On the dispersion relation and spectral properties of surface gravity waves

Tore Magnus Arnesen Taklo

Thesis submitted for the degree of Ph.D.

Department of Mathematics

University of Oslo

June 2016

© **Tore Magnus Arnesen Taklo, 2016**

*Series of dissertations submitted to the
Faculty of Mathematics and Natural Sciences, University of Oslo
No. 1765*

ISSN 1501-7710

All rights reserved. No part of this publication may be
reproduced or transmitted, in any form or by any means, without permission.

Cover: Hanne Baadsgaard Utigard.
Print production: Reprosentralen, University of Oslo.

Preface

The thesis is submitted in partial fulfillment of the degree of *philosophiae doctor* (Ph.D.) at the University of Oslo. The work has been performed from August 2012 to June 2016. The work is part of the research project *Inversion of radar remote sensing images and deterministic prediction of ocean waves* funded by the research Council of Norway and the University of Oslo.

All the work presented in the thesis has been carried out in collaboration with my supervisors Professor Karsten Trulsen, Professor emeritus Harald Elias Krogstad, associate Professor José Carlos Nieto Borge and Professor Atle Jensen.

Oslo, June 2016

Tore Magnus Arnesen Taklo

Acknowledgments

First I would like to express my gratitude to my principal supervisor Professor Karsten Trulsen. His patience, support, broad knowledge on nonlinear waves and scientific experience inspired me throughout the work.

I would like to express my gratitude to my supervisor associate Professor José Carlos Nieto Borge. His ideas, creativity and broad knowledge on spectral analysis and radar images has been of great value. I would also like to thank him for his hospitality and Spanish sense of humour during a three month exchange in Alcalá, Spain in the spring of 2015. I would like to express my gratitude to my supervisor Professor emeritus Harald Elias Krogstad. His scientific experience and broad knowledge on spectral analysis and stochastic theory has been of invaluable importance for the work. Furthermore, I would like to express my gratitude to my supervisor in the laboratory, Professor Atle Jensen. His broad experience on experiments has inspired me in the laboratory work.

I would like to thank the staff at the Department administration for support and help with applications and computer problems. Thanks also to my scientific colleagues at the Mathematics Institute at the University of Oslo and at the University of Alcalá, Spain. In particular I would like to thank my Ph.D. colleagues, Abushet Simaneseuw and Susanne Støle-Hentschel, for the memorable stay in Alcalá in the spring of 2015.

I am grateful to my family, friends and my love and best friend Marianne who have cheered for me.

Contents

Preface	iii
Acknowledgments	v
Introduction	1
Papers	15
Taklo, T. M. A. & Trulsen, K. & Gramstad, O. & Krogstad, H. E. & Jensen, A. 2015 Measurement of the dispersion relation for random surface gravity waves. <i>J. Fluid Mech.</i> 766 , 326–336	19
Taklo, T. M. A. & Trulsen, K. & Krogstad, H. E. & Nieto Borge, J. C. 2016 On disperion of directional surface gravity waves. <i>Submitted to J. Fluid Mech.</i> . . .	33
Taklo, T. M. A. & Nieto Borge, J. C. & Krogstad, H. E. & Trulsen, K. 2016 Cross-spectral evolution of unidirectional surface gravity waves.	53

Introduction

Background

Describing ocean waves mathematically and understanding their physics is significant for the safety and development of maritime industries and environments.

Linear wave theory is a cornerstone in the mathematical description of ocean waves and well-known applications include the response of ocean structures and ships to waves, see e.g. Goda (2000) and Tucker & Pitt (2001) and the references therein. The theory assumes that the ocean surface can be modeled as a random linear superposition of non-interacting regular free waves of different wave lengths λ and directions.

Nevertheless, to be able to improve the understanding of the physics of ocean waves, nonlinearity must be introduced. For waves propagating on deep water where gravity is the restoring mechanism we distinguish two types of nonlinearities; non-resonant and resonant wave interactions.

In the following a brief theoretical background about linear and nonlinear surface gravity waves on infinitely deep water is given with particular emphasis on the results of the papers in this thesis.

The governing equations for surface gravity waves on an inviscid, incompressible fluid of infinite depth is given by

$$\nabla^2 \phi = 0, \quad -\infty < z < \eta, \quad (1a)$$

$$\phi_t + \frac{1}{2}(\nabla \phi)^2 + g\eta = 0, \quad z = \eta, \quad (1b)$$

$$\eta_t + \nabla \phi \cdot \nabla \eta - \phi_z = 0, \quad z = \eta, \quad (1c)$$

$$\phi_z = 0, \quad z \rightarrow -\infty, \quad (1d)$$

where $\phi(\mathbf{x}, z, t)$ is the velocity potential in the fluid domain, $\eta(\mathbf{x}, t)$ is the free surface, $\mathbf{x} = (x, y)$ is the horizontal position vector, z is the vertical coordinate, t is time, g is the acceleration due to gravity, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ is the gradient vector and the subscripts denote partial derivatives.

The system of equations (1) is complicated since the boundary conditions, (1b) and (1c), are evaluated at the free surface, which itself is an unknown in the equations. Consequently, it is convenient and for practical purposes necessary to simplify (1). Simplifications typically rely on certain assumptions about the specific problem under investigation. In particular, for many applications it is natural to assume that the waves are weakly nonlinear. This implies that the

wave steepness $\epsilon = ka \ll 1$ where $k = 2\pi/\lambda$ is the wavenumber, λ is the wave length and a is the wave amplitude. All the papers in this thesis deals with weakly nonlinear waves.

Under the assumption of weak nonlinearity, the leading order approximation of (1) is obtained by neglecting all nonlinear terms, in which case it simplifies to

$$\nabla^2 \phi = 0, \quad -\infty < z < 0, \quad (2a)$$

$$\phi_t + g\eta = 0, \quad z = 0, \quad (2b)$$

$$\eta_t - \phi_z = 0, \quad z = 0, \quad (2c)$$

$$\phi_z = 0, \quad z \rightarrow -\infty. \quad (2d)$$

The equations (2a), (2c) and (2d) have the simple periodic solutions

$$\eta(\mathbf{x}, t) = \frac{a}{2} e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} + *, \quad \phi(\mathbf{x}, z, t) = \frac{-ia\omega}{2k} e^{kz} e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)} + *, \quad (3)$$

of a monochromatic free wave where $\mathbf{k}$ is the wave vector. The angular frequency $\omega = 2\pi/T$ where T is the wave period, $i = \sqrt{-1}$ is the imaginary unit and $*$ denotes complex conjugate. From superposition of components in the form (3), with random amplitudes and phases, a random linear wave field can be constructed. With an additional assumption about a Gaussian spectrum the theory characterizes the stochastic properties of linear waves.

By substituting the solutions (3) into the boundary condition (2b) the linear dispersion relation

$$\omega(\mathbf{k}) = \sqrt{g|\mathbf{k}|} \quad (4)$$

is obtained. Here, $|\mathbf{k}| = k$ is the wavenumber. By substituting $\omega = ck$ into (4) it can be shown that the phase speed

$$c = \sqrt{\frac{g\lambda}{2\pi}} \quad (5)$$

i.e. the phase speed of waves with long wave length is higher than for waves with shorter wave length. This type of dispersion is a fundamental property of surface gravity waves on deep water.

Higher-order approximations of the equations (1) can be obtained through perturbation expansion of ϕ and η in terms of small wave steepness. The nonlinear solution corresponding to the monochromatic free wave (3) is the well-known Stokes wave expansion (Stokes, 1847), which in addition to the leading order terms (3) contains nonlinear corrections in terms of bound harmonic waves appearing through non-resonant nonlinear interactions.

In contrast to a random linear wave field, where all wave components are independent, one cannot directly construct a superposition of Stokes wave components since the different wave components will undergo mutual nonlinear interactions. However, to second order in steepness the Stokes wave expansion may be generalized to apply to a sum of wave components, and expressions for all mutual interactions between wave components can be found (Longuet-Higgins, 1963). This is possible since to second order the nonlinear interactions do not affect the evolution of the free waves.

To third order in wave steepness nonlinear interactions not only generate bound waves, but also resonant and quasi-resonant interactions, first described mathematically in the pioneering work of Phillips (1960). As a result of the resonant interactions, transfer of energy between free waves occurs while the waves evolve in time and space. The nonlinear evolution may be described by model equations which represent approximations to, and can be derived from, the equations (1). For waves on deep water, two very important model equations are the Zakharov equation and the nonlinear Schrödinger equation, both first derived by Zakharov (1968) for waves on infinitely deep water. These model equations have played important roles in the understanding of weakly nonlinear waves, and both models are important parts of the work presented in this thesis.

While the Zakharov equation is valid for wave fields with energy distributed over a wide range of wave lengths and directions the nonlinear Schrödinger equation is based on the assumption of narrow bandwidth i.e. that the energy in the wave field is distributed within a narrow band or spectral radius around a characteristic wave vector $\mathbf{k}_c$. Here, we define narrow bandwidth as $\Delta k/k_c \ll 1$ where the bandwidth Δk is a measure of the width of the wave spectrum around the characteristic wavenumber k_c .

Under the narrow bandwidth assumption the surface elevation $\eta(\mathbf{x}, t)$ may be written

$$\eta = \bar{\eta} + \frac{1}{2} \left(B e^{i\chi} + B_2 e^{2i\chi} + B_3 e^{3i\chi} + \dots + \text{c.c.} \right) \quad (6)$$

where c.c. denotes complex conjugate. Under this assumption, $\bar{\eta}$, B , B_2 and B_3 are slowly varying functions of $\mathbf{x}$ and t relative to the more rapidly varying phase function $\chi = (\mathbf{k}_c \cdot \mathbf{x} - \omega_c t)$ where $\omega_c = \omega(\mathbf{k}_c)$ is the characteristic frequency. By assuming the steepness and bandwidth to be small parameters one can from (1) derive equations describing the nonlinear evolution of the complex amplitude B . These types of equations are known as nonlinear Schrödinger (NLS) equations for surface gravity waves. The functions $\bar{\eta}$, B_2 and B_3 represent bound waves and can be expressed in terms of B , see Toffoli *et al.* (2010) equation (3.4).

The first derivation of an NLS equation for surface gravity waves on infinite depth was given by Zakharov (1968) as a narrow band limit of the Zakharov equation. The NLS equation is valid to third order in steepness and bandwidth. Dysthe (1979) extended the NLS equation to fourth order. The fourth order NLS equation is often referred to as the modified nonlinear Schrödinger (MNLS) equation or Dysthe equation. The MNLS equation describes more physical effects than the NLS equation such as the wave induced mean flow and the asymmetric evolution of wave packets and side bands.

In addition to being important theoretical models, the Zakharov and NLS equations are also important tools for numerical simulations of surface gravity waves. In particular, efficient numerical methods exist for the NLS equations, which make them convenient for numerical studies.

Numerical simulations of the NLS equation has revealed that a wave field with an initially narrow banded spectrum can suffer energy leakage to high wavenumbers, whereby the model breaks down due to violation of its own bandwidth constraint (Martin & Yuen, 1980; Yuen & Lake, 1980). After this discovery the NLS equation became somewhat unfashionable for the study of the physics of surface gravity waves.

Lo & Mei (1985, 1987) solved the MNLS equation numerically using an efficient pseudo-

spectral, split-step Fourier method. Although the MNLS equation also can suffer energy leakage the simulations showed that the energy leakage was not a problem in practice provided the spectrum was discretized only within a radius around the characteristic wave vector k_c . On this background the MNLS equation is physically and numerically attractive in the sense that it describes the necessary physical properties to fourth order in steepness and bandwidth, is numerically stable within a spectral radius around the peak and is efficient to solve numerically. In the past years the MNLS equation has been used to study statistical properties (Socquet-Juglard *et al.*, 2005; Gramstad & Trulsen, 2007) and spectral evolution (Dysthe *et al.*, 2003) of surface gravity waves.

Recently, with the improvement of computational power and development of parallel computing, fully nonlinear models have been used to study statistical properties of surface gravity waves (Xiao *et al.*, 2013). These models solve (1) with higher accuracy and do not assume bandwidth limitation. On the other hand higher order models, including the Zakharov equation, are computationally expensive to solve relative to lower order models such as the MNLS equation. Although higher order models solve (1) with higher accuracy and show discrepancy to lower order models for long evolution, numerical simulations have revealed that the initial evolution is relatively similar, see Xiao *et al.* (2013).

During the seventies and eighties, the applicability of the linear dispersion relation (4) was questioned after ocean measurements by Von Zwick (1970), Yefimov *et al.* (1972), Grose *et al.* (1972) and Ramamonjiarisoa & Giovanangeli (1978), laboratory measurements by Ramamonjiarisoa (1974), Ramamonjiarisoa & Coantic (1976), Lake & Yuen (1978), Rikiishi (1978), Masuda *et al.* (1979), Mitsuyasu *et al.* (1979) and Donelan *et al.* (1985) as well as theoretical analysis by Huang & Tung (1977), Mollo-Christensen & Ramamonjiarisoa (1978), Crawford *et al.* (1981), Phillips (1981) and Barrick (1986).

The early ocean and laboratory measurements mentioned here typically consisted of a couple or a handful number of measurements locations where time series were collected at a sufficient sampling rate and the phase speed was calculated from the correlation between pairs of measurement locations. It was found that the spectral contribution above the peak had higher phase speed than that predicted by the linear dispersion relation (4).

Within pragmatic temporal and spatial scales for real data, and regardless of any wave theory, it is usually reasonable to assume that an open ocean wave field is stationary in time and spatially homogeneous. This minimal assumption enables introduction of a general three-dimensional $(\mathbf{k}, \omega)$ -spectrum obtained by discrete Fourier transform of measured time t - and space x - and y -series of sufficient resolution and coverage. Here, sufficient resolution is defined in terms of the Nyquist sampling theorem; the spatio-temporal sampling of the surface elevation η should satisfy the Nyquist theorem. Sufficient coverage is defined in terms of the Benjamin-Feir (BF) scale $(\epsilon^2 \omega_c)^{-1}$; the spatio-temporal coverage of η should fulfill the BF scale which is the typical evolution scale for the resonant wave interactions. Depending on ϵ the BF scale typically covers several characteristic periods T_c and characteristic wave lengths λ_c .

In practice, ocean waves are observed by a multitude of systems ranging from compact instruments like buoys and pressure and current rigs, that provide point measurements, to instrumentation that covers local areas in the ocean such as marine and HF radar and systems with global coverage such as satellites. There is currently no measurement system able to measure

the full (k, ω) -spectrum in the ocean with high accuracy, the main limitation being caused by the spatial extension such measurements would require. The space-borne Synthetic Aperture Radar (SAR), although having excellent area coverage, provides data with a temporal extension limited by the traverse time of the synthetic aperture. In addition, the ocean-to-image mapping for the SAR is by itself highly nonlinear, although based on linear theory for the wave field (Hasselmann *et al.*, 1985; Krogstad, 1992; Hasselmann *et al.*, 1996).

The only operational system today which utilize real space and time measurements and three-dimensional (k, ω) -spectral analysis is based on sequences of marine radar imagery. Nevertheless, the inversion of the imagery currently relies on linear wave theory as a prerequisite. The spectral analysis of the image sequences consists of applying a three-dimensional band-pass filter that suppresses those spectral contributions that do not satisfy the linear dispersion relation (Nieto Borge *et al.*, 2004).

In the theoretical works by Huang & Tung (1977) and Crawford *et al.* (1981) extensions of the linear dispersion relation (4) with regard to nonlinearity were derived. Using a third-order equation Huang & Tung (1977) found analytically that directional waves deviated from the linear dispersion relation above the peak and that the deviation was dependent on the directional spread of the wave field with increased deviation for decreasing directional spread. Using the Zakharov equation Crawford *et al.* (1981) found analytically that unidirectional waves deviated from the linear dispersion relation above the peak and that the deviation was more pronounced for narrow bandwidths than for broad bandwidths. Crawford *et al.* (1981) also considered wave fields with different steepness showing that the deviation was most pronounced for the wave fields with high steepness.

In the past decade, (k, ω) -spectra from ocean measurements with relatively low resolution and limited coverage were presented by Hara and Karachintsev (2003) and Wang and Hwang (2004). Around the peak the spectra did not indicate deviation from the linear dispersion relation.

More recently, Gibson & Swan (2006), Krogstad & Trulsen (2010) and Houtani *et al.* (2015) performed simulations of unidirectional waves with sufficient spatio-temporal resolution and coverage to obtain accurate (k, ω) -spectra. Krogstad & Trulsen (2010) used the NLS and MNLS equations, Gibson & Swan (2006) used the Zakharov equation and a fully nonlinear wave model by Bateman *et al.* (2001) and Houtani *et al.* (2015) used a higher order spectral method. From the spectra it was found that the spectral contribution of the free waves deviated from the linear dispersion relation above the spectral peak due to nonlinear evolution of resonant and quasi-resonant wave interactions.

While the frequency spectrum is obtained from a time interval at a specific spatial location the cross-spectrum is obtained from two locations. Consequently, the cross-spectrum can be used to obtain information about the dependence between time series at different locations. Within the stochastic approach of linear theory the coherence and phase, computed from the cross-spectrum, can be used as a measure of the linear dependence between stochastic processes at different locations, see e.g. Bendat & Piersol (2010) and Lindgren (2013).

Mitsuyasu *et al.* (1979) measured the coherence of wind generated random waves and compared the results with linear theory and a third order nonlinear numerical model accounting for resonant interactions (Masuda *et al.*, 1979). They found that the coherence of spectral contributions near the peak was close to that given by linear theory. Above the peak they observed

a local decrease of the coherence at frequencies close to two times the peak and attributed this effect to second order bound harmonic waves coexisting with the first order waves. Regarding the second bound harmonic the nonlinear numerical model (Masuda *et al.*, 1979) agreed with the experimental observations.

Benjamin & Feir (1967) showed that a Stokes wave is unstable to small side band perturbations. This physical mechanism has been named the Benjamin-Feir (BF) instability and has been devoted much attention in the past decades. Alber (1978), Alber & Saffman (1978) and Crawford *et al.* (1980) studied the instability of random wave fields and found that a requirement of instability is that the ratio of wave steepness to spectral bandwidth is sufficiently high. The effect of the instability on the statistical properties of waves were reported by Onorato *et al.* (2001) using numerical simulations of the NLS equation for unidirectional waves. They found that the probability of freak waves depends on the ratio of steepness to bandwidth. The ratio was later named the Benjamin-Feir index (BFI) by Janssen (2003).

Main new findings

The fundamental motivation for the work in this thesis has been to improve the understanding of how nonlinear evolution affects the dispersion relation and spectral properties of surface gravity waves on deep water. This has been investigated by comparing high-resolution spatio-temporal measurements of laboratory waves and numerical simulations of weakly nonlinear waves with linear wave theory.

The first and the second paper in this thesis compares the linear dispersion relation (4) with experiments and simulations of weakly nonlinear waves.

The first paper (Taklo *et al.*, 2015) examines unidirectional waves exclusively. In this paper we performed experiments with an array of sixteen ultrasonic probes in a long and narrow wave-tank at the Hydrodynamic Laboratory at the University of Oslo. The array is shown in figure 1. Using the principle of a synthetic aperture, measurements from the array were collected along the entire wavetank providing a spatio-temporal resolution and coverage sufficient to obtain an accurate (k, ω) -spectrum. Nine different wave fields with different bandwidths and fixed steepness were measured. For narrow banded wave fields the experiments confirmed the deviation from the linear dispersion relation numerically predicted by Gibson & Swan (2006), Krogstad & Trulsen (2010) and Houtani *et al.* (2015). For broad banded wave fields the experiments showed that the linear dispersion relation was satisfied, suggesting validity of linear wave theory. Numerical simulations of the Zakharov (1968) equation were performed and agreed with the experiments.

The second paper (Taklo *et al.*, 2016b) extends the work of the first paper to directional waves. The numerical efficiency of the MNLS equation enabled us to perform a large number of simulations to study the dispersion of directional waves. We propose a parameter for describing the deviation between the dispersion relation obtained from the simulated (k, ω) -spectra and the linear dispersion relation. For short crests the magnitude of the deviation parameter is low while for long crests the magnitude of the deviation parameter is high and the magnitude depends on the BFI.

In the second paper (Taklo *et al.*, 2016b) we also consider directional data from the Seakeeping and Manoeuvring Basin at the Marine Institute of the Netherlands (MARIN). The MARIN

Figure 1: Array of 16 ultrasonic probes in the long wavetank at the Hydrodynamic Laboratory at the University of Oslo. Photo by Tore Magnus Arnesen Taklo.

experiments were initiated with the purpose of developing and testing a system which could predict the motion of a vessel ahead of time. The project was named the On-board Wave and Motion Estimator Joint Industry Project (OWME JIP) and several companies participated i.e. Gusto, Kongsberg, SBM, Seaflex, Sirehna, Statoil and Total.

The MARIN data were made available for our project at the University of Oslo. The data are unique in the sense that they have a relatively dense spatial coverage and to our knowledge similar measurements have not been performed or published. The purpose of the MARIN experiments was not to perform measurements with a dense and uniform spatio-temporal coverage, required to compute an accurate (k, ω) -spectrum. The spatial coverage used for the experiments was nonuniform. Nevertheless, we were able to compute the (k_x, ω) and (k_y, ω) -spectra from the experiments by using a nonuniform Fourier transform (Fessler & Sutton, 2003) for the space series. Here, k_x and k_y are the longitudinal and transversal wavenumbers in the x and y -direction, respectively. The spectra showed no indication of deviation from the linear dispersion relation because of the relatively low BFI and short crests employed and this result was confirmed by numerical simulations of the MNLS equation. Even though the experiments did not provide evidence for the deviation the unique data gave us valuable insight into the properties of directional waves.

In the third paper (Taklo *et al.*, 2016a) the experiments are identical to those presented in the first paper. From the cross-spectrum we compute the coherence and phase between the closely separated locations in our experiments and compare the experiments with numerical simulations of linear and weakly nonlinear theory. From the experiments and nonlinear simulations we find that the spatio-temporal evolution of the coherence and phase is in accordance with lin-

ear theory for broad bandwidths while for narrow bandwidths discrepancies from linear theory are observed. At two times the peak we also find an increase in the coherence likely caused by the second harmonic waves being phase locked to the free waves. The observation of the second harmonic is confirmed by the nonlinear simulations reconstructed with third order bound harmonics. For the phase we find approximately agreement with linear theory for broad bandwidths while for narrow bandwidths we find that the phase of spectral components above the peak deviates from linear theory. Additional linear and nonlinear Monte-Carlo simulations are performed which give estimates of the coherence and phase with lower statistical uncertainty and variability than from the experiments. The Monte-Carlo simulations confirm the behaviour of the coherence and phase observed from the experiments.

The results of the coherence and phase in the third paper Taklo *et al.* (2016a) indicate that the effect of the nonlinearity is most pronounced when the bandwidth is narrow, as according to Alber (1978), Alber & Saffman (1978) and Crawford *et al.* (1980), while for broader bandwidths the effect of the nonlinearity is less pronounced and the wave field is expected to behave more linearly. This result could not have been anticipated from the relatively broad banded wave fields of Mitsuyasu *et al.* (1979) and Masuda *et al.* (1979) which were more broad banded than our most narrow banded experimental and numerical wave fields.

From a different point of spectral view the result of the third paper is qualitatively similar to that of the first paper. Both the (k, ω) -spectrum used in the first paper and the cross-spectrum used in the third paper show that weakly nonlinear unidirectional waves deviate from linear theory when the bandwidth is narrow while for broader bandwidths the behaviour of the waves is more in accordance with linear theory.

One of the most important contributions from this thesis has been the accurate and dense spatio-temporal laboratory measurements of long crested wave fields with different bandwidths covering the BF scale. From the measurements we have been able to compute accurate (k, ω) -spectra (Taklo *et al.*, 2015) and study the cross-spectrum between multiple closely separated measurement locations (Taklo *et al.*, 2016a).

Accurate (k, ω) -spectra have been obtained from simulations (Gibson & Swan, 2006; Krogstad & Trulsen, 2010; Houtani *et al.*, 2015), but we are not aware of other experiments than ours where accurate (k, ω) -spectra have been measured. The laboratory and ocean measurements mentioned from the seventies reported phase speed above the peak higher than that predicted by the linear dispersion relation. These observations were based on the correlation between individual measurement locations. In this thesis our accurate and dense spatio-temporal measurements over several locations on the BF scale has enabled us to compute (k, ω) -spectra and compare the spectra directly to the linear dispersion relation. Hence, we have not only confirmed the observations from the seventies but removed uncertainty and speculations around these relatively limited measurements by interpreting a more comprehensive picture of the spatio-temporal evolution of a wave field.

Our directional simulations in the second paper Taklo *et al.* (2016b) have shown that the deviation from the linear dispersion relation eventually vanishes when the crests become short and the BFI is low. As mentioned the spectral analysis of marine radar image sequences consists of applying a three-dimensional band-pass filter that suppresses those spectral contributions that do not satisfy the linear dispersion relation (Nieto Borge *et al.*, 2004). Our directional simulations are therefore valuable for quantifying wave conditions where the band-pass filter is erroneous

relative to the actual dispersion of the wave field. On the other hand it should be mentioned that there are several effects in the radar images that are not taken into account in our simulations such as shadowing, tilt modulation and the position of the antenna in the middle of the measured sea surface area. As a consequence of the positioning of the antenna our simulations cover a larger spatial domain than what is achievable from a radar image. On this background our simulations are not directly comparable to the radar image. For future work it would be valuable to improve the radar filter such that the actual dispersion of the wave field is taken into account.

Brief extracts from each paper

Paper I - Measurement of the dispersion relation for random surface gravity waves

Paper I (Taklo *et al.*, 2015) presents experiments conducted at the Hydrodynamic Laboratory at the University of Oslo designed to have sufficient resolution and coverage to obtain accurate (k, ω) -spectra of unidirectional waves. For narrow banded wave fields the (k, ω) -spectra show deviation from the linear dispersion relation. For broad banded wave fields the (k, ω) -spectra show that the linear dispersion relation is satisfied, suggesting validity of linear wave theory. Numerical simulations of the Zakharov (1968) equation were performed and agreed with the experiments.

Paper II - On dispersion of directional surface gravity waves

Paper II (Taklo *et al.*, 2016b) presents numerical simulations of directional waves using the modified nonlinear Schrödinger (MNLS) equation of Dysthe (1979). Laboratory data of directional waves from the Marine Research Institute of the Netherlands (MARIN) is also considered. We show that the deviation from the linear dispersion relation depends on the BFI and the crest length of the waves. The MARIN data confirm the simulations for three cases of BFI and crest length.

Paper III - Cross-spectral evolution of unidirectional surface gravity waves

Paper III (Taklo *et al.*, 2016a) presents the cross-spectrum, the coherence and the phase, of unidirectional waves. Experiments are compared with simulations of linear and nonlinear waves. The coherence and phase is in accordance with linear wave theory for broad bandwidths while for narrow bandwidths discrepancies from linear theory are observed.

References

- ALBER, I. A. 1978 The effects of randomness on the stability of two-dimensional surface wavetrains. *Proc. R. Soc. Lond. A*. **363**, 525–546.
- ALBER, I. A. & SAFFMAN, P. G. 1978 Stability of random nonlinear deep water waves with finite bandwidth. *TWR Defense and Spacesystems Rep.* pp. 31326–6035–RU–00, 89.
- BARRICK, E. 1986 The role of the gravity-wave dispersion relation in HF radar measurements of the sea surface. *IEEE J. Oceanic Eng.* **11**, 286–292.

- BATEMAN, W. J. D., SWAN, C. & TAYLOR, P. H. 2001 On the efficient numerical simulation of directionally-spread surface water waves. *J. Comput. Phys.* **174**, 277–305.
- BENDAT, J. S. & PIERSON, A. G. 2010 *Random Data: Analysis and Measurement Procedures*. Wiley, New Jersey.
- BENJAMIN, T. B. & FEIR, J. E. 1967 The disintegration of wave trains on deep water part 1. theory. *J. Fluid Mech.* **27**, 417–430.
- CRAWFORD, D. R., LAKE, B. L., SAFFMANN, P. G. & YUEN, H. C. 1981 Effects of non-linearity and spectral bandwidth on the dispersion relation and components phase speeds of surface gravity waves. *J. Fluid Mech.* **112**, 1–32.
- CRAWFORD, D. R., SAFFMAN, P. G. & YUEN, H. C. 1980 Evolution of a random inhomogeneous field of nonlinear deep-water gravity waves. *Wave Motion* **2**, 1–16.
- DONELAN, M. A., HAMILTON, J. & HUI, W. H. 1985 Directional spectra of wind-generated waves. *Philos. Trans. R. Soc. A.* **315**, 509–562.
- DYSTHE, K. B. 1979 Note on a modification to the nonlinear Schrödinger equation for application to deep water waves. *Philos. Trans. R. Soc. A.* **369**, 105–114.
- DYSTHE, K. B., TRULSEN, K., KROGSTAD, H. E. & SOCQUET-JUGLARD, H. 2003 Evolution of a narrow-band spectrum of random surface gravity waves. *J. Fluid Mech.* **478**, 1–10.
- FESSLER, J. A. & SUTTON, B. P. 2003 Nonuniform fast Fourier transform using min-max interpolation. *IEEE Transactions on Signal Processing* **51**, 560–574.
- GIBSON, R. S. & SWAN, C. 2006 The evolution of large ocean waves: the role of local and rapid spectral changes. *Proc. R. Soc.* **463**, 21–48.
- GODA, Y. 2000 *Random seas and design of maritime structures*. World Scientific, Singapore.
- GRAMSTAD, O. & TRULSEN, K. 2007 Influence of crest and group length on the occurrence of freak waves. *J. Fluid Mech.* **582**, 463–472.
- GROSE, P. L., WARSH, K. L. & GARSTANG, M. 1972 Dispersions relations and wave shapes. *J. Geophys. Res.* **77**, 3902–3906.
- HASSELMANN, K., RANEY, R. K., PLANT, W. J., ALPERS, W., SHUCHMAN, R. A., LYZENGA, D. R., RUFENACH, C. L. & TUCKER, M. J. 1985 Theory of synthetic aperture radar ocean imaging: A MARSSEN view. *J. Geophys. Research* **90** (C3), 4659–4686.
- HASSELMANN, S., BRÜNING, C., HASSELMANN, K. & HEIMBACH, P. 1996 An improved algorithm for the retrieval of ocean wave spectra from synthetic aperture radar image spectra. *J. Geophys. Research* **101** (C7), 16615–16629.
- HOUTANI, H., WASEDA, T., FUJIMOTO, W., KIYOMATSU, K. & TANIZAWA, K. 2015 Freak wave generation in a wave basin with HOSM-WG method. In *ASME 2015 34th Int. Conf. on Ocean, Offshore and Arctic Eng. American Soc. of Mech. Engineers*. **V007T06A085**, 1–15.

- HUANG, N. E. & TUNG, C. C. 1977 The influence of the directional energy distribution on the nonlinear dispersion relation in a random gravity wave field. *J. Phys. Oceanog.* **7**, 403–414.
- JANSSEN, P. A. E. M. 2003 Nonlinear four-wave interactions and freak waves. *A. Met. Soc.* **33**, 863–884.
- KROGSTAD, H. E. 1992 A simple derivation of Hasselmann’s nonlinear ocean-synthetic aperture radar transform. *J. Geophys. Research* **97** (C2), 2421–2425.
- KROGSTAD, H. E. & TRULSEN, K. 2010 Interpretations and observations of ocean wave spectra. *Ocean Dynamics* **60**, 973–991.
- LAKE, B. M. & YUEN, H. C. 1978 A new model for nonlinear wind waves. Part 1. Physical model and experimental evidence. *J. Fluid Mech.* **88**, 33–62.
- LINDGREN, G. 2013 *Stationary Stochastic Processes - Theory and Applications*. CRC Press.
- LO, E. & MEI, C. C. 1985 A numerical study of water-wave modulations based on a higher-order nonlinear Schrödinger equation. *J. Fluid Mech.* **150**, 395–415.
- LO, E. & MEI, C. C. 1987 Slow evolution of nonlinear deep water waves in two horizontal directions: A numerical study. *Wave Motion* **9**, 245–259.
- LONGUET-HIGGINS, M. S. 1963 The effect of non-linearities on statistical distributions in the theory of sea waves. *J. Fluid Mech.* **17**, 459–480.
- MARTIN, D. U. & YUEN, H. C. 1980 Quasi-recurring energy leakage in the two-space-dimensional nonlinear Schrödinger equation. *Phys. Fluids* **23**, 881.
- MASUDA, A., KUO, Y. Y. & MITSUYASU, H. 1979 On the dispersion relation of random gravity waves. Part 1. Theoretical framework. *J. Fluid Mech.* **92**, 717–730.
- MITSUYASU, H., KUO, Y. Y. & MASUDA, A. 1979 On the dispersion relation of random gravity waves. Part 2. An experiment. *J. Fluid Mech.* **92**, 731–749.
- MOLLO-CHRISTENSEN, E. & RAMAMONJIARISOA, A. 1978 Modeling the presence of wave groups in a random wave field. *J. Geophys. Res.* **83**, 4117–4122.
- NIETO BORGE, J. C., RODRÍGUEZ, G., HESSNER, K. & IZQUIERDO, P. 2004 Inversion of marine radar images for surface wave analysis. *J. Atmos. Ocean Tech.* **21**, 1291–1300.
- ONORATO, M., OSBORNE, A. R., SERIO, M. & BERTONE, S. 2001 Freak waves in random oceanic sea states. *Phys. Rev. Letters* **86**, 5831–5834.
- PHILLIPS, O. M. 1960 On the dynamics of unsteady gravity waves of finite amplitude. Part 1. The elementary interactions. *J. Fluid Mech.* **9**, 193–217.
- PHILLIPS, O. M. 1981 The dispersion of short wavelets in the presence of a dominant long wave. *J. Fluid Mech.* **107**, 456–485.

- RAMAMONJIARISOA, A. 1974 Thèse de Doctorat d'Etat, université de Provence, enregistrée au C.N.R.S. no. A.O.10.023 .
- RAMAMONJIARISOA, A. & COANTIC, M. 1976 Loi expérimental de dispersion des vagues produites par le vent sur une faible longueur d'action. *C. R. Acad. Sci. Paris B.* **282**, 111–113.
- RAMAMONJIARISOA, A. & GIOVANANGELI, J. P. 1978 Observations de la vitesse de propagation des vagues engendrées par le vent au large. *C. R. Acad. Sci. Paris B.* **287**, 133–136.
- RIKIISHI, K. 1978 A new method for measuring the directional wave spectrum. Part II. measurement of the directional spectrum and phase velocity of laboratory wind waves. *J. Phys. Oceanog.* **8**, 518–529.
- SOCQUET-JUGLARD, H., DYSTHE, K., TRULSEN, K., KROGSTAD, H. E. & LIU, J. D. 2005 Probability distributions of surface gravity waves during spectral changes. *J. Fluid Mech.* **542**, 195–216.
- STOKES, G. G. 1847 On the theory of oscillatory waves. *Trans. Camb. Philos. Soc.* **8**, 441–455.
- TAKLO, T. M. A., NIETO BORGE, J. C., KROGSTAD, H. E. & TRULSEN, K. 2016a Cross-spectral evolution of directional surface gravity waves. *Under preparation* .
- TAKLO, T. M. A., TRULSEN, K., GRAMSTAD, O., KROGSTAD, H. E. & JENSEN, A. 2015 Measurement of the dispersion relation for random surface gravity waves. *J. Fluid Mech.* **766**, 326–336.
- TAKLO, T. M. A., TRULSEN, K., KROGSTAD, H. E. & NIETO BORGE, J. C. 2016b On dispersion of directional surface gravity waves. *Under consideration of publication in J. Fluid Mech.* .
- TOFFOLI, A., GRAMSTAD, O., TRULSEN, K., MONBALIU, J., BITNER-GREGERSEN, E. & ONORATO, M. 2010 Evolution of weakly nonlinear random directional waves: laboratory experiments and numerical simulations. *J. Fluid Mech.* **664**, 313–336.
- TUCKER, M. J. & PITT, E. G. 2001 *Waves in ocean engineering*. Elsevier Science & Technology, Oxford.
- VON ZWECK, O. H. 1970 Observations of propagation characteristics of a wind driven sea. Doctorate thesis, Massachusetts Institute of Technology .
- XIAO, W., LIU, Y., WU, G. & YUE, D. K. P. 2013 Rouge wave occurrence and dynamics by direct simulations of nonlinear wave-field evolution. *J. Fluid Mech.* **720**, 357–392.
- YEFIMOV, V. V., SOLOV'YEV, Y. P. & KHRISTOFOROV, G. N. 1972 Observational determination of the phase velocities of spectral components of wind waves. *Atmospheric and Oceanic Phys.* **8**, 435–446.
- YUEN, H. C. & LAKE, B. M. 1980 Instabilities of waves on deep water. *Annu. Rev. Fluid Mech.* **12**, 303.

ZAKHAROV, V. E. 1968 Stability of periodic waves of finite amplitude on the surface of a deep fluid. *J. Appl. Mech. Tech. Phys.* **9**, 190–194.

Papers

- Paper I** **Taklo, T. M. A. & Trulsen, K. & Gramstad, O. & Krogstad, H. E. & Jensen, A. 2015**
Measurement of the dispersion relation for random surface gravity waves
J. Fluid Mech. **766**, 326–336.
- Paper II** **Taklo, T. M. A. & Trulsen, K. & Krogstad, H. E. & Nieto Borge, J. C. 2016**
On dispersion of directional surface gravity waves
Submitted to J. Fluid Mech.
- Paper III** **Taklo, T. M. A. & Nieto Borge, J. C. & Krogstad, H. E. & Trulsen, K. 2016**
Cross-spectral evolution of unidirectional surface gravity waves