Fast and robust common-reflection-surface parameter estimation

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ABSTRACT

The common-reflection-surface (CRS) method offers a stack with higher signal-to-noise ratio at the cost of a time-consuming semblance search to obtain the stacking parameters. We have developed a fast method for extracting the CRS parameters using local slope and curvature. We estimate the slope and curvature with the gradient structure tensor and quadratic structure tensor on stacked data. This is done under the assumption that a stacking velocity is already available. Our method was compared with an existing slope-based method, in which the slope is extracted from prestack data. An experiment on synthetic data shows that our method has increased robustness against noise compared with the existing method. When applied to two real data sets, our method achieves accuracy comparable with the pragmatic and full semblance searches. Our method has the advantage of being approximately two and four orders of magnitude faster than the semblance searches.

INTRODUCTION

The common-reflection-surface (CRS) method (Höcht et al., 1999; Mann et al., 1999) offers a stack with improved signal-to-noise ratio (S/N) compared to the traditional common-midpoint (CMP) method (Mayne, 1962). When computing the CRS stack, traces from nearby midpoints are stacked together with traces from a central midpoint. This requires three parameters for 2D and eight parameters for 3D. The parameters represent the first- and second-order derivatives of the traveltimes and can be related to the emergence angle of the normal ray, the curvature of the reflector at the measurement surface, and the normal moveout (NMO) velocity. Each location in the zero-offset

(ZO) stack has one set of CRS parameters. Obtaining these parameters is a challenge in terms of robustness and computation time.

The CRS parameters are commonly obtained by parameter searches to maximize the semblance measure (Neidell and Taner, 1971). One strategy is to apply a search for all three parameters simultaneously. This is often referred to as the full (global) search. If a velocity guide is available from a previous velocity analysis, it is possible to use the guide to restrict the search space. The search is then conducted over a wide range of values for the emergence angle and curvature, but it is limited to velocities close to the guide. This is computationally less expensive because there are fewer parameter combinations to be evaluated. In the pragmatic search, the three parameters are found sequentially (Mann et al., 1999; Jäger et al., 2001). The first step is to search for the velocity parameter in the CMP gathers. Also here, the search space can be limited if an interpreter-provided velocity guide is available. The resulting velocity is then used to construct the CMP stack. In the stacked section, a search for the emergence angle is conducted, followed by a search for the curvature. Garabito et al. (2001) suggest the hybrid search as an extension of the pragmatic search. In this approach, the emergence angle and velocity are searched for simultaneously from prestack data. Finally, the curvature parameter is obtained with a one-parametric semblance search. After the pragmatic or hybrid search, a global optimization search can be applied to refine the parameters. The search space is then restricted by the initial parameter estimates. If the initial estimates are good, the optimization search will find the global optimum.

In addition to the search strategies, there exist different heuristic methods for how the semblance searches can be conducted. Pragmatic and hybrid searches are commonly executed using brute-force searches. Here, all parameter combinations in a discretized search space are evaluated. This strategy can also be applied to the full global search. This is computationally expensive, but it is guaranteed to find the global maximum, given that the search space has a sufficiently high sampling rate. Other heuristics are faster than the

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brute-force approach, but they are not guaranteed to find the global optimum. The most popular are simulated annealing (Müller, 2003; Garabito et al., 2012; Minato et al., 2012) and differential evolution (Barros et al., 2015).

By default, the CRS method cannot handle conflicting dips satisfactory. Ideally, one set of CRS parameters should be obtained for each of the conflicting events. Coimbra et al. (2015) suggest using the derived CRS parameters to create a new stack from the initially created CRS stack. The amplitudes at each location in the initial stack are spread out along the CRS operator defined by the parameters at the given point and added to the new stack. Thus, the amplitude for a given point is the result of the constructive or destructive interference of its neighbors. This method gives a better S/N and produces a stack in which the conflicting events are better resolved. Mann (2001) and Müller (2009) propose a workflow for handling conflicting events with the pragmatic semblance search. In this method, multiple events are detected by selecting multiple semblance maxima when searching for the emergence angle. Then, one individual set of CRS parameters is obtained for each of the conflicting events. This results in multiple sections with CRS parameters. One stack is created from each of these sections and superimposed to construct the final stack.

Besides the mentioned semblance-based search strategies, it is possible to extract the CRS parameters without semblance searches. Santos et al. (2011) present a method for obtaining the CRS parameters using local slopes. The slopes are extracted for every location in the prestack data using plane-wave destructors (PWDs) (Claerbout, 1992; Schleicher et al., 2009). When the parameters are estimated, they can be used to obtain the CRS stack directly, or they can be used to guide a global optimization search. In Santos et al. (2011), this method was evaluated on two synthetic data sets with good results. The slope-based method was found to be orders of magnitude faster than the pragmatic search. In Hellman (2014) and Hellman and Boyer (2015), the concept of estimating CRS parameters using local slopes is reformulated as a linear regression problem. A hyperplane is fitted to the NMO-corrected data, and the CRS parameters are derived from the coefficients describing the fitted hyperplane. Local slopes have also been used to extract the velocity-related parameters for conventional CMP stacking with hyperbolic and nonhyperbolic moveout (Ottolini, 1983; Fomel, 2007; Stovas and Fomel, 2016).

In this paper, we build on the work of Santos et al. (2011) and propose a new method for extracting the CRS parameters, aiming for increased robustness against noise while retaining efficiency. Our method is based on extracting the emergence angle with the gradient structure tensor (GST) (Bigun and Granlund, 1987) and the midpoint-related curvature with the quadratic structure tensor (QST) (Bakker, 2002). We assume that the velocity parameter is available from previously conducted velocity analysis. We use the velocity guide to construct a CMP stack and estimate the remaining parameters from the stacked section. This is contrary to the method by Santos et al. (2011), where the slopes are estimated in prestack data. The hypothesis is that slope extraction on stacked data is more robust than slope extraction on prestack data because the stacked section has a higher S/N. Our method is compared against other relevant methods on synthetic and real data. This shows that our method has advantages in terms of computational cost and robustness.

THE CRS STACK AND PARAMETERS

The CRS stack can be regarded as an extension of the CMP stack. In the CMP stack, multiple traces covering the same source-receiver

midpoint, but obtained with different offsets, are stacked together to increase the S/N. The traces are time-corrected with the hyperbolic CMP traveltime formula:

$$t_{\text{CMP}}^2 = t_0^2 + \frac{4}{v_{\text{NMO}}^2} h^2, \quad (1)$$

where v_{NMO} is the NMO velocity, h is the half-offset, and t_0 is the ZO two-way traveltime. In the CRS method, this concept is extended. To obtain a stacked trace for a given midpoint, traces with midpoints close to the midpoint of interest are also included. This leads to an even higher S/N in the final stack. The CRS method was developed from a ray-tracing point of view and requires eight parameters for 3D data sets and three parameters for 2D data sets. These parameters are related to the curvatures of wavefronts from the normal incidence point and the normal wave (exploding reflector) experiments. This is commonly referred to as the model-space formulation of the parameters. It is also possible to regard the CRS method as a second-order Taylor approximation of the time-reflection surface in the recorded data. In this case, the parameters describe the events by first- and second-order derivatives of the traveltime. This is the data-space formulation of the parameters. Although the model space parameters are related to the physical model and can be useful as attributes, it is more convenient to use the data-space formulation when estimating the parameters. For this reason, we use the data-space formulation of the parameters in this paper. After the parameters are extracted, they can be converted to the model-space formulation if desired.

The 3D CRS traveltime is given by

$$t_{\text{CRS}}^2(\mathbf{x}, \mathbf{h}) = (t_0 + \mathbf{a}^T \Delta \mathbf{x})^2 + \Delta \mathbf{x}^T \mathbf{B} \Delta \mathbf{x} + \mathbf{h}^T \mathbf{C} \mathbf{h}, \quad (2)$$

where $\mathbf{h}$ is a vector representing the half-offsets in the x - and y -directions:

$$\mathbf{h} = \begin{bmatrix} h_x \\ h_y \end{bmatrix}, \quad (3)$$

and $\Delta \mathbf{x}$ is the distance vector from a central midpoint (x_0, y_0) to the nearby midpoints:

$$\Delta \mathbf{x} = \mathbf{x} - \mathbf{x}_0 = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}. \quad (4)$$

The elements in vector $\mathbf{a}$ are related to the emergence angle at the measurement surface and are expressed by the first-order derivatives:

$$\mathbf{a} = \left. \begin{bmatrix} \frac{\partial t}{\partial x} \\ \frac{\partial t}{\partial y} \end{bmatrix} \right|_{\mathbf{x}=\mathbf{x}_0, \mathbf{h}=0}. \quad (5)$$

$\mathbf{B}$ is a matrix related to the curvature of the event and is expressed by the time-scaled second-order derivatives:

$$\mathbf{B} = t_0 \left. \begin{bmatrix} \frac{\partial^2 t}{\partial x^2} & \frac{\partial^2 t}{\partial x \partial y} \\ \frac{\partial^2 t}{\partial x \partial y} & \frac{\partial^2 t}{\partial y^2} \end{bmatrix} \right|_{\mathbf{x}=\mathbf{x}_0, \mathbf{h}=0}, \quad (6)$$

and $\mathbf{C}$ is a matrix related to the moveout:

$$\mathbf{C} = t_0 \left[\begin{array}{cc} \frac{\partial^2 t}{\partial h_x^2} & \frac{\partial^2 t}{\partial h_x \partial h_y} \\ \frac{\partial^2 t}{\partial h_x \partial h_y} & \frac{\partial^2 t}{\partial h_y^2} \end{array} \right] \bigg|_{\mathbf{x}=\mathbf{x}_0, \mathbf{h}=0}, \quad (7)$$

where t is the two-way reflection time. In the case of a 2D data set, the y -dimension is omitted and then $\Delta \mathbf{x} = \Delta_x = x - x_0$ and $\mathbf{h} = h_x = h$. The CRS traveltimes then becomes

$$t_{\text{CRS}}^2(x, h) = [t_0 + A(x - x_0)]^2 + B(x - x_0)^2 + Ch^2, \quad (8)$$

with the parameters:

$$\begin{aligned} A &= \frac{\partial t}{\partial x} \bigg|_{x=x_0, h=0}, \\ B &= t_0 \frac{\partial^2 t}{\partial x^2} \bigg|_{x=x_0, h=0}, \\ C &= t_0 \frac{\partial^2 t}{\partial h^2} \bigg|_{x=x_0, h=0}. \end{aligned} \quad (9)$$

The CRS traveltimes (equation 8) is reduced to the CMP traveltimes (equation 1) if $x = x_0$. Consequently, C can be expressed by the NMO velocity:

$$C = \frac{4}{v_{\text{NMO}}^2}. \quad (10)$$

Each trace in the ZO stack is constructed by summing the traces covering all offsets and midpoints within an aperture centered around the central midpoint (see Figure 1). The traces are time corrected with the parameters obtained at the central midpoint. Because the CRS traveltimes are only accurate within a small region close to the central midpoint, the stacking aperture should not be too large (Facciopieri et al., 2016).

THE LOCAL SLOPES METHOD

The chosen reference method was proposed by Santos et al. (2011). It shows that the CRS parameters can be extracted using local slopes extracted from prestack data. The slopes are defined as the estimates (hat notation) of the derivatives of the reflection traveltimes:

$$p = \frac{\widehat{\partial t}}{\partial h}, \quad q = \frac{\widehat{\partial t}}{\partial x}, \quad (11)$$

In the following section, we will present how this method can be used to find A , B , and C . Using the notation from Santos et al. (2011); a , b , and c are the prestack estimates of the parameters and A , B , and C are the final estimates obtained for the ZO section.

In the reference paper, three different traveltimes are introduced. The traveltimes in common-offset (CO) sections is denoted by t , in the CMP sections by t_{CMP} , and in the ZO section by t_0 . The latter is also the traveltimes in the stacked section because the stack is an approximation of the ZO section. The prestack estimate for C is obtained from the slope in the CMP sections ($p(h, t_{\text{CMP}})$):

$$c(h, t_{\text{CMP}}) = t_{\text{CMP}} \frac{2p(h, t_{\text{CMP}})}{h}. \quad (12)$$

To move the $c(h, t_{\text{CMP}})$ estimates to ZO, a mapping between t_{CMP} and t_0 is established:

$$t_0 = \sqrt{t_{\text{CMP}}^2 - 2hp(h, t_{\text{CMP}})t_{\text{CMP}}}. \quad (13)$$

When this mapping is applied to c , the final C estimate is given as the weighted average over all $c(h, t_0)$ values with the same traveltime t_0 . The weights are the coherency value related to the slope estimate.

Santos et al. (2011) present two techniques for obtaining the remaining parameters A and B . The first technique uses the second-order derivative with respect to x . The other technique estimates B directly from q . We will concentrate on the latter because this is claimed to be better (Santos et al., 2011). This is referred to as technique 2 in the reference paper.

As for the C parameter, we need to establish a mapping between the traveltimes at an arbitrary point (x, t) to a central point in the ZO section (x_0, t_0) . The slope $q(x, t)$ needs to be transferred to the CMP section at $x = x_0$ first. A mapping from a midpoint x to a central midpoint x_0 is done using the relation

$$t_{\text{CMP}} = \sqrt{t^2 - t(x - x_0)[q(x, t) - q(x_0, t)]}. \quad (14)$$

After this, the mapping in equation 13 is used to transfer the estimate to (x_0, t_0) . Now that these mappings are established, we will see how the A and B parameters are derived.

The A parameter is extracted in two steps. First, an intermediate fourth parameter d is computed:

$$d(x_0, t) = tq(x_0, t). \quad (15)$$

Figure 1. The stacking CRS surface for a central midpoint. The aperture is centered around the midpoint. To construct the stacked ZO sample for this point, samples from all traces within this aperture are used.

This parameter is transferred to ZO using equations 13 and 14. Note that equation 14 reduces to $t_{\text{CMP}} = t$ for $x = x_0$. The a estimate is then given as

$$a(x_0, t_0) = \frac{d(x_0, t_0)}{t_0}, \quad (16)$$

where t_0 is given by equation 13.

The B parameter is obtained by considering the slope at points in the vicinity of the central point ($x \neq x_0, t$). We define an aperture around the central point to select the midpoints that should be included. An intermediate fifth parameter is calculated

$$e(x, t) = \frac{t[q(x, t) - q(x_0, t)]}{x - x_0}, \quad (17)$$

and transferred to (x_0, t_0) using equations 13 and 14. The b estimates are then given as

$$b(x_0, t_0) = e(x_0, t_0) - a(x_0, t_0)^2. \quad (18)$$

The final A and B parameters are the weighted average over all $a(x_0, t_0)$ and $b(x_0, t_0)$ with the same (x_0, t_0) coordinates. The weights are given by the coherence associated with the extracted slope.

The slopes used in Santos et al. (2011) were extracted using PWDs (Claerbout, 1992). There have been multiple improvements to the original PWD algorithm. In our work, we have used the same approach as used by Santos et al. (2011), which is a windowed implementation (Schleicher et al., 2009) of the algorithm proposed by Fomel (2002). Here, the slope at a given point (x, t) is computed as

$$q(x, t) = -\frac{\delta_t \sum_{(i,j) \in W} g_x(x_i, t_j) g_t(x_i, t_j)}{\delta_x \sum_{(i,j) \in W} g_t(x_i, t_j)^2}, \quad (19)$$

where g_x and g_t are the estimates of the local derivatives of the seismic image in the x - and t -directions, δ_t and δ_x are the sampling intervals, and W is a window centered around (x, t) . The slope in the offset direction (p) is obtained in the same manner, by replacing x with h and g_x with g_h in equation 19. The summations can be efficiently computed by convolving with the window function.

It should be noted that the reference method was developed to estimate A , B , and C whereas our method only estimates A and B . We assume that C is available from the velocity analysis. To have a fair comparison, only A and B were extracted using the method by Santos et al. (2011). When estimating A and B , instead of using the p slope estimated with PWDs, we use the slope that corresponds to the velocity guide. To convert the velocity guide to $p(h, t)$ values, the following steps must be done. First, the velocity is converted to $C(h = 0, t_0)$ using equation 10. An inverse NMO correction using the velocity guide is applied to obtain the prestack $c(h, t_{\text{CMP}})$ values. The slope can then be obtained by rearranging equation 12.

A schematic overview of the reference method as used in this paper is given in Figure 2a.

Applying PWD on a stacked section

One of our hypotheses is that by estimating slopes in the stacked section, the estimation becomes more robust against noise. To investigate this claim, we apply the method of Santos et al. (2011) directly on a stacked section. The slopes are extracted using PWD on the stacked section.

The formulation of the poststack PWD approach can be derived directly from the prestack formulation by setting $h = 0$. The first mapping (equation 13) is reduced to $t_{\text{CMP}} = t_0$. The d estimate (equation 15) is already obtained in the ZO section, and the A parameter can be found directly:

$$A(x_0, t_0) = \frac{d(x_0, t_0)}{t_0} = \frac{t_0 q(x_0, t_0)}{t_0} = q(x_0, t_0). \quad (20)$$

The B estimate is derived using the intermediate parameter e :

$$e(x, t_0) = \frac{t_0[q(x, t_0) - q(x_0, t_0)]}{x - x_0}. \quad (21)$$

The estimate is mapped to (x_0, t_0) using equation 14, assuming $t = t_0$. The b estimate is then given as

$$b(x_0, t_0) = e(x_0, t_0) - A(x_0, t_0)^2. \quad (22)$$

a) Method proposed by Santos et al. (2011)

Convert velocity guide to C
Extract slope (q) using PWD on pre-stack data
Obtain ZO estimates for A and B :
Compute temporary parameters d and e
Transfer d and e estimates to ZO section by mappings from CO to CMP and from CMP to ZO
Compute a and b , and average to get A and B

b) Poststack PWD - approach

Make CMP stack using velocity guide
Convert velocity guide to C
Extract slope (q) from stack using PWD
Compute A and B :
Compute A directly from q
Compute d and transfer to ZO
Compute b and average to obtain B

c) GST/QST - approach

Make CMP stack using velocity guide
Convert velocity guide to C
Extract A using GST
Extract B using QST

Figure 2. A schematic overview of the methods for fast parameter extraction investigated in this paper. To have a fair comparison, the methods are only used to find A and B , while C is derived from the interpreter-provided velocity guide.

The final B parameter is the weighted average of the b values with the same x_0, t_0 . The C parameter is derived from the velocity guide.

In the remainder of the paper, this approach will be referred to as the poststack PWD approach. A schematic overview is given in Figure 2b.

OUR METHOD

Our method is based on extracting local slope and curvature using the GST and QST. It is assumed that a velocity guide is available and the GST and QST are applied directly to the CMP stack. This method is referred to as the GST/QST approach in the remaining part of this paper. The extracted slopes (q_x and q_y) are estimates of the first-order derivatives:

$$q_x = \frac{\widehat{\partial t}}{\partial x}, \quad q_y = \frac{\widehat{\partial t}}{\partial y}, \quad (23)$$

and the extracted curvatures (κ_x , κ_y , and κ_{xy}) are estimates of the second-order derivatives:

$$\kappa_x = \frac{\widehat{\partial^2 t}}{\partial x^2}, \quad \kappa_y = \frac{\widehat{\partial^2 t}}{\partial y^2}, \quad \kappa_{xy} = \frac{\widehat{\partial^2 t}}{\partial x \partial y}. \quad (24)$$

Because the slopes and curvatures are estimated in the ZO section, they can be used directly in the estimate of $\mathbf{a}$ and $\mathbf{B}$. If the estimated parameters are not sufficiently accurate, a global optimizing search can be applied. A schematic overview of the GST/QST approach is provided in Figure 2c.

Obtaining C

Equation 10 gives an expression that relates the C parameter to the velocity guide (V_{NMO}). For the 3D case, the velocity is a symmetric 2×2 matrix, commonly called the NMO ellipse ($\mathbf{W}$) (Grechka and Tsvankin, 1998; Grechka et al., 1999). The elements of $\mathbf{W}$ are by definition equal to the elements of $\mathbf{C}$. We therefore use this as an initial value for $\mathbf{C}$:

$$\hat{\mathbf{C}} = \mathbf{W}. \quad (25)$$

Obtaining $\mathbf{A}$

After the CMP stack is constructed, the slopes are extracted using the GST (Bigun and Granlund, 1987). Consider a ZO section with three dimensions x , y , and t . The GST is computed using estimates of the gradients in the x -, y -, and t -directions, namely, g_x , g_y , and g_t . The gradient estimates are calculated using the central difference operator ($D_x = [-(1/2), 0, (1/2)]$) convolved with a Gaussian smoothing kernel for increased robustness. The kernel is defined as

$$\mathcal{N}(\mathbf{x}, N) = \exp \left[-\frac{1}{2} \left(\frac{3x}{N\delta_x} \right)^2 - \frac{1}{2} \left(\frac{3y}{N\delta_y} \right)^2 - \frac{1}{2} \left(\frac{3t}{N\delta_t} \right)^2 \right], \quad (26)$$

where δ_x , δ_y , and δ_t are the data-sampling rates and $\mathbf{x} = [x, y, t]$ is the space/time vector. The Gaussian function is constructed such

that three standard deviations of the Gaussian function fit inside a window with size $(2N + 1)^3$ samples. The gradient operator can be written as the difference of two kernels:

$$-\frac{1}{2} \mathcal{N}(\mathbf{x} + \frac{\mathbf{u}}{2}, N_{\text{grad}}) + \frac{1}{2} \mathcal{N}(\mathbf{x} - \frac{\mathbf{u}}{2}, N_{\text{grad}}), \quad (27)$$

where $\mathbf{u}$ defines the direction of the gradient (e.g., $\mathbf{u} = [1, 0, 0]$ for the x -direction). The term N_{grad} is the size of the smoothing function. By adjusting this parameter, we control how large patterns in the seismic data that the gradient estimate is sensitive to. This parameter should be set to approximately half the thickness of the typical reflection events (given in number of samples).

After the gradients are extracted, the GST is constructed for each location in the data:

$$\mathbf{T} = \begin{bmatrix} g_x^2 & g_x g_y & g_x g_t \\ g_y g_x & g_y^2 & g_y g_t \\ g_t g_x & g_t g_y & g_t^2 \end{bmatrix}. \quad (28)$$

This matrix holds information about the local structure, but the elements are still quite sensitive to noise. For increased robustness, the tensor elements are averaged in a local neighborhood using the Gaussian window function:

$$\bar{\mathbf{T}} = \mathbf{T} * \mathcal{N}(\mathbf{x}, N_{\text{smooth}}) = \begin{bmatrix} \overline{g_x^2} & \overline{g_x g_y} & \overline{g_x g_t} \\ \overline{g_y g_x} & \overline{g_y^2} & \overline{g_y g_t} \\ \overline{g_t g_x} & \overline{g_t g_y} & \overline{g_t^2} \end{bmatrix}, \quad (29)$$

where N_{smooth} is the size of the smoothing window used to smooth the tensor elements. The bar indicates that the tensor elements are averaged in the local neighborhood. The parameter N_{smooth} should typically be at least two times larger than N_{grad} .

When the tensor is computed, meaningful information about the structure at the given location is extracted by performing an eigenvalue decomposition. The eigenvalue decomposition can be efficiently implemented by applying the Cayley-Hamilton theorem (Smith, 1961) due to the symmetry in $\bar{\mathbf{T}}$. The eigenvector associated with the largest eigenvalue ($\mathbf{e}_1 = [e_x, e_y, e_t]$) is perpendicular to the locally dominant structure, and $\mathbf{a}$ can therefore be derived from this vector. Because we are considering a plane-like structure, there will exist two possible normal vectors to the plane, pointing in opposite directions. To obtain a consistent slope field, the vectors that have a negative t -component are multiplied with -1 . The $\mathbf{a}$ vector is estimated as

$$\hat{\mathbf{a}} = \begin{bmatrix} q_x \\ q_y \end{bmatrix} = \begin{bmatrix} \frac{e_x \delta_t}{e_t \delta_x} \\ \frac{e_y \delta_t}{e_t \delta_y} \end{bmatrix}. \quad (30)$$

In the case of $e_t \rightarrow 0$, the slope goes to infinity, which indicates a vertical (and unphysical) event. This could be handled by thresholding $\mathbf{a}$ to a chosen maximum value.

The PWD slope (equation 19) used in Santos et al. (2011) can be compared with the GST (equation 28). Considering the 2D case, we observe that the PWD slope is given as a fraction of some of the tensor elements:

$$q_x^{(\text{PWD})} = -\frac{\overline{g_x g_t} \delta_t}{g_t^2 \delta_y}. \quad (31)$$

Obtaining B

Now that the slopes are extracted, it is possible to extract the curvatures using the QST method (van de Weijer et al., 2001; Bakker, 2002). Note that the curvatures discussed here are related to the data-space formulation of CRS. These should not be confused with the wavefront curvatures used in the model-space formulation of CRS.

To obtain all elements in the **B** matrix, we need to extract four curvatures from the reflection surface. The curvatures with respect

Figure 3. The reflector-oriented coordinate system. The vector **u** is normal to the reflector, whereas **v** and **w** are orthogonal to **u** and oriented along the **x**- and **y**-axes when projected onto the **x**-**y**-plane. The illustration is reproduced from Bakker (2002).

Table 1. The parameters for the CRS parameter estimation methods used to generate the results in this paper. The same gradient and smoothing windows were used for the reference method, the poststack PWD approach, and the GST/QST approach. The initial estimates for *A* and *C* are denoted by $\hat{A}$ and $\hat{C}$.

Half-gradient window size	$N_{\text{grad}} = 2$ samples
Half-smoothing window size	$N_{\text{smooth}} = 15$ samples
Half-stacking aperture	Five midpoints
All semblance searches:	
Half-search aperture	Five midpoints
Half-window height	Five samples
Pragmatic search:	
<i>A</i> values	61 values $(\pm 30^\circ)^4$
<i>B</i> values	31 values $(\pm 10^{-6} \text{ s/m}^2)$
<i>C</i> values	Five values $(\hat{C} \pm 200 \text{ m/s})^5$
Small search for <i>A</i>:	
<i>A</i> values	11 values $(\hat{A} \pm 5^\circ)^4$
Full search	
<i>A</i> values	61 values $(\pm 30^\circ)^4$
<i>B</i> values	31 values $(\pm 10^{-6} \text{ s/m}^2)$
<i>C</i> values	Five values $(\hat{C} \pm 200 \text{ m/s})^5$

⁴The parameter range for $\hat{A}$ is converted from angle (model space) to slope (data space).

⁵Initial $\hat{C}$ estimates are converted to velocity before the perturbation is applied and then converted back to second-order derivatives.

to the *x*- and *y*-directions (κ_x and κ_y) give the diagonal elements in **B**. To obtain the nondiagonal elements of **B**, we also need the curvatures with respect to the axes in the 45° rotated coordinate system ($\kappa_{x'}$ and $\kappa_{y'}$). The QST method does not extract these curvatures directly, but the curvatures in the reflector oriented local coordinate system. This coordinate system has the coordinates (*u*, *v*, *w*) and is spanned by the vectors **u**, **v**, and **w** (see Figure 3). Here, **u** is the normal vector to the reflector surface and is given by

$$\mathbf{u} = [-q_i, -q_j, 1], \quad (32)$$

where q_i and q_j are the slopes obtained by the GST with units given in samples:

$$q_i = q_x \frac{\delta_x}{\delta_t}, \quad q_j = q_y \frac{\delta_y}{\delta_t}. \quad (33)$$

The extracted curvatures will be associated with **v** and **w**; $\partial u^2 / \partial^2 v$ and $\partial u^2 / \partial^2 w$. Therefore, we choose **v** and **w** to be oriented along the *x*- and *y*-axes, so that the curvatures can be transformed to $\partial^2 / \partial^2 x$ and $\partial^2 / \partial^2 y$ once they are extracted:

$$\mathbf{v} = [1, 0, q_i], \quad \mathbf{w} = [0, 1, q_j]. \quad (34)$$

For each location in the stack, the curvatures are obtained by considering the derivatives in a local neighborhood around this point ($u_0 = v_0 = w_0 = 0$):

$$\kappa_v = \frac{-v g_u g_v}{v^2 g_u^2}, \quad \kappa_w = \frac{-w g_u g_w}{w^2 g_w^2}, \quad (35)$$

where *v* and *w* are the coordinates in the local reflector-oriented coordinate system. The gradient estimates (g_u, g_v, g_w) are computed with the same gradient operator as used for the GST (equation 27), but they are oriented along the axes in the reflector-oriented coordinate system. We use the same smoothing function as used for the GST.

After the curvatures in the reflection-oriented coordinate system are found, they are corrected for the rotation and expressed with the proper dimensions:

$$\kappa_x = \kappa_v (1 + q_i^2)^{\frac{3}{2}} \frac{\delta_t}{\delta_x^2}, \quad \kappa_y = \kappa_w (1 + q_j^2)^{\frac{3}{2}} \frac{\delta_t}{\delta_y^2}. \quad (36)$$

The extraction of $\kappa_{x'}$ and $\kappa_{y'}$ is done in the same manner, but by replacing **v** and **u** with **o** and **p**, which spans the rotated coordinate system (Figure 3):

$$\mathbf{o} = \frac{1}{\sqrt{2}} [1, 1, q_j + q_i], \quad \mathbf{p} = \frac{1}{\sqrt{2}} [-1, 1, q_j - q_i]. \quad (37)$$

The mixed derivative is then given as the difference between the curvatures in the rotated coordinate system:

$$\kappa_{xy} = \frac{\kappa_{y'} - \kappa_{x'}}{2}. \quad (38)$$

Finally, we obtain the estimate for $\mathbf{B}$ as the time-scaled curvatures:

$$\hat{\mathbf{B}} = t_0 \begin{bmatrix} \kappa_x & \kappa_{xy} \\ \kappa_{xy} & \kappa_y \end{bmatrix}. \quad (39)$$

There is one problem when computing the curvature using QST. Because we are considering the reflector-oriented coordinate system, the orientation of this coordinate system is dependent on the slope, which is different for different locations in the stack. This means that equation 35 cannot be computed for the full stack using global convolutions, but it has to be computed locally. In Bakker (2002), it is shown how this can be avoided using a linear combination of 52 spatially invariant convolutions.

One of the advantages of the GST/QST approach is the efficient computation of gradients and smoothing operations using convolutions. Because the filter kernels are separable, each 3D convolution is implemented using three 1D convolutions, which is significantly faster than 3D convolutions.

Figure 4. The setup for the experiment on synthetic data. (b) The data were constructed in NORSAR-2D using (a) a simple model. The error in A was obtained along the dipping reflector (the second from the top) and B at the apex of the curved reflector. In (c-f), Gaussian noise is added to the prestack data before stacking. The images show the stacked section indicated by the rectangle in (b).

RESULTS AND DISCUSSION

In this section, we compare our method (GST/QST approach) with the reference method by Santos et al. (2011) and the poststack PWD approach. As a reference for the semblance-based methods, we used the pragmatic search and the full search. The pragmatic search is the fastest semblance-based method, and the full search is the most accurate. We use the brute-force strategy for both searches. The full search is however often implemented using other heuristics, which is significantly faster than the brute-force approach. To make a fair comparison, we use the velocity guide to constrain the searches for C in the pragmatic and full searches.

In the experiments on the second real data set, we study how the GST/QST-method performs in the case of conflicting events. In this experiment, we use the pragmatic search modified to handle conflicting dips. Details on how the pragmatic semblance search is implemented can be found in Mann (2001) and Müller (2009). In our implementation of the pragmatic search, we used the CMP stack instead of the path-summation step because the data set has strong coherent noise. The GST/QST approach smooths the parameter estimates which can be problematic in regions with rapidly varying structures. To improve the parameters extracted with the GST/QST, we applied a small semblance search in the CMP stack to refine the initial A estimate. This is similar to how the search for A in the pragmatic workflow is conducted. This was only done for the second data set. We also used the spreading approach suggested by Coimbra et al. (2015) for the experiments on the second data set.

The search ranges and window/aperture sizes used in to generate the results are summarized in Table 1. The same gradient estimates (equation 27) and Gaussian smoothing window (equation 26) are used for all slope-based methods. An increasing aperture with depth is often used when estimating the CRS parameters. This is more robust against noise, but it gives less detail. For the GST/QST method, this could be implemented by dividing the stacked section into smaller regions and applying an increasingly larger smoothing

Figure 5. Error in the parameter estimation as a function of noise levels.

Figure 6. A comparison of CRS stacks is obtained with different methods for the Tacutu data set. The aperture size used for the CRS stacks is ± 5 midpoints (± 137.5 m) around the central midpoint. The red square indicates the smaller area studied in Figure 7. The same prestack data were used for the CMP and CRS stacks, and no postprocessing was applied to the stacks. Normally, poststack noise filtering is applied to the final stack CMP stack before presentation. Therefore, the stacks should not be used to compare the quality of the CMP method versus the CRS method.

window. Because the effect of an increasing aperture is the same for all methods, and the focus of this paper is on the differences between methods, we chose to use a fixed aperture/smoothing window for all methods.

Experiment on synthetic data

To investigate how the different methods perform in noisy conditions, they are applied to a simple 2D synthetic data set. The synthetic data set was generated using the dynamic ray-tracing technique using a simple model (Figure 4a). Edge diffractions are not modeled.

Figure 4b shows the nearest offset section of a total of 23 CO sections. Random Gaussian noise with increasing power (Figure 4c–4f) was added to the synthetic data. The experiments were repeated 20 times with different noise realizations. A smoothed velocity guide created from the interval velocities was used to obtain the C parameter for all methods. The true values of A and B were found by manually picking the horizons and curve fitting a polynomial to the picked points. From this polynomial, we derived the analytical derivatives. We have evaluated the error in A on the dipping reflector and in B on the curved reflector. The presented error is the mean relative error along the given horizon.

Figure 7. A small section of the stacks in Figure 6 (the red square) and the corresponding parameter panels.

Figure 8. A comparison of CRS stacks obtained with the GST/QST approach and the pragmatic search modified for handling conflicting dips. The red square indicates the smaller area studied in Figure 9. The aperture size is set to ± 5 midpoints (± 62.5 m) around the central midpoint. No post-processing was used on the stacks, except muting in the regions above the seawater. All stacks were produced with the spreading approach in Coimbra et al. (2015).

Figure 9. A magnification of the red square in Figure 8. The CMP stack is included for reference. All stacks were produced with the spreading approach in Coimbra et al. (2015).

The results for the estimation of A (Figure 5a) show that the GST/QST approach is more robust against noise than the method by Santos et al. (2011) for low to moderate noise levels. An important difference between the PWD and the GST slope estimate is that the PWD-slope goes to zero when there is no dominant direction, whereas the GST-slope gives a random estimate. This explains why the error for the GST method rises quickly when the noise level is high. This does not mean that the PWD method is better. In regions where the structure is chaotic, the PWD slope goes to zero, which makes the CRS method smooth horizontally. This can generate horizontal structures in the stack where, in reality, there is no coherent structure. Therefore, the random estimate from the GST/QST approach is preferable. Neither of the methods gives a completely accurate estimate, even for the low-noise case. This is because the accuracy of the gradient estimates used in PWD and GST is limited by the sampling resolution.

The results for the estimation of B (Figure 5b) show similar tendencies to those for the estimation of A . The GST/QST approach performs better than the method by Santos et al. (2011) and the PWD poststack approach. In general, the B estimates are more sensitive to noise than the A estimates independent of the choice of method. Because the B estimates are dependent on the A estimates, any error in A will also affect the accuracy in B . It is not possible to obtain a good B estimate when the noise levels are high using the slope-based methods. This is where the semblance-based methods are significantly better. However, the effect of B on the CRS stack is also much smaller than the effect of A (Faccipieri et al., 2016). Note that the error in B estimates for all slope-based methods converges toward 100%. Although the slope-based methods put the curvature to zero when the noise levels are high, the semblance searches often set the B -value to a (high) random value. When the B estimate is very high, we get artifacts. This is discussed in the next section.

Experiment on real data

In this section, the various approaches are applied to real data sets to further investigate the differences between the methods. The results were obtained using the parameters in Table 1.

The Tacutu data set is a 2D land data set from the Tacutu Basin (line 50-RL-90). It contains a considerable amount of noise. The results show that the method by Santos et al. (2011) fails to detect the events in the noisy regions (the blue square in Figure 6). The poststack PWD approach is considerably better here. This shows that slope-based poststack estimation of A and B is more robust than prestack estimation. The difference between the poststack PWD approach and the GST/QST approach can be seen in the magnification of the stacks (Figure 7). The GST/QST approach captures the highly dipping structures significantly better than the two other slope-based methods. This confirms the results from the synthetic experiment: that the GST slopes are more robust against noise than the PWD slopes. The parameter panels displaying A show that the parameters produced using the GST/QST method are closer to those produced by the full search than those produced by the PWD.

When compared with the semblance searches, the GST/QST performs almost equally good. The largest difference is in the area where there are highly dipping structures. These structures are almost invisible in the CMP stack, but an interpretation of the horizons (Garabito et al., 2005) shows that there should be highly dipping events in this area. To obtain a CRS stack that is consistent with the interpretation, a manual approach where guides are con-

structed for all parameters (such as that proposed by Faccipieri, 2016) must be used. Still, the semblance searches recover more of the dipping events than the GST/QST method does. The CRS method has a fundamental problem in regions where there is no underlying structure. What should the stacking parameters be in these regions? The pragmatic semblance search often gives a noisy parameter estimate in such regions. This can give noisy artifacts in the final stack, sometimes referred to as “CRS worms.” The same effect is present, but less distinct, for the full search. The results show that the GST/QST method does not produce such artifacts. This is, as discussed before, because the GST/QST method puts the B estimate to zero in regions where there is no coherent structure and that the A parameter is relatively smooth.

The second data set is a marine 2D line from the Jequitinhonha Basin in Minas Gerais, Brazil. This data set is used to investigate how the GST/QST approach performs on data sets with complex structures (Figure 8). As a reference, we used the full search and the pragmatic semblance search modified to handle conflicting events. In regions without conflicting events, the results of the GST/QST approach are very similar to those obtained with the semblance searches. In regions with conflicting events or rapid changes (Figure 9), the difference is larger. There are two properties of the tensor-based methods that can explain why. There is a limitation to tensor-based representation of seismic signals because it can only represent signals with one dominant direction. Therefore, only parameters for one of the conflicting events can be obtained. In addition, the

Figure 10. (a) The computation time for the different steps in the GST/QST approach compared with the CRS stacking operation. (b) Comparison of the relative computation time for the different methods compared with the GST/QST approach. The results were obtained when processing the Tacutu data set (2D) using a laptop with a 2.9 GHz Intel Core i5 (2015) CPU. The calculation time of the semblance value for one set of parameters was 3.1×10^{-5} s per sample for the pragmatic search, and the stack consists of 1605×1024 samples. Parameter intervals used in the searches can be found in Table 1. We used a fixed aperture of 11 cdfs for all methods. The total running time for the semblance searches is dependent on the number of parameters pairs that are evaluated. *The running time for the full search is the approximated running time using very fast simulated annealing heuristic. We assumed 300 semblance evaluations for each parameter pair, which is consistent with the results in Garabito et al. (2012).

Figure 11. The GST/QST approach was applied to a 3D land data set with a maximum fold of 168. A CMP stack is provided to show the input data to the GST/QST method. The figure shows an inline and a crossline section with time slices between 0 and 2 s. The same prestack data were used for the CMP and CRS stacks, and no postprocessing was applied to the stacks. Normally, poststack noise filtering is applied to the final CMP stack before presentation. Therefore, the stacks should not be used to compare the quality of the CMP method versus the CRS method.

window function used in the GST/QST approach smooths the parameter estimates, which makes it harder to get good estimates in regions with rapidly changing events. When a small perturbation of A is applied, the smoothing effect in regions with rapidly changing structures becomes less visible. The results now are very close to those obtained with the full search and the modified pragmatic search, but still some conflicting dips are not perfectly resolved.

Computation time

The main motivation for using the GST/QST approach over a semblance search is the decreased computation time. To evaluate the efficiency of the different methods, they were applied and timed on the Tacutu data set. Figure 10 offers a breakdown of the computation time for each of the different steps in the GST/QST method. For comparison, we included the running time for constructing the CRS stack. We see that extracting the parameters is much faster than constructing the CRS stack. This shows that the GST/QST

approach is very efficient. The largest contribution to our method, in terms of running time, comes from extracting B .

When comparing the total running times for the different methods, we observe that the post-stack PWD approach is very comparable with the GST/QST approach. The method by Santos et al. (2011) is considerably slower due to the need for extracting slopes and curvatures in the prestack data. In our example, the pragmatic search needs to evaluate 92 different parameter combinations (see Table 1) and the running time grows linearly with the number of parameter combinations. The GST/QST method is more than 10^2 times faster than the pragmatic search when using this setup. The full search is 10^4 times slower than the GST/QST method. Our method with the small perturbation of A is still cheaper than applying the pragmatic search. However, because the parameters from the pragmatic search and our method may be subject to a global optimization search, this perturbation may not be needed. If the global search is applied, the difference in total running time between our method and the pragmatic search will be smaller. Because the estimation using the GST/QST is slightly less accurate, the optimization search may be more often used with our method than with the pragmatic search. Regardless of this, because the estimation of CRS parameters using the GST/QST approach is less computationally demanding than computing the CMP stack, it can be very useful as a quick test of how a CRS stack will look like. When comparing this initial CRS stack with the CMP stack, it is possible to get an impression of whether a semblance search is needed to get a satisfying CRS stack.

EXAMPLE ON 3D DATA SET

In Figure 11, the GST/QST approach was applied to a small 3D data set. The data set covers an area of 1.5×1.2 km and consists of, respectively, 81 inline and 103 crossline slices with a sampling interval of 30 m. The geology in this area is dominated by horizontally layered structures. The computation time of estimating $\mathbf{a}$ and $\mathbf{B}$ was 3 min on a laptop (2.9 GHz Intel Core i5 [2015] CPU). The computation of the NMO stack lasted for 1 min and the CRS stack for 10 min. The input stack has a high level of noise. In the resulting CRS stack, the events are preserved and the noise is significantly attenuated.

CONCLUSION

In this study, we proposed a new method for fast extraction of CRS attributes. The basic assumption is that an interpreter-provided velocity guide is available to obtain the moveout-related parameter matrix $\mathbf{C}$. The angle- and curvature-related CRS-parameter vector/matrix $\mathbf{a}$ and $\mathbf{B}$ are estimated directly from the CMP stack. This is done using the GST and QST to avoid time-consuming semblance

searches. The method is efficiently implemented using convolutions and vectorized eigenvalue decomposition.

Our method was compared with the reference method on a synthetic data set. The experiment showed that the GST/QST approach is more robust than the reference method without being slower. To investigate the effect of stacking the data before the slope-based parameter estimation, we reformulated the reference method to be used on the stacked section (poststack PWD approach). The results showed that stacking is an effective way of increasing the robustness against noise for the slope-based methods.

Finally, the methods were applied to two 2D data sets. This showed that the GST/QST approach was significantly better than the reference method. The stacks produced by the GST/QST approach recovered structures buried in high noise levels that were not recovered with the poststack PWD approach. The difference between the semblance searches and the GST/QST method was very small unless the noise levels were very high. In these cases, semblance searches were better. The GST/QST method has the advantage of not producing artifacts that are commonly observed when using semblance searches. In regions with rapidly changing structures, the GST/QST method smoothes the parameter estimates and a small optimization of the parameters is needed. In regions with conflicting dips, it is not possible to obtain an estimate for both conflicting events with our method. Further work is needed to detect these scenarios and develop a strategy to handle multiple dipping events.

The total running time for the GST/QST approach was less than 1 min for a 2D data set on a laptop (2.9 GHz Intel Core i5 [2015] CPU) compared with more than 1 h for the pragmatic search. Our method can be especially useful to produce a quick CRS stack. This can be used to decide whether a CRS stack will give good results and assist the user to determine if a semblance search is needed.

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