the rim v_R is:

$$v_R = v_{R_P}(\Delta p, \bar{\tau}) + v_{R_S}(\bar{\tau}) \quad (54)$$

where v_{R_P} and v_{R_S} are respectively the contributions from the pressure and shear end members. This linear approach ensures that the end members are reproduced correctly as well as the perfectly linear case ($n=1$). v_{R_P} does not only depend on the pressure load Δp but also on the shear load $\bar{\tau}$. It is based on the observation that for β values larger than 0.1, the far field shear $\bar{\tau}$ limits the viscosities at infinity (Figure 20). We use this apparent far field viscosity to replace the material viscosity limit μ_0 . Of the two other parameters characterizing a Carreau viscosity (Eq.(10)), the stress exponent remains unchanged but the transition rate D_{II}^* has to be evaluated using Eq.(12) and the new $A_L = 2\mu_0$ value. The shear contribution v_{R_S} in the proxy Eq.(54) is simply the solution from Eq.(50) evaluated for the load $\bar{\tau}$.

Figure 21 Maximum closure velocities computed numerically normalized by the analytical velocity defined by Eq.(54) as a function of β for different stress exponents. The results are presented in four plots, each one corresponding to a specific load level $\Delta p/\tau_{II}^*$.

In Figure 21, the maximum velocity at the rim is plotted normalized using the proxy from Eq.(54) for different load levels $\Delta p/\tau_{II}^*$ and stress exponents. The radial velocities are computed numerically all along the rim and the largest one is kept. Hence, the reported maximum velocity does not necessarily occur along the x axis. In all the plots, the end members are correctly reproduced except for the case $\Delta p/\tau_{II}^* = 100$ and $n = 5$. This is a numerical artifact: the domain size used to carry out the numerical simulations is too small to be able to reproduce the analytical solution derived for an infinite body for large stress exponents. All the values in Figure 21 are comprised between 1 and 1.5 showing that, in the worst case, there is less than 50% error between our prediction and the actual closure velocity. Moreover, the more linear the problem (small stress exponent or load level $\Delta p/\tau_{II}^*$), the better the prediction of the proxy Eq.(54). The almost linear case $n = 1.1$ reproduces almost exactly the solution for all β values so the discrepancies for $n = 3$ and 5 in Figure 21 are due to the complex hydrostatic-deviatoric splits that occur when using non-linear constitutive laws as illustrated by the viscosity distributions in Figure 19. The proxy given in Eq.(54) assumes that the fastest closing point is always along the x axis (Figure 11) but this may not be the case and it can explain the small discrepancies observed in Figure 21.

2.7 Discussion

2.7.1 Constitutive law

The creep of geomaterials and metals is frequently modelled using a power law model (Berest and Brouard, 1998; Kirby and Kronenberg, 1987; Mackerle, 2000) and it raises the question of the effect of considering a linear diffusion process in addition in the constitutive law. In Figure 22, we plot the closure velocities obtained using a Carreau viscosity as a function of the load level $\Delta p/\tau_{II}^*$ and of the load ratio β . The velocities

computed using a power law rheology are used as normalization to show the effect of having a two mechanisms constitutive law. This contour plot shows that the effect of diffusion creep is very important for low loads ($\Delta p / \tau_{II}^* = 0.01$) of the pressure end member ($\beta = 0.01$). Under these conditions, the closure velocity is five orders of magnitude higher than if only dislocation creep was considered. From this point, increasing either the load level $\Delta p / \tau_{II}^*$ which controls the degree of non-linearity (Figure 12) or the load ratio β reduces the speed up due to diffusion creep: the material behaves more non-linearly. For a pressure load equal to τ_{II}^* and no shear, a Carreau fluid still yields a closure speed that is ten times faster than if only power law creep was considered. It is therefore critical to use a two-deformation constitutive law if one wants to accurately predict the closure speed of a cavity and it is especially important to do so for small pressure loads.

Figure 22 Closure velocities normalized by the power law end member as a function of $\Delta p / \tau_{II}^*$ and β for $n = 3$. The $\log_{10}$ values of the normalization are plotted.

2.7.2 Shear enhancement

In many applications in geoscience, it is easier to evaluate the pressure load and to dismiss the potential far field shear stress by setting it to zero. The question arises of

how much the closure velocities are underestimated by not taking the shear into account.

Figure 23 Maximum rim velocities normalized by the pressure end member velocities plotted as a function of $\Delta p / \tau_H^*$ and β for $n = 3$. The $\log_{10}$ values of the normalization are plotted.

In Figure 23 we plot the maximum velocities at the rim normalized by the solutions for the pressure end member as a function of the load ratio β and load level $\Delta p / \tau_H^*$. The figure shows the additional closure due to shear for a stress exponent of 3. It appears that the more non-linear the material behavior (the larger $\Delta p / \tau_H^*$), the bigger the effect of the shear stress on the closure. In the case where the material is purely non-linear and the far field shear is ten times larger than the pressure load, the closure velocity is five orders of magnitude faster than if only pressure had been taken into account. A less dramatic configuration where the material still behaves almost non-linearly but where the shear is equal to the pressure load gives almost two orders of magnitude speed up. This shows that it is important to take shear into account when assessing long-term cavity closure and that not doing it can lead to orders of magnitude underestimation of the closure speed.

2.7.3 Importance of two deformation mechanisms

The proxy for the closure velocity presented in Eq.(54) is as general as possible for a two deformation mechanisms steady state law so it can be used for various applications. If the material behaviour can be described using a Carreau model, then the proxy from Eq.(54) makes an overprediction of maximum 50% for a stress exponent of 5 independently of the loading conditions. This mismatch is small compared to the orders of magnitude underestimations that occur if diffusion creep or shear are not taken into account (Figure 22 and Figure 23).

The main advantage of the Carreau model is its versatility to model steady state creep. Depending on the application considered, different diffusion mechanisms are dominant and have to be chosen. At high temperatures, the diffusion mechanism is either the Coble or Nabarro-Herring creep (Turcotte and Schubert, 2014) which describe the diffusion of atoms, respectively, either along grain boundaries or through the crystal. At low temperature, on the other hand, pressure solution is a potential diffusion mechanism. This mechanism involves the dissolution, transport and precipitation of minerals and is especially strong in salt at low stresses (Spiers et al., 1990; Urai and Spiers, 2007). Pressure solution creep is commonly encountered in formations like salt and ice and many applications like borehole closure, long-term cavity closure and wellbore sealing require the estimation of cavity closure in these materials. Applications concerned with Coble or Nabarro-Herring creeps require temperatures above half the melting temperature of the material. Shear enhanced compaction in partially molten rocks (McKenzie, 1984) or in high temperature sediments (Xiao and Evans, 2003) are examples of applications where our study can be used to give a two-dimensional estimate of the closure velocity of a single pore under general loads.

2.8 Conclusion

We revisit the problem of a circular hole in an infinite body loaded under general stresses. The constitutive law is incompressible non-linear viscous and a Carreau viscosity model is used to take into account both dislocation creep and a diffusion process. It is described by only three parameters: μ_0 the Newtonian viscosity, D_{II}^* the transition rate and n the stress exponent.

- Under pressure loads $\Delta p = \bar{p} - p_w$, we establish that the velocity at the rim v_R is found from:

$$\frac{\Delta p}{2\mu_0 D_{II}^*} = -\frac{1}{2} B\left(\frac{v_R^2}{RD_{II}^* + v_R^2}, \frac{1}{2}, -\frac{1}{2n}\right) \quad (55)$$

where B is the incomplete beta function and R is the hole radius.

- Under far field pure shear $\bar{\tau}$, the maximum closure velocity $v_{R\max}$ is:

$$\frac{v_{R\max}}{RD_{II}} = 1 + \bar{M}_s^{-1/2} \quad (56)$$

where $\bar{M}_s = \left(1 + \bar{D}_{II}^2 / nD_{II}^{*2}\right) / \left(1 + \bar{D}_{II}^2 / D_{II}^{*2}\right)$. $\bar{D}_{II}$ is the second invariant of the far field deformation rate.

- Based on the pressure and shear end member we propose a new approximation for the maximum closure velocity at the rim under general loads. Benchmarking this expression with a finite element numerical code shows that it is 50% off in the worst case.
- Several orders of magnitude underestimation can be done if either diffusion creep or shear enhancement is dismissed. The proposed solution therefore insures that this level of error is not reached even when diffusion creep and shear stresses occur.

Acknowledgements

We would like to thank the University of Oslo and more precisely the PGP group for their support. This work is part of the Tight Rocks research program funded by Statoil ASA at the University of Oslo.

2.9 Appendix

2.9.1 Appendix 1

Analytic solution for power law viscosity under pressure loads derived following the method used by Dabrowski et al. (2015).

Writing force balance when gravity and inertia terms are neglected and using the plane strain assumption, we have:

$$-\frac{\partial p}{\partial r} + \frac{\partial \tau_{rr}}{\partial r} + 2\frac{\tau_{rr}}{r} = 0 \quad (57)$$

We can now express $\frac{\partial \tau_{rr}}{\partial r}$ as:

$$\frac{\partial \tau_{rr}}{\partial r} = 2\frac{\partial \mu_{app}}{\partial r} D_{rr} + 2\mu_{app} \frac{\partial D_{rr}}{\partial r} \quad (58)$$

With:

$$\frac{\partial D_{rr}}{\partial r} = -\frac{2}{r} D_{rr} \quad (59)$$

So

$$\frac{\partial \tau_{rr}}{\partial r} = 2\frac{\partial \mu_{app}}{\partial r} D_{rr} - \frac{2}{r} \tau_{rr} \quad (60)$$

Now, the force balance becomes:

$$-\frac{\partial p}{\partial r} + 2\frac{\partial \mu_{app}}{\partial r} D_{rr} = 0 \quad (61)$$

For power law, μ_{app} is given by Eq.(8) and $\frac{\partial \mu_{app}}{\partial r}$ can be computed as:

$$\frac{\partial \mu_{app}}{\partial r} = \frac{A^{NL}}{2} (1-n) |\tau_{rr}|^{-n} \text{sgn}(\tau_{rr}) \frac{\partial \tau_{rr}}{\partial r} \quad (62)$$

Reusing the constitutive law Eq.(8) we get:

$$2\frac{\partial \mu_{app}}{\partial r} D_{rr} = (1-n) \frac{\partial \tau_{rr}}{\partial r} \quad (63)$$

which simplifies the force balance in:

$$-\frac{\partial p}{\partial r} + (1-n) \frac{\partial \tau_{rr}}{\partial r} = 0 \quad (64)$$

which is straightforwardly integrated using the boundary conditions at infinity:

$$p - \bar{p} = -(n-1) \tau_{rr} \quad (65)$$

From the definition of the stress we get:

$$\sigma_{rr} = -\bar{p} + n\tau_{rr} \quad (66)$$

Finally, using the boundary condition at the rim $\sigma_{rr} = -p_w$, we get that:

$$\tau_{rr}(R) = \frac{\Delta p}{n} \quad (67)$$

This is the radial deviatoric stress at the rim. The velocity at the rim, $r = R$, is then computed to be:

$$v_R = -\frac{1}{A^{NL}} \left(\frac{\Delta p}{n} \right)^n R \quad (68)$$

From which we deduce the velocity and stress distributions:

$$\begin{aligned} v_r &= -\frac{(\Delta p / n)^n}{A^{NL}} \frac{R^2}{r} \\ \tau_{rr} &= \frac{\Delta p}{n} \left(\frac{R}{r} \right)^{\frac{2}{n}} \\ \sigma_{rr} + \bar{p} &= \Delta p \left(\frac{R}{r} \right)^{\frac{2}{n}} \\ p - \bar{p} &= -\frac{n-1}{n} \Delta p \left(\frac{R}{r} \right)^{\frac{2}{n}} \end{aligned} \quad (69)$$

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Paper II: Long-term creep closure of salt cavities

Published in *International Journal of Rock Mechanics and Mining Sciences*, Is. **103**,
2018, pp. 96-106.

doi: <https://doi.org/10.1016/j.ijrmms.2018.01.025>

Paper III: Non-linear viscoelastic closure of salt cavities

Submitted to *Rock Mechanics and Rock Engineering*

5 Appendix A: PhD activities

5.1 Publications

5.1.1 Presented in this thesis

Cornet J.S., Dabrowski M., Schmid D. W., 2017, Long-term cavity closure in non-linear rocks, *Geophysical Journal International*.

Cornet J.S., Dabrowski M., Schmid D. W., 2017, Long-term creep closure of salt cavities, submitted to *International Journal of Rock Mechanics and Mining Sciences* and awaiting final decision for the revised manuscript.

Cornet J.S. and Dabrowski M., 2017, Non-linear viscoelastic closure of salt cavities, submitted to *Rock Mechanics and Rock Engineering*.

5.1.2 Other publications

Cornet J.S., Dabrowski M., Schmid D. W., 2016, Shear enhanced borehole closure, 2016 ARMA, American Rock Mechanics Association

5.2 Conference contributions

EGU, Vienna, 2015

Cornet J.S., Dabrowski M., Schmid D.W., Viscoelastic effective properties of two types of heterogeneous materials (Poster)

PhD day, Oslo, 2015

Cornet J.S., To flow or not to flow? (Poster)

Geology of geomechanics, London, 2015

Cornet J.S., Dabrowski M, Schmid D.W., Effective viscoelastic properties of shales (Poster)

EGU, Vienna, 2016

Cornet J.S., Dabrowski M, Schmid D.W., On the closure of a circular hole in a creeping medium (Poster)

EGU, Vienna, 2016

Cornet J.S., Dabrowski M, Schmid D.W., On the dependence of stress states on viscoelastic rheologies (Poster)

ARMA, Houston, 2016

Cornet J.S., Dabrowski M, Schmid D.W., Shear enhance hole closure (Talk)

EGU, Vienna, 2017

Cornet J.S., Dabrowski M, Schmid D.W., Long term creep closure of salt cavities (Poster)

EGU, Vienna, 2017

Cornet J.S., Dabrowski M, Schmid D.W., Effective viscoelastic properties of shales (Poster)

5.3 Courses

FYS-GEO9510	Introduction to Mechanical Geomodelling (10 Credits)
INF9650	Numerical Methods for Partial Differential Equations 2 (10 Credits)
INF9063	Programming Heterogeneous Multi-Core Architectures (10 Credits)
MNSES9100	Science, Ethics and Society (5 Credits)