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Anticipating the Uncertain:
Economic Modeling and
Climate Change Policy



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Introduction

Almost all economic activity today heavily relies on fossil energy in one way or the other. The combustion of fossil fuels releases carbon dioxide and is the single largest source of anthropogenic greenhouse gas emissions. Emissions of greenhouse gases accumulate in the atmosphere and contribute to global warming through the greenhouse effect. Consequently, the mean surface temperature is expected to rise and alter the earth's ecosystems over the coming decades and centuries (Socolow and Lam, 2007). While the physical laws and regularities governing the climate are well known, empirical predictions about the severity of climate change are extremely hard as a result of the system's complexity: Finding an accurate model and determining its parameters is currently not within our reach (Millner et al., 2010). Scientific consensus is that global warming most probably will cause a multitude of adverse effects for large parts of the world's population (IPCC, 2007). The gains from burning coal or oil are private, the costs are public. The climate is a global public good, and so climate change is the ultimate tragedy of the commons. The goal of climate policy should be to break the link between economic activity and global warming. Unless we doubt the natural science or believe that natural ecosystems are very robust and human adaptation to altered climate simple and cheap, only two choices remain: Reduce emissions by replacing carbon as the main source of energy or capturing the carbon dioxide, or directly manipulate the global climate to counteract the warming through geo-engineering. All of these alternatives are unknown territory, and the scale of the challenge is unprecedented. There are many diverse economics questions we have to ask. Among them are: For which level of greenhouse gas concentrations should we aim? How can we create international agreement on the target and split the costs involved between nations? How do we best implement those targets at manageable costs, and when should they be implemented? How to prepare for the climate change to come?

With this thesis I wish to contribute to the understanding of how uncertainty and the anticipation of future events by economic actors affect climate policies. Two aspects of economic analysis therefore feature prominently in my thesis: Intertemporal decision making and decision making under uncertainty. First, timing matters: The benefits of burning fossil fuels are immediate, but the damages will only be felt decades or even centuries from now. A practical reason for paying particular attention to the time

dimension are the characteristics of fossil fuels. Coal, gas and oil are non-renewable resources and as such command economic rents. The owner of a coal mine or an oil well has a long planning horizon and has to make the decision about when to extract the resource. He therefore anticipates future developments and acts today.¹

Secondly future climate conditions are unknown, and so are technological options, the level of economic well-being, and the political situation. Economic analysis of climate change hence has to properly model uncertainty and social attitudes to it (Traeger, 2012; Gollier et al., 2000; Weitzman, 2009). We usually assume that people and societies dislike that they do not know what happens next: They are risk averse. But such a preference alone offers no behavioral advice.² The specific conditions matters for the optimal decision. As a simple thought experiment, suppose you are engaging in an joyful activity that may cause you harm in the future. There is also a chance of an unrelated negative event. You are risk averse. But since there is a chance for trouble anyway, why be careful and cut down on the joy today just to avoid harm that you may never experience? If on the contrary your indulgence affects the chances of the adverse event, you act prudently and reduce or abandon the joyful activity.

My thesis consists of four papers. The first two are analytical models in which emissions are caused by extracting a non-renewable resource. The latter two are numerical integrated assessment models with a focus on uncertainty. For both paper pairs I first discuss the common features jointly before summarizing each paper by itself.

1 The Supply Side of CO_2 and Carbon Taxes

My first two papers treat fossil fuel extraction explicitly as the source of carbon dioxide emissions. I model the supply side of carbon dioxide and investigate its effects on the design of climate policy. I restrict myself to one particular instrument, a tax on carbon dioxide emissions. A universal carbon tax set at Pigou level to correct the externalities of emissions is generally viewed by economists as an efficient way to allocate abatement activities.³ It illustrates how the problem could be solved, and is thus a useful benchmark for politically feasible policies. The early literature established how the supply side dynamics of fossil fuels influence the optimal emissions tax.⁴ More recently, attention has shifted to some of the imperfections in a tax regime and unintended consequences

¹Here I rely on the theoretical literature. Empirical evidence on intertemporal firm decisions is inherently difficult to produce (Krautkraemer, 1998; Kronenberg, 2008).

²As a simple example, consider adding uncertainty to a given consumption path. The volatility has two effects, an income and a substitution effect. The decision maker dislikes that the future is uncertain and *ceteris paribus* shifts consumption to the (more) certain present. But as expected future welfare decreases, she also wants to shift some consumption into the future.

³At least in theory, when disregarding common imperfections such as market power, other, distortionary taxes, externalities or dynamic issues such as commitment problems.

⁴Sinclair (1994); Ulph and Ulph (1994); Withagen (1994); Hoel and Kverndokk (1996); Tahvonen (1997).

that follow if there are areas or time periods not covered by the policy: Carbon leakage is the effect a climate policy in place \mathcal{A} has on place \mathcal{B} . For example, if a carbon tax in \mathcal{A} lowers the profitability of fossil fuels in that region, the resource owner may instead sell them in \mathcal{B} , hence increasing emissions there (see Eichner and Pethig, 2011, for a recent example). The Green Paradox refers to a similar effect over time: Making the sale of fossil fuels less profitable in the future means it is more attractive to extract them today. In the extreme case, this may imply that a future carbon tax harms the climate (Sinn, 2008; Gerlagh, 2011).

In these two papers I also pay attention to two technological solutions to climate change: Renewable energy and carbon capture and storage. These technologies are currently the focus of substantial research and development efforts. Investment in renewables may influence the rate of fossil fuel extraction and vice versa: Anticipated low cost renewable energy means it is more profitable to extract most of the resource today, whereas huge reserves of cheap fossil fuels render investments in renewables unattractive. A policy maker may ideally want to provide different incentives to the two market actors: A low future tax to slow down extraction, and a high future tax to foster innovation today. Related, but not the focus of my work, is the commitment problem: A policy maker would wish to announce a high future carbon tax to foster investment in green R&D, but when the technology exists she would want it widely disseminated. If the carbon tax is low, this lowers the price of the new technology's patents and increases dissemination rates (Requate, 2005; Scotchmer, 2010).

1.1 Cutting Costs of Catching Carbon (with M. Hoel)

The first paper, which is co-authored with Michael Hoel, asks whether some climate friendly technologies are preferable to others. In particular, should policy makers discriminate between subsidizing cost reductions in renewable energy sources such as wind or solar power on the one hand, and abatement technologies such as carbon capture and storage (CCS) on the other? Adding to the many conceivable arguments for and against differentiation, we suggest one more: in a world with imperfect climate policies, developing each of these technologies alters the incentives that fossil fuel owners face in a different way. While cheaper future renewables cause extraction to speed up, lower costs of CCS may delay it.

To cease GHG emissions and at the same time secure sufficient energy supplies requires the development and deployment of new, low emission power sources. Two promising options are electricity generation from renewable sources such as wind and solar, and carbon capture and storage (CCS). Wind and solar energy are from a pure physics perspective available at a sufficient scale to replace fossil fuel power generation (MacKay, 2008). However, they remain noncompetitive on costs and various technolog-

ical challenges remain. CCS is a technology under development meant to abate carbon dioxide emissions by capturing large point source emissions and storing them underground. Currently, both these options raise costs relative to regular fossil fuel energy. Consequently climate policies are necessary if they are to be deployed. At the global level, a comprehensive international agreement that can achieve this has yet to emerge.⁵ This provides owners of fossil fuels with a window of unknown length during which they can sell fossil fuels without a carbon tax or price-competitive, low-emission alternatives. Such intertemporal reallocation undermines policy objectives as more carbon dioxide is emitted in early periods, and potentially total emissions remain unchanged. This supply side effect has become known as the ‘green paradox’ (Sinn, 2008).

The present paper contributes to the literature on fossil fuel supply under imperfect climate policies by focusing on differences in prospective climate friendly technologies. In particular, we ask how reductions in the costs of CCS technology affects the market outcome. We contrast this with improvements in renewable energy technology. To that end, we build a simple analytical two period model. In period one, emission free technologies play no role, only conventional energy is available. However, actors know about the arrival of alternatives in the second period. By that time, three types of energy technology are available: conventional fossil energy, fossil energy with CCS technology and renewable energy. Fossil fuel suppliers optimize dynamically and sell fossils to conventional and (in period two) CCS power generators. Those sell power competitively in the same market as renewable energy suppliers to energy end users, who are indifferent with regards to the source of their energy. Climate policy is enacted either in both periods (as a first best benchmark) or in the second period only. Our main finding is that imperfect climate policies, cost reductions related to CCS may be more desirable than comparable cost reductions related to renewable energy. The finding rests on the incentives fossil resource owners face. With regulations of emissions only in the future, cheaper renewables speed up extraction, whereas CCS cost reductions potentially make fossil resources more attractive for future use, leading to postponed extraction. Further, it is possible that renewable energy innovations lower social welfare, whereas lower costs of CCS are in our setting always beneficial.

1.2 Carbon Tax Uncertainty, Fossil Energy, and Green R&D

The first essay does not model uncertainty explicitly, but shows how various beliefs on the part of fossil fuel suppliers may affect climate change and social welfare. In the second paper, I explicitly analyze how uncertainty about future carbon taxes impacts investment in renewable energy and extraction of fossil resources. Does it matter whether the uncertainty reflects our limited knowledge about the climate system or whether it

⁵Barrett (2005) has a game theoretical treatment of international climate agreements, while Røgeberg et al. (2010) offer more of a political economy approach.

derives from political processes?

Besides the difficulty of predicting the magnitude of the climate's response to increased atmospheric GHG concentrations, current policy makers also need to take into account the uncertainty regarding future policy makers: To what extent will they be able to cooperate and cap global emissions - and what importance will they place on the global climate issue in their policies? As climate change threatens to cause catastrophic outcomes (Lemoine and Traeger, 2010), it would seem reasonable to expect stringent climate policy in response. Existing climate policies, however, seem more in line with modeling results which assume purely self-interested non-cooperative behavior (Anthoff, 2011). The unwillingness of most nations to enter a binding agreement on greenhouse gas emissions has consequently been explained as a manifestation of the tragedy of the commons (Barrett, 2005). Since international cooperation on climate change so far has failed to substantially reduce greenhouse gas emissions, it is a priori equally reasonable to expect weak climate policies in the future.

I investigate how these two sources of uncertainty impact the decisions of current policy makers regarding fossil fuel extraction and R&D investments in renewable energy. I assume that the energy market today is supplied by scarce fossil fuels. Investments can be undertaken to make new, non-polluting energy sources available in the future. Distinguishing between political and scientific uncertainty, I ask how uncertainty alters the actions of current policy makers. I assume in the model that current and future policy makers view scientific uncertainty in the same way given the same information. Political uncertainty, on the other hand, stems from successive decision makers giving different weight to the climate problem in their policies. Current regulators may then attempt to manipulate the options available to future regulators to shift their actions towards those desired by the current regulators. The three papers most closely related to my contribution are Ulph and Ulph (2011), Hoel (2012a) and (Hoel, 2012b). Ulph and Ulph (2011) investigate a situation where a government cannot set future taxes but decides on R&D in renewable energy. They find that uncertainty about future taxes affects the incentives of the current government and clean technology investors in opposite directions: Whereas investors want to reduce investments, governments would like to see them increased. Their model, however, does not feature a non-renewable resource. Hoel (2012b) investigates political and scientific uncertainty in a setting without resources and investment in a physical capital stock rather than innovation. Finally, Hoel (2012a) adds a non-renewable resource and studies the effect of expected carbon tax rates, but his analysis does not treat uncertainty explicitly. The present work expands on these by going beyond their two-period models and treating the two types of uncertainty explicitly within a fully dynamic model.

Considering investment and extraction separately, I find that scientific uncertainty encourages a social planner decreases early extraction and increases investment in renew-

ables. Political uncertainty has the opposite effect: Because the present social planner loses control over her investments in green innovation and fossil fuel stock, she extracts faster and invests less in renewable energy. Importantly, when considered simultaneously, both sources of uncertainty have ambiguous effects on green investments and the extraction profile for fossil fuels. The social planner solutions can be implemented by carbon taxes. In the case of scientific uncertainty, the optimal carbon tax equals marginal damages. Political uncertainty on the contrary demands a tax below marginal damages that decreases over time.

2 Optimal Climate Policy under Uncertainty

The second half of the thesis is devoted to investigating the impact of uncertainty and learning on optimal climate policy in a numerical integrated assessment model framework. The two papers in this part of the thesis set fossil fuels aside in favor of a more detailed look the climate system. Both papers are joint work with Christian Traeger. An integrated assessment model embeds a model of the economy in a simplified representation of the climate system, making it possible to analyze the interactions of the two systems and to derive economic climate policy recommendations. Several such models have been used to calculate the optimal global social cost of carbon over the next centuries. The social cost of carbon is the cost that needs to be internalized in order to optimally trade off the benefits of emissions (from production) and the costs they cause in form of climate damages. In principle, the optimal emission path can be achieved by a global carbon tax. While a global carbon tax or any equivalent policy is not realistic any time soon, it is an important benchmark. With few exceptions, these integrated assessment models do not treat uncertainty properly.⁶ They are solved by simultaneous solution methods such as optimal control, making the inclusion of uncertainty in multiple time periods computationally infeasible.⁷ We construct a recursive dynamic programming version of the DICE model by Nordhaus (2008). DICE is possibly the most popular economic integrated assessment model of climate change. The recursive structure enables us to model decision making under uncertainty properly. It allows us to include uncertainty at all future dates when new decisions must be made.

⁶Exceptions are Kelly and Kolstad (1999); Leach (2007); Golosov et al. (2011); Crost and Traeger (2011)

⁷This is in particular true of the most popular and most widely used ones: DICE (Nordhaus, 2008), FUND (Anthoff and Tol, 2010), and MERGE (Richels et al., 2004).

2.1 Growth Uncertainty in the Integrated Assessment of Climate Change (with C. Traeger)

Assuming modest sustained income growth is tantamount to believing that future generations will be far more affluent than today's. This viewpoint is consistent with the experience of the world economy over the last century. However, extrapolating this tremendous growth for another century is a bold assumption. Any economic assessment of long term problems must by necessity be strongly influenced by how it treats economic growth. This is particularly true of the global warming challenge, as the benefits of preventive actions taken today will be reaped over the course of centuries. The DICE-2007 model (Nordhaus, 2008) illustrates the importance of economic growth assumptions well: Even in the absence of climate policy, it suggests that future generations living 100 years from now will be five times richer than today's generation even after the welfare costs of the uncontrolled global warming have been taken into account. In such a setting, any costly climate action today is a redistribution of a wealth from relatively poor current generations to affluent future generations.

In our opinion, this should not be seen as the central issue of climate economics. Growth may slow, and we need to acknowledge the substantial uncertainties in estimates of future economic growth. Take the ongoing economic crisis in Southern Europe for example. It may lead to the current young generation being the first one not to surpass their parents' living standards since World War II. At the same time, hopes are high that several countries in East Africa finally 'catch up', eradicating poverty at a more rapid pace than ever before. In an analytic model Traeger (2010) shows that growth uncertainty can have a major impact on the social discount rate when time and risk preferences are modeled correctly.⁸ But theoretical work alone cannot determine the effect uncertainty has in the more elaborate environment of an integrated assessment model. The model structure is too complex, and a priori one can argue that uncertainty should both in- and decrease the social discount rate.

This paper is the first to consistently analyze how growth uncertainty impacts optimal climate policies in the integrated assessment of climate change. In particular, we ask how optimal abatement effort and the optimal social cost of carbon are impacted by future economic conditions being unknown. In studying how optimal policy with a long time horizon depends on growth uncertainty, it is important to capture both time and risk preferences correctly. People exhibit different behavior when exposed to risk and when making allocations over time. Also from a normative perspective there is no reason why the attitude towards risk and the propensity to smooth consumption over time should coincide in economic analysis (Traeger, 2010). The recursive structure of

⁸The social (or consumption) discount rate is a summary measure of how much a marginal unit of consumption today is valued tomorrow. It hence determines the optimal trade-off between investment costs today and benefits in the future.

the model used permits us to separate concerns about the allocation of consumption over time from risk aversion by using Epstein-Zin preferences. The Epstein-Zin model is a generalization of the common time additive expected utility (EUT) model. The latter expresses time and risk attitudes by one single parameter, whereas the Epstein-Zin model features a parameter for each preference. It is the preferred model in macroeconomics and finance whenever recursive methods are employed. The reason is simple: the model explains observed investment behavior far better. With Epstein-Zin preferences, the actual risk premia and the average market returns can be explained by the same model (Bansal and Yaron, 2004).⁹ The analysis also allows interesting insights in precautionary savings behavior. Most theoretical work discusses savings through the accumulation of manmade capital only. In our model, the decision maker in addition has the ability to save by abating carbon dioxide emissions, thus investing in ‘environmental capital’. Those two types of savings have different characteristics, and their interactions are too complex to analyze theoretically.

We find that uncertainty in the growth rate has a substantial impact on optimal abatement and the social cost of carbon. The size and direction of the effect depend on the specification of the risk and the time preference: If they coincide at the standard values ($RRA = \eta = 2$), the social cost of carbon increases slightly under uncertainty, but the effect is negligible in size. Raising risk aversion ($RRA = 10$) and keeping consumption smoothing constant, increases the effect to modest levels: After 100 years, the abatement rate is approximately 4 percentage points higher. With a lower aversion to intertemporal consumption smoothing ($RRA = 10, \eta = 2/3$), the effect is reversed: Abatement is lower under uncertainty than under certainty.

2.2 Optimally Climate Sensitive Policy (with C. Traeger)

Whereas the greenhouse effect has been well known for decades, the relationship between greenhouse gas (GHG) concentrations in the atmosphere and global average temperatures is still not quantified. The time series observations of both quantities permit for no precise estimate of the so called climate sensitivity.¹⁰ One main reason are the stochastic fluctuations in temperature that add random noise to the relationship. Yet climate sensitivity is at the core of the climate change problem: If GHG emissions cause a strong reaction in temperatures, the costs of climate change are high. If temperatures only react moderately, the benefits of reducing emissions are likely too. In this paper we

⁹The risk premia (the return demanded for an asset’s risk) observed in markets cannot be captured by the standard EUT model without accepting a very strong preference for immediate gratification, i.e. very high interest rates. Such time preferences contradict the observed low risk free market interest rates. These seemingly contradictory observations have been coined the equity premium puzzle and risk free rate puzzle respectively.

¹⁰Climate sensitivity is most commonly defined as the equilibrium response of global average temperatures to a doubling of carbon dioxide concentrations in the atmosphere

analyze how temperature stochasticity, uncertainty about climate sensitivity and learning affect optimal climate policy. In particular, we disentangle the contributions of each of the three phenomena to the overall impact on emission reduction effort. We employ a recursive version of the prominent integrated assessment model DICE by (Nordhaus, 2008). By using an integrated assessment model that is calibrated to economic and climate data, we aim for realistic orders of magnitudes in the effects. We are also able to investigate the interaction between the climate system and the economy, and how the relation influences adjustments in both optimal abatement policies and investment choices. Our knowledge about climate sensitivity is fundamentally different from what we know about temperature stochasticity. Climate sensitivity is an uncertain parameter. Due to the poor data availability, no single, well founded probability distribution exists that describes it. On the contrary, temperature fluctuations can be accurately characterized by statistical methods. We extend our baseline model by allowing for different attitudes to different types of uncertainty. Employing Klibanoff et al. (2009)'s smooth ambiguity model, we let the decision maker be more averse to the subjective climate sensitivity than to temperature volatility. This preference specification is referred to as ambiguity aversion.

Our analysis is closely related to the work by Kelly and Kolstad (1999). They also use a recursive dynamic climate-economy model based on (a more dated) version of DICE to analyze learning about climate sensitivity. They find that learning the true parameter is slow (in the order of 100 years) and that (with reservations for numerical accuracy) the decision maker keeps abatement low to speed up learning about the true value. Their work is constrained by the numerical possibilities of the day and thus does not disentangle stochastic temperature contributions from climate sensitivity uncertainty and learning. They do not consider ambiguity aversion and, from today's perspective the underlying climate model is incorrect, suggesting abatement rates for the coming century of at most 13%. Leach (2007) expands their investigation of learning. His model features a second uncertain parameter, the warming delay. He also focuses on the learning process, in particular errors and the learning speed. His central result is that learning times increase by an order of magnitude when realistically considering multiple parameters of the climate model as uncertain.

We find that temperature stochasticity does not alter optimal abatement but increases investment. Over time the higher capital stock leads to more production and rising emissions. Secondly, climate sensitivity uncertainty causes higher abatement efforts, while investment rates remain unchanged. We confirm that learning by observing temperatures and GHG stocks is too slow to make a relevant contribution to climate policy over the coming decades. Importantly, it is still optimal to increase abatement to insure against the possibility of high damages. Active learning does not take place. Finally, ambiguity aversion has, surprisingly, virtually no impact on optimal decisions.

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Chapter I

Cutting Costs of Catching Carbon: Intertemporal effects under imperfect climate policy (with Michael Hoel)

Abstract

We use a two-period model to investigate intertemporal effects of cost reductions in climate change mitigation technologies for the power sector. With imperfect climate policies, cost reductions related to carbon capture and storage (CCS) may be more desirable than comparable cost reductions related to renewable energy. The finding rests on the incentives fossil resource owners face. With regulations of emissions only in the future, cheaper renewables speed up extraction (the 'green paradox'), whereas CCS cost reductions make fossil resources more attractive for future use and lead to postponement of extraction.

Keywords

climate change, exhaustible resources, carbon capture and storage, renewable energy, green paradox

A revised version of this paper is forthcoming in Resource and Energy Economics.

1 Introduction

Are some ‘climate friendly’ technologies preferable to others? Should policy makers discriminate between supporting renewable energy sources such as wind or solar power and carbon capture and storage (CCS)? Adding to the many conceivable arguments for and against differentiation, we suggest one more: in a world with imperfect climate policies, developing these technologies alters the incentives fossil fuel owners face differently. While cheaper renewables cause extraction to speed up, lower costs of CCS may delay extraction.

Climate change is to be expected as a result of human activity. On aggregate, it will almost certainly affect the human condition adversely. Carbon dioxide emissions from producing power are the single largest contribution to this process. In order to stabilize greenhouse gas (GHG) concentrations in the atmosphere at a level likely to avoid the most harmful damages, emissions need to be reduced and eventually to stop. A concentration of 450 parts per million carbon dioxide-equivalent for example is estimated to give a 50 per cent chance of limiting the rise in global average temperature to 2 degrees Celsius (Solomon et al., 2007). This target temperature would leave about half a trillion tonnes of carbon to be burned (Allen et al., 2009).¹

Quitting emitting GHGs and at the same time securing sufficient energy supplies requires the development and deployment of new, climate friendly technologies. Two promising options are electricity generation from renewable sources such as wind and solar, and carbon capture and storage (CCS). Wind and solar energy are in principle physically available at a sufficient scale to replace fossil fuel power generation (MacKay, 2008). They are however at present not fully competitive,² and various technological challenges remain.³ CCS is a technology under development meant to abate carbon dioxide emissions from large point sources by capturing them and storing them underground.⁴

As using those technologies is more expensive than fossil fuel energy, climate policies are necessary to encourage their deployment. But a comprehensive international agreement to limit GHG concentrations does not exist today.⁵ The best one therefore can expect is a future commitment to limit climate change. This lack of strong climate

¹See also Meinshausen et al. (2009). For an accessible introduction to climate science, see Socolow and Lam (2007).

²Barrett (2009) reports that the best locations are at present competitive if a ton of carbon dioxide is priced at about 35 US Dollars (2006 value).

³Examples are the lack of adequate power storage possibilities, buffering varying wind speeds and sun hours, or the need for distributed transmission networks (Heal, 2009).

⁴So far, no full scale test plants are operating. Golombek et al. (2011) review several studies and find that the most promising types of CCS plants could be competitive at about 30 USD (2007 value) per ton carbon dioxide.

⁵Barrett (2005) has a game theoretical treatment of international climate agreements, while Røgeberg et al. (2010) offer more of a political economy approach.

policy today gives owners of fossil fuels a possibility to sell some of their exhaustible resources prior to climate policies being implemented and climate friendly technologies being competitive. Such intertemporal reallocation undermines policy objectives as more carbon dioxide is emitted in early periods, and potentially total emissions remain unchanged. This supply side effect has become known as the ‘green paradox’ (Sinn, 2008).

The present paper contributes to the literature on fossil fuel supply under imperfect climate policies by focusing on differences in prospective climate friendly technologies. In particular, we ask how reductions in the costs of the abatement technology CCS affect the market outcome. We contrast this with improvements in renewable energy technology. To that end, we build a simple analytical two period model. In period one, emission free technologies play no role, only conventional energy is available. However, actors know about the arrival of alternatives in the second period. By that time, three types of energy technology are available: conventional fossil energy, fossil energy with CCS technology and renewable energy. Fossil fuel suppliers optimize dynamically and sell fossils to conventional and (in period two) CCS power generators. Those sell power competitively in the same market as renewable energy suppliers to energy end users, who are indifferent with regards to the source of their energy. Climate policy is enacted either in both periods (as a first best benchmark) or in the second period only.

We find that with imperfect climate policies, cost reductions related to CCS may be more desirable than comparable cost reductions related to renewable energy. The finding rests on the incentives fossil resource owners face. With regulations of emissions only in the future, cheaper renewables speed up extraction, whereas CCS cost reductions potentially make fossil resources more attractive for future use, leading to postponed extraction.

1.1 Literature

The current work belongs to the literature on the interaction of fossil fuel extraction and climate change. In particular it is part of the so called ‘green paradox’ literature. As it investigates CCS, it also links to economic analyses of this technology in other contexts.⁶

Early work on the interaction of fossil resource extraction and climate change focuses primarily on optimal carbon taxes.⁷ The contributors investigate how varying assumptions on accumulation of pollutants in the atmosphere, damage functions, back-

⁶Various aspects of the CCS technology are analyzed without taking the dynamics of the supply side into account. Cost estimates are established, and the optimal use of the technology is investigated in numerical models, see Al-Juaied and Whitmore (2009); Gerlagh and van der Zwaan (2006); Golombek et al. (2009); İşlegen and Reichelstein (2011); Lohwasser and Madlener (2009).

⁷Sinclair (1994); Ulph and Ulph (1994); Withagen (1994); Hoel and Kverndokk (1996); Tahvonen (1997).

stop technologies, extraction costs, etc. impact the optimal tax. Both rising and falling tax paths are possible, mirroring the net present value of future damages by emissions today. Also the transition to backstop technologies can take different shapes, dependent on cost functions and pollution accumulation. One recent paper in this line of research explicitly considers CCS. Ayong Le Kama et al. (2009) determine cost conditions under which it is optimal to use CCS as an abatement technology and describe the optimal path of usage.

Other contributions relax the assumption of an optimal carbon tax. With varying specifications of climate policy, new trade-offs are added. Of most interest for the current analysis is the work by Chakravorty et al. (2006). They investigate the optimal use of abatement and renewable technologies, given a cap on the total stock of pollution in the atmosphere at any time. Their results are that renewable and fossil energy may be produced simultaneously, while (constant unit cost) abatement is never used before the cap is reached, or at the same time as renewable energy is produced. With a different policy specification, namely a time invariant cap on emissions at each point in time, Smulders and van der Werf (2008) analyze substitution between clean and dirty fossil fuels. They find that such a policy may lead to increased usage of the relatively dirty fossil fuel in early periods. Finally, with a time invariant constant carbon tax, Chakravorty et al. (2011) show that the presence of learning by doing in renewable energy technologies may speed up the extraction of fossil fuels.

While those contributions relax the assumption of optimal balancing of climate damages and abatement costs, they still presume a global policy is in place at all times. However, the assumption of comprehensive climate policy implemented today is hard to defend as descriptive. Another line of research therefore investigates incomplete policies. When climate policy is implemented in the future only, present emissions remain unpriced.⁸ Long and Sinn (1985) investigate reactions of fossil fuel owners to surprise changes in current and expected future prices. Sinn (2008) points explicitly at the role of the supply side in climate policy. He shows that fossil fuel owners respond to taxation by re-allocating extraction over time: If high future taxes are expected, extraction takes place earlier, and, under extreme assumptions, the total amount extracted remains unaltered (the ‘green paradox’).

Di Maria et al. (2008) model multiple fossil resources differing in their carbon content, finding that policy announcements may lead to more extraction of the relatively dirty resource earlier. Van der Ploeg and Withagen (2010) show that expensive but not cheap backstops cause the green paradox to occur. Strand (2007) focuses on climate friendly technologies. He demonstrates that if a technology policy today leads to fossil fuel becoming superfluous in the future and other policies are absent, present carbon

⁸Another imperfection arises when not all judicial entities participate in a climate agreement (carbon leakage), see Eichner and Pethig (2011); Van der Werf (2009).

dioxide emissions will increase. Hoel (fcm) shows that such an effect will also be observed if an incomplete climate policy is in place, even if alternative energy technologies become only marginally cheaper. He illustrates that it is possible for such a technological improvement to lower social welfare.

Several authors have examined conditions under which the green paradox arises. Gerlagh (2011) differentiates between a ‘weak’ (an increase in current emissions) and a ‘strong’ (higher cumulative damages) green paradox. Increasing extraction costs counteract the strong version, while imperfect substitutes counteract both. Independently, Grafton et al. (2010) define a weak green paradox in the same way, while they call a rise in total atmospheric carbon dioxide a ‘strict’ green paradox. They look into effects of biofuel subsidies under both linear and nonlinear demand schedules, and with constant and rising extraction costs. They find numerically that the weak green paradox may arise for a wide range of specifications.

Our contribution is to introduce CCS into the literature on imperfect climate policies. It is fundamentally different from other abatement options such as reduced production or renewable energy in that it requires fossil fuels.

2 The basic model

Consider a two period model⁹ of demand and supply of energy. In the first period conventional energy x_1 is supplied by power generators at price P_1 . The equilibrium condition is

$$x_1 = \tilde{D}_1(P_1) \quad (1)$$

\tilde{D}_1 is the first period demand for energy and has standard properties.

In the second period, energy comes from three sources. In addition to conventional energy x , renewable energy x^{RE} and carbon capture and storage (CCS) energy x^{CCS} are available, so that¹⁰

$$x + x^{CCS} + x^{RE} = D(P) \quad (2)$$

Renewable energy is supplied competitively. Marginal costs are increasing (for example because costs are location dependent), so we have a standard supply function S . The equilibrium condition for renewable energy is

$$x^{RE} = S(P - b) \quad (3)$$

⁹The first period is the near future, when CCS is not yet available on full scale. A rough estimate is 10–20 years. The second period is everything beyond that.

¹⁰The second period time index is dropped for notational convenience.

Here, b is a shift parameter that changes the costs for all units equally.¹¹ We can write $x + x^{CCS} = D(P) - S(P - b)$. The energy market is competitive and thus power suppliers earn zero profits.

The energy price in period one is

$$P_1 = p_1 + \tau_1 \quad (4)$$

Here, p_1 is the price of fossil fuels and τ_1 is a tax levied per unit of carbon.¹² The second period price of conventional energy similarly is (for $x > 0$)

$$P = p + \tau \quad (5)$$

We assume that taxes are not ‘too high’ such that $p > 0$. A zero resource price would mean no resource scarcity, a case we ignore (for further discussion, see Hoel, 2011). If $x^{CCS} > 0$, the price for CCS energy is

$$P = p(1 + \gamma) + c \quad (6)$$

The non-energy costs per unit of CCS energy are c , whereas γ is the extra energy required for CCS (the ‘energy penalty’). We assume that both conventional and CCS energy are supplied. They must trade at the same price, so the conditions (5) and (6) both hold.

There is a fixed, given stock of fossil fuel F which can be extracted at no cost.¹³ Fossil fuel owners maximize over the two periods. Arbitrage requires that the price grows at the rate of interest r . We get the Hotelling rule

$$p = p_1(1 + r) \quad (7)$$

Absent extraction costs, we can conclude that resource owners want to extract everything, so

$$x_1 + x + x^{CCS}(1 + \gamma) = F \quad (8)$$

Equality in (8) is conditional on energy demand being high enough ($P > 0$) and CCS being not too expensive. For given carbon taxes, the equations (1) – (8) determine the four energy quantities (the x ’s) and the four prices (the P ’s and p ’s) for the two periods,

¹¹We thus have a cost function $K(x^{RE}) = bx^{RE} + g(x^{RE})$, $g' > 0$, $g'' > 0$.

¹²We choose units such that one unit of fossil fuel is converted to one unit of conventional energy and causes one unit of carbon emissions.

¹³This assumption will be relaxed in section 6.

for given carbon taxes τ_1 and τ . We assume these taxes are set such that

$$x_1 + x = G \tag{9}$$

where G is a target level for total carbon in the atmosphere.¹⁴ To avoid a trivial problem we assume that $G < F$. Note that without CCS ($x^{CCS} = 0$), (8) and (9) cannot be fulfilled simultaneously. For (9) to hold, taxes must be set such that fossil fuel prices and hence resource rents are zero ($p_1 = p = 0$). Resource owners are then indifferent to extracting or not. With CCS, the amount of CCS energy is determined from (8) and (9) to be $x^{CCS} = \frac{F-G}{1+\gamma}$.

We explore two different scenarios. The first one is optimal taxation. Only GHGs in period two are of concern and they accumulate linearly, so¹⁵

$$\tau = \tau_1(1 + r) \tag{10a}$$

In the second scenario, climate policy is in place only in period two, implying

$$\tau_1 = 0 \tag{10b}$$

Either assumption in combination with the policy target (9) determines the taxes in both periods.

3 A perfect world – taxation in both periods

Now assume the GHG constraint is implemented by a intertemporally cost efficient taxation scheme, i.e. $\tau = \tau_1(1 + r)$. Demand in period one can then be written as $\tilde{D}_1 ([p + \tau](1 + r)^{-1}) = D_1(p + \tau) = D_1(P)$. End users face the same energy price in net present value terms in both periods. The model from section 2 can after some simplifications be written in reduced form

$$F + \gamma G = (1 + \gamma) [D_1(P) + D(P) - S(P - b)] \tag{11}$$

$$p = \frac{P - c}{1 + \gamma} \tag{12}$$

$$\tau = \frac{\gamma P + c}{1 + \gamma} \tag{13}$$

¹⁴Thus we focus on the share of GHGs that can be regarded as remaining in the atmosphere indefinitely, see Socolow and Lam (2007).

¹⁵See for example Hoel and Kverndokk (1996). With no decay of atmospheric carbon and an exogenous limit on the stock, the net present values of marginal abatement costs are equalized. This follows from maximizing the social planner problem $P_1(x_1) + (1 + r)^{-1}[P(x + x^{CCS} + x^{RE}) - cx^{CCS} - PRE(x^{RE})] - \mu(x_1 + x + x^{CCS}(1 + \gamma)) - \tau_1(x_1 + x)$, where PRE is the inverse of S .

What happens to prices and the emission profile if one of the cost parameters c , b or γ is reduced while taxes are adjusted such that the GHG target is still fulfilled? First note that (11) implies that lower non-energy CCS costs c have no effect on the energy price P , even though it is now cheaper to abate. Looking at p and τ ('breaking up' P) helps with the intuition. Knowing that $\frac{dP}{dc} = 0$, one sees from (12) and (13) that

$$\frac{dp}{dc} = -\frac{1}{1+\gamma} \quad (14)$$

$$\frac{d\tau}{dc} = \frac{1}{1+\gamma} \quad (15)$$

The reason is that even though CCS energy is now cheaper, the amount optimally used is unaffected: it is given by the limit on GHGs, the available resource and the technology. But if the amount of CCS energy produced remains unchanged, no adjustments in allocation are desirable and the energy price is the same as before. What happens though is that limiting GHGs gets cheaper and fossil fuel resources become more valuable. Taxes go down and fossil fuel prices increase by the same amount. Some of the economic surplus shifts from the regulator to the fossil fuel owners.

Turning next to the energy cost of CCS γ , the effect on P is obtained from (11). The reactions by p and τ are then retrieved from (12) and (13). Recall that $G = x_1 + x$, $D_1 = x_1$ and $D - S = x + x^{CCS}$. We get

$$\frac{dP}{d\gamma} = \frac{G + S - D_1 - D}{(1+\gamma)[D'_1 + D' - S']} = \frac{-x^{CCS}}{(1+\gamma)[D'_1 + D' - S']} > 0 \quad (16)$$

$$\frac{dp}{d\gamma} = \frac{1}{1+\gamma} \frac{dP}{d\gamma} - \frac{P-c}{(1+\gamma)^2} \quad (17)$$

$$\frac{d\tau}{d\gamma} = \frac{\gamma}{1+\gamma} \frac{dP}{d\gamma} + \frac{P-c}{(1+\gamma)^2} > 0 \quad (18)$$

A decrease in the extra energy required for CCS lowers energy prices, has an ambiguous impact on fossil fuel prices and decreases the carbon tax.

The price of energy for both periods (P) has to go down. Less energy is needed for CCS to reduce GHGs, so more is available to end users. Extraction is in response shifted forward in time. The tax has to be lowered too. Preventing GHG emission has become cheaper, so a lesser opportunity cost is needed. The effect on fuel price is indeterminate. More CCS energy is supplied from the same amount of fossil fuels, lowering demand for fossils. But the fall in energy price means less renewable energy is supplied, increasing demand. After some manipulations of (17) one gets the following condition (recall that $F - G = x^{CCS}(1 + \gamma)$)

$$\frac{dp}{d\gamma} > 0 \Leftrightarrow F - G > (1 + \gamma)(P - c)[S' - D'_1 - D'] \quad (19)$$

Large carbon reserves in the ground (F) work in favor of a dropping fuel price, and so does a strict limit on carbon emissions (G). On the contrary, high CCS energy costs, a high equilibrium fossil fuel price (recall that $P - c = (1 + \gamma)p$), steep demand curves and a steep supply curve for renewables pull in the direction of a rising fuel price as the energy cost (γ) declines.

What are the distributional consequences? Consumer surplus increases, the regulator's revenues decrease while the effect on the Hotelling rent is ambiguous. Owners of renewable power production lose some Ricardian rent.

Finally, for a shift in the cost curve of renewables we get

$$\frac{dP}{db} = -\frac{S'}{D'_1 + D' - S'} \in (0, 1) \quad (20)$$

$$\frac{dp}{db} = \frac{1}{1 + \gamma} \frac{dP}{db} \in \left(0, \frac{1}{1 + \gamma}\right) \quad (21)$$

$$\frac{d\tau}{db} = \frac{\gamma}{1 + \gamma} \frac{dP}{db} \in \left(0, \frac{\gamma}{1 + \gamma}\right) \quad (22)$$

The positive derivative indicates that in response to a lower b , P is again reduced, and also p and τ go down. A source of energy becoming cheaper leads to a falling energy price. As it is a substitute for fossil energy, the derived value of the fossil resource is decreased. And the tax is reduced to make sure that the resulting fall in opportunity costs is reflected. Some of the fossil fuel is re-allocated to the first period ($x_1 = D_1(P), D'_1(P) < 0$). Hotelling rent and regulator revenues are decreased, while consumer surplus and the Ricardian rent for owners of renewable power go up.

Summing up, recall that all adjustments in response to technological changes in the current section are socially cost efficient. Taxation in both periods allows policy makers to price emissions correctly. Table 1 summarizes the results. All improvements lower the carbon tax path: it becomes cheaper for society to 'solve' the climate problem. For renewables and CCS energy costs, the consumers benefit from lower energy prices in both periods. One major difference is how the fossil fuel price is affected by changes in non-energy costs of CCS and more efficient renewables: the former makes a complement to fossil resources in energy production cheaper, the latter a substitute. Total emissions are given exogenously by G . But emissions are accelerated by a lower γ or b and left unchanged by a reduction in c . Some economic rent is shifted from the government (the taxation revenue falls) to resource owners (the Hotelling rent rises) when c is reduced. A lower γ increases consumer surplus, decreases tax revenues and has no conclusive effect on fossil fuel owners. Renewable energy producers lose some Ricardian rent due to the lower energy price. A cut in costs of renewables finally benefits the owners of renewables and the consumers while it reduces the Hotelling rent and the regulator revenues.

Table 1: *Impacts of changes in cost parameters on prices, taxes and emissions in period one under taxation in both periods*

	lower c (CCS non-energy)	lower γ (CCS energy)	lower b (renewable)
P (energy)	0	–	–
p (resource)	+	+/-	–
τ (tax)	–	–	–
x_1 (early emissions)	0	+	+

4 Plan B – fixing it tomorrow

Now suppose a carbon tax is imposed only in the second period, i.e. $\tau_1 = 0$. This represents for example a scenario where in the medium term the major emitting countries agree upon a target level for GHG concentrations. Demand in period one can be written as $\tilde{D}_1(p(1+r)^{-1}) = D_1(p)$. Note that p is now both: the resource price (for both periods due to the Hotelling rule) and the energy price in period one.¹⁶ P is the energy price for period two only. The equilibrium conditions can now be reduced to

$$F + \gamma G = (1 + \gamma) \left[D_1 \left(\frac{P - c}{1 + \gamma} \right) + D(P) - S(P - b) \right] \quad (23)$$

$$p = \frac{P - c}{1 + \gamma} \quad (24)$$

$$\tau = \frac{\gamma P + c}{1 + \gamma} \quad (25)$$

What do market and policy reactions to technological changes look like now? Implicit derivation yields

$$\frac{dP}{dc} = \frac{D'_1}{D'_1 + (1 + \gamma)[D' - S']} \in (0, 1) \quad (26)$$

$$\frac{dp}{dc} = \frac{1}{1 + \gamma} \left(\frac{dP}{dc} - 1 \right) \in \left(\frac{-1}{1 + \gamma}, 0 \right) \quad (27)$$

$$\frac{d\tau}{dc} = \frac{1}{1 + \gamma} \left(\gamma \frac{dP}{dc} + 1 \right) \in \left(\frac{1}{1 + \gamma}, 1 \right) \quad (28)$$

A lower c now decreases second period energy price P . Given standard supply and

¹⁶More precisely, the energy price in the first period is $p(1+r)^{-1}$.

demand function properties the fraction is positive. Intuitively: the amount of CCS used remains the same.¹⁷ To keep conventional energy competitive in period two, the regulator must lower the tax. This makes fossil fuel sales in period two more attractive, fuel prices (and hence energy prices in period one) rise and extraction is postponed.¹⁸

Secondly, changes in the energy requirement of CCS plants (γ) lead to

$$\frac{dP}{d\gamma} = \frac{G + S - D_1 - D + D'_1 \cdot \frac{(P-c)}{1+\gamma}}{D'_1 + (1+\gamma)[D' - S']} = \frac{-x^{CCS} + D'_1 \cdot \frac{(P-c)}{1+\gamma}}{D'_1 + (1+\gamma)[D' - S']} > 0 \quad (29)$$

$$\frac{dp}{d\gamma} = \frac{1}{1+\gamma} \frac{dP}{d\gamma} - \frac{P-c}{(1+\gamma)^2} \quad (30)$$

$$\frac{d\tau}{d\gamma} = \frac{\gamma}{1+\gamma} \frac{dP}{d\gamma} + \frac{P-c}{(1+\gamma)^2} > 0 \quad (31)$$

The cost drop for CCS energy requires that the regulator adjusts the tax downward. The energy price in period two must thus fall, and renewable supplies decrease. Both effects make fossil fuel sales in period two more attractive. But as more energy is derived from the constant amount of fossil fuel used for CCS energy, residual demand drops. Thus the final effect on fossil fuel prices is indeterminate. Manipulating (30) yields a condition for lower energy costs of CCS leading to a drop in fossil fuel price, which is very similar to the one derived in section 3

$$\frac{dp}{d\gamma} > 0 \quad \Leftrightarrow \quad F - G > (1+\gamma)(P-c)(S' - D') \quad (32)$$

Finally, a change in the cost of renewable energy b gives the following changes

$$\frac{dP}{db} = -\frac{(1+\gamma)S'}{D'_1 + (1+\gamma)[D' - S']} \in (0, 1) \quad (33)$$

$$\frac{dp}{db} = \frac{1}{1+\gamma} \frac{dP}{db} \in \left(0, \frac{1}{1+\gamma}\right) \quad (34)$$

$$\frac{d\tau}{db} = \frac{\gamma}{1+\gamma} \frac{dP}{db} \in \left(0, \frac{\gamma}{1+\gamma}\right) \quad (35)$$

The expressions are formally almost identical to those in section 3,¹⁹ but mind the difference in interpretation. A cut in b increases supply of renewable energy, and P drops. For fossil based energy to be sold in period two, p must go down. The reduction is worth γ more to CCS than to conventional power producers, so τ must be reduced too. As p is also the energy price in period one, more fossils are allocated to the first period.

In summary, the policy instrument in this scenario is incomplete: Conventional power

¹⁷The GHG cap G , the resource stock F and the conversion factor γ are unchanged.

¹⁸Also, more fossil energy supply in period two decreases P and lowers x^{RE} .

¹⁹The denominator is by $\gamma D'_1$ smaller.

producers in period one pay a zero carbon tax. Hence from the outset more of the fossil resource than socially optimal is extracted in the first period. Table 2 summarizes the results. All improvements again lower the tax and the second period energy price as it

Table 2: *Impacts of changes in cost parameters on prices, tax and emissions in period one under taxation in period two only*

	lower c (CCS non-energy)	lower γ (CCS energy)	lower b (renewable)
P (price $t = 2$)	—	—	—
p (price $t = 1$)	+	+/-	—
τ (tax)	—	—	—
x_1 (emissions $t = 1$)	—	+/-	+

becomes cheaper to solve the climate problem. Of most interest is the difference in cost cuts in non-energy costs of CCS c and renewables b on the extraction profile: a smaller c shifts extraction to the second period, while lower b does the opposite.

5 Welfare effects of technological changes

In our basic model, earlier extraction is not worse for the climate. Damages are caused by the accumulated carbon stock in period two, which is given. Is it still possible (and likely) that a reduction in c is preferable to a reduction in b due to the intertemporal inefficiency? Yes, provided that the comparison is between parameter changes that give the same total cost reductions. To see this, consider first the welfare effects of an incremental change Δc in the cost of CCS. The total effect on social welfare W (discounted to period 1) is found by differentiating total welfare with respect to c ²⁰

$$\frac{dW}{dc} = -(1+r)^{-1} x^{CCS} + P_1 \frac{dx_1}{dc} + (1+r)^{-1} P \frac{dx}{dc} \quad (36)$$

The first term is the direct cost effect. Initial social CCS costs are cx^{CCS} . The second and third term give the welfare effects of changes in conventional fuel use in period one

²⁰ $W = \int_0^{x_1} P_1(y)dy + (1+r)^{-1} \left(\int_0^{x+x^{CCS}+x^{RE}} P(y)dy - cx^{CCS} - K(x^{RE}) \right)$, where $K(x^{RE}) = bx^{RE} + g(x^{RE})$ are the costs of renewable energy, and $P(y)$, $P_1(y)$ are the inverse demand functions. Noting that $P = K'$ yields the following results.

and two. The changes in these quantities are multiplied by the consumer prices, i.e. the marginal utilities. Changes in the two other energy sources in period two are not included. For renewable energy, consumer price minus marginal costs is equal to zero. Recall that CCS energy use is determined by $x^{CCS} = \frac{F-G}{1+\gamma}$. It thus does not change with changes in c ($dx^{CCS}/dc = 0$). Since $dx/dc = -dx_1/dc$ (from $x_1 + x = G$), we may rewrite this expression as

$$\frac{dW}{dc} = -(1+r)^{-1} x^{CCS} + [P_1 - (1+r)^{-1} P] \frac{dx_1}{dc} \quad (37)$$

In the social optimum (section 3) the term in square brackets is zero, so the total welfare effect consists only of the direct effect $-(1+r)^{-1} x^{CCS} \Delta c$. However, when there is no carbon tax in the first period, the model in section 2 implies²¹ that the term in square brackets equals $(1+r)^{-1} (-\tau)$, giving

$$\frac{dW}{dc} = -(1+r)^{-1} \left[x^{CCS} + \tau \frac{dx_1}{dc} \right] \quad (38)$$

We know that a reduction in c decreases extraction in period 1, i.e. $dx_1/dc > 0$. The second term in square brackets thus adds to the direct positive effect on welfare of reduced costs.

Proceeding in exactly the same way with a change in b , we find

$$\frac{dW}{db} = -(1+r)^{-1} \left[x^{RE} + \tau \frac{dx_1}{db} \right] \quad (39)$$

We know that a reduction in b increases extraction in period one, i.e. $dx_1/db < 0$. The second term in square brackets thus reduces the direct positive effect on welfare of reduced costs. For decreases in c and b that give the same total cost reductions, i.e. $\Delta c x^{CCS} = \Delta b x^{RE}$, it follows that reduced costs of CCS increase welfare more than reduced costs of renewables.

Notice also that the term $[x^{RE} + \tau(dx_1/db)]$ can be negative if x^{RE} is sufficiently small and $S'(C'(x^{RE}) - b) \geq \bar{s} > 0$ for all $x^{RE} \geq 0$ (where $C' - b$ is the marginal cost of renewables). To see this, rewrite dx_1/db

$$\frac{dx_1}{db} = \frac{1}{db} \frac{1}{D'_1} dp = \frac{1}{db} \frac{1}{D'_1} \frac{dp}{db} db \quad (40)$$

which after inserting from (34) gives

$$\frac{dx_1}{db} = \frac{1}{-D'_1} \frac{S'}{D'_1 + (1+\gamma)[D' - S']} \quad (41)$$

²¹Recall that $P_1 = p_1$, $p = p_1(1+r)$ and $P = p + \tau$.

The term dx_1/db will have an upward bound that is below zero provided that $S'(C'(x^{RE}) - b) \geq \bar{s} > 0$ for all $x^{RE} \geq 0$. For sufficiently small values of x^{RE} the term $[x^{RE} + \tau(dx_1/db)]$ must therefore be negative (for $\tau > 0$), implying that social welfare declines as a response to reduced costs of renewable energy. The intuition is that for a sufficiently low initial value of renewable energy, the direct effect of the reduced cost is so small that it is dominated by the indirect negative welfare effect of reallocating extraction from the future to the present.

6 The general model

The model is so far quite rigid. First, it has been assumed that there are no residual emissions from CCS power stations. However, CCS is expected to remove only about 90 per cent of carbon dioxide emissions (IPCC, 2005). Second, we simplified by assuming a fixed amount of fossil fuel resources available, all of which can be extracted at the same constant unit cost (set to zero). Third, in the basic model the level of GHG concentration is exogenous and the timing of emissions is irrelevant. We now relax all of these assumptions for the case of taxation in period two only ($\tau_1 = 0$). We show that improvements in renewables still speed up extraction, while cheaper CCS slows it down.

6.1 Extensions

Equations (1) – (5) from the model in section 2 remain unchanged

$$x_1 = \tilde{D}_1(P_1) \quad (1^*)$$

$$x + x^{CCS} + x^{RE} = D(P) \quad (2^*)$$

$$x^{RE} = S(P - b) \quad (3^*)$$

$$P_1 = p_1 \quad (4^*)$$

$$P \geq p + \tau \quad (= p + \tau \quad \text{if } x > 0) \quad (5^*)$$

The extensions concern the equations (6) – (10). First, the price of CCS energy has to account for *residual emissions*. Let producing one unit of CCS energy cause δ units of emissions ($\delta \in (0, 1)$). CCS energy producers now pay $\tau\delta$ in carbon tax per unit of energy. The new price of CCS energy is

$$P \geq p(1 + \gamma) + c + \tau\delta \quad (6^*)$$

Secondly, *extraction costs* change the price of fossils. A complete formal treatment of extraction costs is deferred to appendix A. We assume that the extraction costs are

independent of the extraction rate, but increase with accumulated extraction.²² To simplify, extraction costs are assumed zero for all extraction up to a level f which is larger than the equilibrium extraction in period one. Beyond that level, extraction costs are positive and rising. We call the costs in period two $A(F)$. With those changes, the Hotelling rule remains valid, but the second period resource price is determined by marginal extraction costs

$$A'(F) = p = (1 + r)p_1 \tag{7*}$$

The total amount extracted F becomes endogenous. It is a function of the resource price, $F(p)$ with $F' > 0$. The case treated previously was the limiting case of $F' = 0$. The constraint on total extraction (8) hence becomes

$$x_1 + x + x^{CCS}(1 + \gamma) = F(p) \tag{8*}$$

G now includes residual emissions from CCS energy production, so

$$x_1 + x + \delta x^{CCS} = G \tag{9*}$$

The amount of CCS used is now $x^{CCS} = \frac{F-G}{1+\gamma-\delta}$. Finally, total emissions G become endogenous by including *climate costs*. According to Allen et al. (2009), the peak temperature increase is approximately insensitive to the timing of emissions. However, we would expect this peak temperature increase to occur earlier the more of the emissions occur at an early stage. It also seems reasonable to expect climate costs to be higher the more rapidly the temperature increases. We therefore model climate costs as increasing in the two variables G, x_1 . To simplify, but without changing anything of substance, we assume that the climate cost function is given by $E(G + \sigma x_1)$, where E' and σ are positive. The optimal (Pigovian) carbon tax in period two thus is

$$\tau = E'(G + \sigma x_1) \tag{10b*}$$

This gives G as $G = E'^{-1}(\tau) - \sigma x_1 = \Lambda(\tau) - \sigma x_1$.²³

6.2 Results

As the calculations follow the same steps for the basic model, we defer most of them to appendix B. Again, after some calculation the model can be written in reduced form

²²The basic model in section 2 treated the limiting case, where we assumed constant (zero) unit cost of extraction combined with an absolute upper limit on accumulated extraction.

²³Note that $x_1 = D_1(p)$ as there is no carbon tax in period one. If taxes were set optimally also in period one, x_1 would depend on this tax. If $\sigma = 0$ as before $\tau_1 = (1 + r)^{-1}\tau$. However, if $\sigma > 0$, the optimal tax in period one would be higher.

as three equations in three unknowns (P, p, τ) . We focus on the effects of reducing the CCS non-energy costs c and the costs of renewables b . We show in appendix B that

$$\frac{dp}{dc} < 0 \quad (42)$$

$$\frac{dp}{db} > 0 \quad (43)$$

The first period energy price ($= P_1 = p_1 = (1+r)^{-1}p$) increases with cheaper CCS and falls with cheaper renewables. In response to the higher price, demand falls, thus making extraction go down in period one (and vice versa). Thus the main result obtained from the basic model is robust to the extensions introduced here: While lower costs for renewables aggravate the intertemporal inefficiency caused by taxation being unavailable in period one, lower costs of CCS dampen it. Overall, for changes in b , the generalizations do not change the results found in the basic model. The direction of effects on prices and carbon tax remain the same, as summarized in table 3. For changes in c , we find

Table 3: *Impacts of changes in cost parameters in the extended model under taxation in period two only*

	lower c (CCS non-energy)	lower b (renewable)
P (price $t = 2$)	+/-	-
p (price $t = 1$)	+	-
τ (tax)	-	-
x_1 (emissions $t = 1$)	-	+
F (extraction)	+	-
G (total emissions)	+/-	-
E (climate damage)	-	-

one relevant difference between the basic and the general model: In the basic model, we found $dP/dc < 0$, while the sign of dP/dc now is indeterminate. It is positive if

$$(1 - \delta)F' - [1 + \gamma(1 + \sigma) - \delta]D'_1 > \gamma\Lambda' \quad (44)$$

In comparison, in the original model the effect was always positive. We observe that the ambiguity stems from the endogenization of the GHG cap G (in the original model we had $\Lambda' = 0$), whereas extraction costs of the form $A(F)$ in period two do not change the qualitative result from the original model.

We may now also ask what happens to the now endogenous total extraction F , stock of carbon in the atmosphere G and climate damage E . Firstly, since $F'(p) > 0$, it follows, not surprisingly, that total extraction goes up with cheaper CCS. Second, the total stock of GHGs is determined by $G = \Lambda(\tau) - \sigma D_1(p)$. As we assume $\sigma > 0$, a reduction in b decreases G (both p and τ fall), while a reduction in c has an ambiguous effect (due to p rising). Less early emissions decreases the additional harm they cause (as expressed by $\sigma > 0$), hence opening up for increased total emissions.²⁴ The effect on marginal climate costs can be read out of (10b*): $\tau = E'(G + \sigma x_1)$. As τ falls in response to a lower b or c , so must E' . Since E is increasing and strictly convex, a lower derivative indicates lower total climate costs. Results are summarized in table 3.

6.3 Welfare effects

Also the welfare analysis from the basic model carries over to the more general version: A cost reduction in c is preferable to a reduction in b if they provide the same total cost reductions. In appendix C we derive the following equation

$$\frac{dW}{dc} = -\frac{1}{1+r} E'(1+\sigma) \frac{dx_1}{dc} + x^{CCS} \quad (45)$$

The two terms on the right hand side have the same sign for a lower c : x^{CCS} is positive, and x_1 decreases in response to lower CCS non-energy costs. Differentiating with respect to b yields

$$\frac{dW}{db} = -\frac{1}{1+r} E'(1+\sigma) \frac{dx_1}{db} + x^{RE} \quad (46)$$

For lower costs of renewables b , the two terms on the right hand side have opposing signs: x_1 goes up in response to lower costs of renewables. So while lower costs of renewables have a beneficial effect for inframarginal units of renewable energy, it worsens the intertemporal misallocation provoked by the policy imperfection. Lower CCS costs c on the contrary have a beneficial effect on the imperfection. For parameter changes that give the same total cost reductions ($\Delta c x^{CCS} = \Delta b x^{RE}$) we see that a cost reduction in CCS is preferable.

²⁴We regard this however not as a very likely outcome.

7 Concluding remarks

We set out to analyze how an improvement in CCS technology influences energy and fossil fuel prices and the timing of GHG emissions, and how it compares to a downwards shift in renewable energy costs. We used a simple two period model that links a market for some stylized fossil fuel to a market for energy. One robust result is that all types of technological improvement give a lower optimal carbon tax in period two. Other effects of technological improvements depend both on the type of technological improvement and on whether climate policy is optimally designed in both periods or only in period two. Key results are summarized in tables 1 to 3. One important conclusion is that if there is no carbon tax in period one, lower non-energy costs for CCS have the opposite effect on period one emissions of lower costs of renewable energy. This is an important difference, as emissions are too high in period one when there is no carbon tax in this period. We showed that the increase in period one emissions resulting from reduced costs of renewable energy might even lead to lower social welfare. A lower non-energy cost of CCS will decrease period one emissions, and therefore always increase social welfare.

As for policy implications, under specific circumstances supporting the development of CCS is preferable from supporting renewables. What are these circumstances? One has to believe that a future climate policy will come into being. Second, fossil fuel producers' ability to reallocate production needs to be large enough for the effect to matter. Also, both technologies are assumed to be available on sufficient scale at the same time. If renewables are ready earlier (or later), the picture changes. And if one believes that climate policy will not even be implemented in the future, supporting renewables may be preferable for another reason: they at least potentially can compete with conventional energy, while CCS will always impose an additional cost.

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Appendix

A Extraction costs

Assume that the extraction costs are independent of the extraction rate, but increase with accumulated extraction. Total extraction F becomes endogenous.²⁵ Formally, we let each unit of the resource be indexed by a continuous variable z , and let $a(z)$ be the cost of extracting unit z , with $a' \geq 0$. In the two-period model x_1 is extraction in period one and $x + x^{CCS}(1 + \gamma) = F - x_1$ is extraction in period two. The cost of extracting x_1 is thus given by $A(x_1) = \int_0^{x_1} a(z)dz$, and cost of extracting $F - x_1$ is $\int_{x_1}^F a(z)dz = \int_0^F a(z)dz - \int_0^{x_1} a(z)dz = A(F) - A(x_1)$. Notice that these relationships imply that $A'(x_1) = a(x_1)$ and $A'(F) = a(F)$. The limiting case of a constant unit cost a of extraction up to an exogenous limit \bar{F} would imply that $A(x_1) = ax_1$ and $A(F) - a(x_1) = a \cdot (F - x_1)$ (up to \bar{F}).

We now simplify and assume that extraction costs are zero for all extraction up to a level f which is larger than the equilibrium extraction in period one, so that $A(x_1) = 0$ for all relevant values of x_1 in period one. Moreover, let extraction costs $a(F)$ be positive and rising for extraction levels above f , so that costs in period two are $A(F)$ which is rising for $F > f$ and strictly convex.

B Solving the general model

With the changes from section (6) the reduced form model is

$$\begin{aligned} [1 - \delta]F \left(\frac{(1 - \delta)P - c}{1 + \gamma - \delta} \right) + \gamma\Lambda \left(\frac{\gamma P + c}{1 + \gamma - \delta} \right) \\ = (1 + \gamma(1 + \sigma) - \delta)D_1 \left(\frac{(1 - \delta)P - c}{1 + \gamma - \delta} \right) + (1 + \gamma - \delta) [D(P) - S(P - b)] \end{aligned} \quad (\text{B.1})$$

$$p = \frac{(1 - \delta)P - c}{1 + \gamma - \delta} \quad (\text{B.2})$$

$$\tau = \frac{\gamma P + c}{1 + \gamma - \delta} \quad (\text{B.3})$$

Implicit differentiation of (B.1) with respect to b gives

$$\frac{dP}{db} = \frac{-(1 + \gamma - \delta)^2 S'}{[1 + \gamma(1 + \sigma) - \delta](1 - \delta)D_1' + (1 + \gamma - \delta)^2 [D' - S'] - (1 - \delta)^2 F' - \gamma^2 \Lambda'} \quad (\text{B.4})$$

The expression is positive (note that $\Lambda' > 0$ and $F' > 0$) and bounded above by one. This mirrors the directions of the effects in the basic model. Likewise do the effects on

²⁵This is a specification frequently used in the resource literature, see e.g. Heal (1976) and Hanson (1980).

p and τ which can be obtained by inserting (B.4) in differentials of equations (B.2) and (B.3) respectively. Again, the first period energy (and resource) price p goes down in response to a reduction of the costs of renewables, and so does the tax τ . Differentiating with respect to c we get

$$\frac{dP}{dc} = \frac{[1 + \gamma(1 + \sigma) - \delta]D'_1 - (1 - \delta)F' + \gamma\Lambda'}{[1 + \gamma(1 + \sigma) - \delta](1 - \delta)D'_1 + (1 + \gamma - \delta)^2 [D' - S'] - (1 - \delta)^2 F' - \gamma^2 \Lambda'} \quad (\text{B.5})$$

The effect on the second period price is ambiguous as the numerator contains negative as well as positive terms. It is positive if

$$(1 - \delta)F' - [1 + \gamma(1 + \sigma) - \delta]D'_1 > \gamma\Lambda' \quad (\text{B.6})$$

The change in p can again be calculated from (B.2)

$$\frac{dp}{dc} = \frac{1}{1 + \gamma - \delta} \left[(1 - \delta) \frac{dP}{dc} - 1 \right] \quad (\text{B.7})$$

The effect is negative: If $\frac{dP}{dc}$ is negative, it follows immediately. If $\frac{dP}{dc}$ is positive, it needs to be true that $(1 - \delta) \frac{dP}{dc} < 1$ for the effect to still be negative, so we require that

$$\begin{aligned} & [1 + \gamma(1 + \sigma) - \delta](1 - \delta)D'_1 - (1 - \delta)^2 F' + (1 - \delta)\gamma\Lambda' \\ & > [1 + \gamma(1 + \sigma) - \delta](1 - \delta)D'_1 + (1 + \gamma - \delta)^2 [D' - S'] - (1 - \delta)^2 F' - \gamma^2 \Lambda' \end{aligned} \quad (\text{B.8})$$

Simplifying

$$[(1 - \delta)\gamma + \gamma^2]\Lambda' > (1 + \gamma - \delta)^2 [D' - S'] \quad (\text{B.9})$$

The last equation is always true, as the left hand side contains only positive terms and the right hand side only negative ones. So the first period energy price rises in response to a fall in non-energy costs of CCS. Thus the main finding of the original model is robust towards the discussed generalizations.

The effect on the carbon tax τ is (from B.3)

$$\frac{d\tau}{dc} = \frac{1}{1 + \gamma - \delta} \left(\gamma \frac{dP}{dc} + 1 \right) \quad (\text{B.10})$$

We see that if $\gamma \frac{dP}{dc} > -1$, then like in the original model the tax decreases in response to lower non-energy costs of CCS. Inserting and rearranging

$$\begin{aligned} & \gamma(1 + \gamma(1 + \sigma) - \delta)D'_1 - \gamma(1 - \delta)F' \\ & < -[1 + \gamma(1 + \sigma) - \delta](1 - \delta)D'_1 - (1 + \gamma - \delta)^2 [D' - S'] + (1 - \delta)^2 F' \end{aligned} \quad (\text{B.11})$$

(B.11) shows that the inequality always holds. The LHS contains only negative terms, the RHS only positive ones. In line with the original model, the tax decreases in response to lower non-energy costs of CCS.

C Welfare effects in general model

Welfare is defined as

$$W = \int_0^{x_1} P_1(y)dy + \frac{1}{1+r} \left[\int_0^{x+x^{CCS}+x^{RE}} P(y)dy - cx^{CCS} - bx^{RE} - g(x^{RE}) - A(x_1 + x + (1+\gamma)x^{CCS}) - E((1+\sigma)x_1 + x + \delta x^{CCS}) \right] \quad (C.1)$$

Differentiating with respect to c

$$\begin{aligned} \frac{dW}{dc} = & P_1(x_1) \cdot \frac{dx_1}{dc} + \frac{1}{1+r} \left[P \cdot \left(\frac{dx}{dc} + \frac{dx^{CCS}}{dc} + \frac{dx^{RE}}{dc} \right) - x^{CCS} - c \frac{dx^{CCS}}{dc} \right. \\ & - b \frac{dx^{RE}}{dc} - g'^{RE} - A' \cdot \left(\frac{dx_1}{dc} + \frac{dx}{dc} + (1+\gamma) \frac{dx^{CCS}}{dc} \right) \\ & \left. - E' \cdot \left((1+\sigma) \frac{dx_1}{dc} + \frac{dx}{dc} + \delta \frac{dx^{CCS}}{dc} \right) \right] \end{aligned} \quad (C.2)$$

Rearranging some and noting that $A' = p = (1+r)p_1$, $E' = \tau$ and using equations (5*) and (6*) we get the result in equation (45)

$$\frac{dW}{dc} = -x^{CCS} - \frac{1}{1+r} E'(1+\sigma) \frac{dx_1}{dc}$$

Differentiating with respect to b and proceeding in the same way as above yields equation (46)

$$\frac{dW}{db} = -x^{RE} - \frac{1}{1+r} E'(1+\sigma) \frac{dx_1}{db}$$

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Chapter II

Carbon Tax Uncertainty, Fossil Energy and Green R&D

Abstract

Using an analytical model, I investigate how uncertainty about future carbon tax levels influences decisions to invest in green innovation and to extract scarce fossil resources. I distinguish between two sources of uncertainty: Scientific uncertainty about the severity of climate change impacts, and uncertainty about future political processes. When future policies are uncertain, the present regulator attempts to impose her preferences on future decision makers by her choices. When investment and extraction decisions are considered independently, I find that the two sources of uncertainty have opposing effects: Scientific uncertainty delays fossil fuel extraction and increases green R&D, whereas political uncertainty has the opposite effect. Most importantly, I find that neither source of uncertainty leads to unambiguous changes in extraction or investment when those decisions are considered simultaneously.

Keywords

climate change, renewable energy, exhaustible resources, green R&D, uncertainty, learning, commitment

1 Introduction

How does uncertainty about future carbon taxes impact investment in renewable energy technologies and extraction of fossil resources? Does it matter whether the uncertainty reflects our limited knowledge about the climate system or whether it derives from political processes? Considering investment and extraction decisions separately, I find that scientific uncertainty delays extraction of fossil fuels and increases investment in renewables technologies. Political uncertainty on the contrary leads to faster extraction and less innovation investments. Considered simultaneously, both sources of uncertainty have ambiguous effects on both green investments and the extraction profile for fossil fuels.

Uncertainty is a central feature of the climate change challenge. There are limits to our understanding of the global warming impact of atmospheric greenhouse gas concentrations. Millner et al. (2010) distinguish between two types of scientific knowledge: Scientific principles and physical laws which may be well known, and empirical knowledge, which is very limited. The climate system is so complex that it is difficult to estimate single causal relations with a high degree of confidence. Politics are an obvious second source of uncertainty: To what extent will nations cooperate and cap global emissions? As climate change threatens to cause catastrophic outcomes (Lemoine and Traeger, 2010), it seems reasonable to expect stringent climate policy in response. Existing climate policies however seem more in line with models assuming purely self-interested, non-cooperative behavior (Anthoff, 2011). The unwillingness of most nations to enter a binding agreement on greenhouse gas emissions has consequently been explained as a manifestation of the tragedy of the commons (Barrett, 2005). Since international cooperation on climate change so far has failed to substantially reduce greenhouse gas emissions, it is a priori equally reasonable to expect weak climate policies in the future.

In this paper I investigate how these two sources of uncertainty impact fossil fuel extraction and R&D investments in renewable energy. I assume that the energy market today is supplied by scarce fossil fuels. Investments can be undertaken to make new, non-polluting energy sources available in the future. I ask how the two types of uncertainty affect incentives of market actors and regulators. I assume that scientific uncertainty is perceived equally by current and future policy makers. Political uncertainty stems from successive regulators having differing ‘preferences’ for the climate.¹ Current regulators may attempt to manipulate the options available to future regulators, who may aim for more or less global warming.

¹I remain deliberately unspecific about the source of those differences since it is not the focus of the current work. A future government may have a different constituency with different interests for example. If one stresses the global nature of climate change instead, the outcome of future climate negotiations is highly uncertain for today’s national decision makers. See for example the non-binding COP17 agreement to reach a binding agreement by 2015 to be implemented by 2020.

The three papers most closely related to my work are Ulph and Ulph (2011), Hoel (2012a) and (Hoel, 2012b). Ulph and Ulph (2011) investigate a situation where a government cannot set future taxes but decides on R&D in renewable energy. They find that uncertainty about future taxes affects the incentives of the current government and clean technology investors in opposite directions: Whereas investors want to reduce investments, governments would like to see them increased. Their model, however, does not feature a non-renewable resource. Hoel (2012b) investigates political and scientific uncertainty in a setting without resources and with investment stock of green physical capital rather than R&D. Finally, Hoel (2012a) adds a non-renewable resource and studies the effect of expected carbon tax rates, but his analysis does not treat uncertainty explicitly. The present work expands on these by going beyond their two-period models and treating the two types of uncertainty explicitly within a fully dynamic model. In the wider sense this work relates to the “Green Paradox” literature (Sinn, 2008). In fossil fuel markets, a known future carbon tax leads to adjustments of the extraction profile that may counteract or even cancel the intent of the climate policy, dependent on extraction costs, tax level and other parameters (Gerlagh, 2011). The same may be true for anticipated renewable energy technologies (Hoel and Jensen, 2010). More remotely related is the literature on incentives to invest in new green technology (Requate, 2005), in particular the work on commitment problems: An ex ante optimal policy to elicit investment in renewable energy is, after the investment is undertaken, not suited to disseminate the technology in a socially efficient manner. Hence the regulator faces a commitment problem. Optimal solutions may therefore require multiple policy instruments (Montero, 2010; Scotchmer, 2010).

The paper proceeds with section 2, which introduces the model and analyzes scientific uncertainty. For expositional reasons, I cast it in terms of a social planner. Section 3 amends the model to accommodate political uncertainty. In section 4 I lay out how the social planner solutions can be implemented by carbon taxes in a market, before section 5 concludes. Some details of the necessary calculations are left to the Appendix A, while some required numerical simulations are found in Appendix B.

2 Scientific uncertainty

A social planner is concerned about welfare, derived from consuming energy and reduced by climate damages from fossil fuel combustion. She chooses an extraction path for a fossil resource and how much to invest in renewable energy technology. Social welfare is determined by energy consumption and climate damages. It is increasing and concave in energy consumption ($U' > 0, U'' < 0$). I distinguish two large scale sources of energy: Fossil fuel and renewable energy. A fixed amount of fossil fuel R_0 is available at the beginning of the planning horizon (measured in energy units). Each point in time,

$x(t)$ units are extracted. Extraction costs are constant, and for simplicity set to zero. Renewable energy y_t has constant unit cost $b(I)$ which depend on the level of R&D effort I prior to the exogenous time T .² I assume that $b(I)$ is decreasing and convex in innovation effort ($b'(I) < 0, b''(I) > 0$). I distinguish two cases of information. First I assume the damages are known at all times, and that the present social planner evaluates them with v . Second, I assume they are uncertain until T when they become known. There are two possible sources of uncertainty concerning damages. First, the social planner does not know for sure what the damages are, but learns so at T . Call this scientific uncertainty. Alternatively, I assume that the current social planner is not in charge in the future (from T onwards). The future social planner's evaluation of climate change V is not known today. Call this political uncertainty.

2.1 Formal setup for scientific uncertainty

The social planner facing scientific uncertainty has the following objective:

$$\begin{aligned} \max_{\{x_t, y_t\}_0^\infty, I} \mathbb{E} \left[\int_0^\infty (\mathcal{U}(x(t) + y(t)) - b(I)y(t) - \tilde{v}x(t)) e^{-rt} dt \right] - I \\ \text{s.t. } \dot{R}(t) = -x(t), \quad R(0) \text{ given .} \end{aligned}$$

The evaluation of climate damages \tilde{v} is unknown before T and known thereafter. I simplify the analysis by splitting the decision problem into two periods, referring to the time before and after the resolution of uncertainty (see Hoel (1978))

$$\max_{S, I} \{g(S) - I + \mathbb{E}G(S, b(I), \tilde{v})\} \quad (1)$$

where

$$g(S) - I = \max_{\{x_t\}} \int_0^T (\mathcal{U}(x(t)) - \mathbb{E}\tilde{v}x(t)) e^{-rt} dt - I \quad (2)$$

$$\text{s.t. } \dot{R}(t) = -x(t), \quad R(0) \text{ given } \quad R(T) = S$$

$$G(S, b(I), v) = \max_{\{x_t, y_t\}} \int_T^\infty (\mathcal{U}(x(t) + y(t)) - b(I)y(t) - vx(t)) e^{-rt} dt \quad (3)$$

$$\text{s.t. } \dot{R}(t) = -x(t), \quad R(T) = S$$

Only two decisions remain to be made under uncertainty: Investment in renewables technology I , and how much of the resource to leave for the time after the uncertainty is resolved, S (equation 1). The maximization in (2) is carried out under certainty: The best the social planner can do is optimize with respect to the expected value of the

² I can be thought of as the net present value of R&D costs during that period. That investment takes place only prior to T is a convenient simplification.

damages. It is a standard Hotelling problem for a given amount of fossil fuels $R(0) - S$. Marginal utility must increase over time at the social discount rate. Call the optimal value of maximization problem (2) $g(S) - I$. Likewise, the optimization problem (3) is equivalent to a standard nonrenewable resource problem with a backstop. After learning the true value of v , the social planner re-optimizes. Again fossil fuel extraction follows a Hotelling path. Only when the resource is exhausted at date τ will the green alternative be used. (see Appendix A for more detail). Call the optimal value of maximization problem (3) $G(S, b(I), \tilde{v})$.

2.2 Analysis of scientific uncertainty

Comparing the first order conditions for (1) under certainty and uncertainty isolates the consequences of scientific uncertainty. When v is known from the beginning the first order conditions read

$$g_S(S) + G_S(S, b(I), v) = 0 \quad (4)$$

$$-1 + G_b(S, b(I), Q)b'(I) = 0 . \quad (5)$$

Under scientific uncertainty (\tilde{v}) I get

$$g_S(S) + E[G_S(S, b(I), \tilde{v})] = 0 \quad (6)$$

$$-1 + E[G_b(S, b(I), \tilde{v})]b'(I) = 0 . \quad (7)$$

I assume that $E\tilde{v} = v$. By Jensen's inequality we know that the impact of introducing uncertainty in \tilde{v} on S and I depends on whether G_S and G_b are convex or concave in \tilde{v} .³ G_S and G_b are given by ⁴

$$G_S = \mu(T) = (b - v)e^{-r\tau(v)} > 0 \quad (8)$$

$$G_b = -\frac{y}{r}e^{-r\tau(v)} < 0 , \quad (9)$$

where the date of exhaustion $\tau(v)$ is defined implicitly by

$$\int_0^\tau \mathcal{U}^{(-1)}(v + e^{-r(\tau-t)}(b - v)) dt = S . \quad (10)$$

The signs of G_{Svv} and G_{bvv} cannot be determined analytically. I hence proceed by numerically simulating those two derivatives. Details on the simulation procedure are found in Appendix B. I find that under a wide range of assumptions G_S is convex and G_b is concave in damages ($G_{Svv} > 0$ and $G_{bvv} < 0$).

³For an *increase* in risk in \tilde{v} one would need the equivalence result by Rothschild and Stiglitz (1970).

⁴See Appendix A for all detailed calculations.

Proceeding under this assumption and making use of Jensen's inequality, I simplify the expressions by denoting the difference between the value functions for $[T, \infty)$ under certainty and uncertainty as $\omega, \xi > 0$.

$$G_S(S, b(I), E\tilde{v}) < E[G_S(S, b(I), \tilde{v})] = G_S(S, b(I), E\tilde{v}) + \omega \quad (11)$$

$$G_b(S, b(I), E\tilde{v}) > E[G_b(S, b(I), \tilde{v})] = G_b(S, b(I), E\tilde{v}) - \xi \quad (12)$$

This gives us the first order conditions for the case of scientific uncertainty (equations (6) and (7)) as

$$g_S(S) + G_S(S, b(I), E\tilde{v}) + \omega = 0 \quad (13)$$

$$-1 + [G_b(S, b(I), E\tilde{v}) - \xi] b'(I) = 0. \quad (14)$$

Differentiating (13) and (14) yields the effects of introducing uncertainty on the optimal choices S and I . Under slight abuse of notation, I will use $d\xi = \xi$ and $d\omega = \omega$ since under certainty $\omega = \xi = 0$.

$$\begin{aligned} g_{SS} dS + G_{SS} dS + G_{Sb} b' dI + \omega &= 0 \\ G_{Sb} b' dS + (G_{bb}(b')^2 + G_b(b'')) dI - b' \xi &= 0 \end{aligned}$$

Rearranging and writing in matrix form

$$\begin{pmatrix} g_{SS} + G_{SS} & G_{Sb} b' \\ G_{Sb} b' & G_{bb}(b')^2 + G_b(b'') \end{pmatrix} \cdot \begin{pmatrix} dS \\ dI \end{pmatrix} = \begin{pmatrix} -\omega \\ b' \xi \end{pmatrix}$$

The first matrix is the Hessian matrix. Call it \mathcal{M} . Making use of Cramer's rule, I find that dS is

$$\begin{aligned} dS &= \frac{\begin{vmatrix} -\omega & G_{Sb} b' \\ b' \xi & (G_{bb}(b')^2 + G_b(b'')) \end{vmatrix}}{|\mathcal{M}|} \\ &= \frac{1}{|\mathcal{M}|} \underbrace{(G_{bb}(b')^2 + G_b(b''))}_{-} \underbrace{(-\omega)}_{-} - \underbrace{(G_{Sb} b')}_{-} \underbrace{(b' \xi)}_{-} \end{aligned} \quad (15)$$

Proceeding the same way for dI

$$\begin{aligned}
 dI &= \frac{\begin{vmatrix} g_{SS} + G_{SS} & -\omega \\ G_{Sb}b' & b'\xi \end{vmatrix}}{|\mathcal{M}|} \\
 &= \frac{1}{|\mathcal{M}|} \underbrace{(g_{SS} + G_{SS})}_{-} \underbrace{(b'\xi)}_{-} - \underbrace{(-\omega)}_{-} \underbrace{(G_{Sb}b')}_{-}
 \end{aligned} \tag{16}$$

For both dS and dI the sign of the effect is ambiguous. To see that, note that the determinant of the Hessian Matrix in the denominator must be positive in a local optimum. Further, by assumption, $b' < 0$ and in Appendix A I show that $G_{Sb} > 0$. Finally, as the Hessian is negative semi-definite, the elements on the main diagonal are negative, i.e. $g_{SS} + G_{SS} < 0$ and $G_{bb}(b')^2 + G_b(b'') < 0$. Without more specific assumptions, for example a simple calibration of the model to real world data, it is not possible to say how a social planner should adjust to scientific uncertainty. To understand how the result comes about, I now look at each decision separately.

2.3 Special cases: Exogenous investment or extraction

If investment in green innovation is exogenously given, the effect of uncertainty on early extraction $R_0 - S$ is found from the equation (12) alone

$$dS = \frac{-\omega}{g_{SS}(S) + G_{SS}(S, b(\bar{I}))} > 0$$

For $\omega > 0$, early extraction decreases ($dS > 0$). This result is in line with findings in the earlier resource literature (Hoel, 1978; Dasgupta and Stiglitz, 1981). Intuitively, it is beneficial to wait with the extraction until a decision can be taken based on more information.

Secondly, if resource extraction is exogenous, we get the effect of uncertainty on investment in renewable technology I from (13)

$$dI = \frac{\xi b'}{G_{bb}(b')^2 + G_b(b'')} > 0.$$

The social planner invests more in renewable resource technologies when she is uncertain about future climate damages. Intuitively, she insures herself against an undesirable outcome by having more of the clean substitute available.

Returning to the case of joint investment and extraction, we observe the following interaction between the two decisions: *Ceteris paribus* uncertainty means more investment in renewable technology. But lower costs of renewable energy in the future depress the profitability of fossil fuels. Hence more is extracted earlier, counteracting the effect

uncertainty has on extraction. Likewise, given that uncertainty *ceteris paribus* implies that more of the fossil resource is available in the future, investment in renewable energy technology becomes less attractive, which counteracts the original effect uncertainty has on investments.

3 Political uncertainty

So far I assumed that the uncertainty was caused by initial lack of knowledge about the climate system. Now I consider political processes as a second possible source of uncertainty. I assume that the current social planner is uncertain about a future planner's evaluation of climate damages. Again she can affect future outcomes in two ways: Via lowering the costs of renewables by investing (I), and via the supply of fossil fuel (S).

3.1 Formal setup for political uncertainty

Modeling political uncertainty requires the introduction of a second social planner who optimizes after T . The social planner prior to T now knows her own valuation v but not the future planner's evaluation V . I assume that she has no systematic bias, so her expectation is $E\tilde{V} = v$. The social planner anticipates the behavior of her successor when optimally choosing extraction and investment

$$\max_{S,I} \left\{ g(S) - I + EG^1(I, b(I), v, \tilde{V}) \right\} \quad (17)$$

where

$$g(S) - I = \max_{\{x\}} \int_0^T (\mathcal{U}(x(t)) - vx(t)) e^{-rt} dt - I \quad (18)$$

$$\text{s.t. } \dot{R}(t) = -x(t), \quad R(0) \text{ given } R(T) = S$$

$$G^1(S, b(I), v, \tilde{V}) = \int_T^\infty (\mathcal{U}(\hat{x}(t) + \hat{y}(t)) - b(I)\hat{y}(t) - v\hat{x}(t)) e^{-rt} dt \quad (19)$$

subject to the second social planner's optimization

$$\{\hat{x}, \hat{y}\}_T^\infty = \arg \max_{\{x,y\}_T^\infty} \int_T^\infty (\mathcal{U}(x(t) + y(t)) - b(I)y(t) - \tilde{V}x(t)) e^{-rt} dt \quad (20)$$

$$\text{s.t. } \dot{R}(t) = -x(t), \quad R(T) = S$$

Call the current social planner's welfare from T onwards G^1 . Under certainty with $\tilde{V} = v$ it is equivalent to the social welfare described in section (3), so $G^1(S, b(I), v, v) = G(S, b(I), v)$. Call the optimized value function of the second social planner in (19)

G^2 . When $V = v$, the evaluations of the successive social planner's coincide and $G^2 = G^1(S, b(I), v, V = v)$. Note that extraction \hat{x} and renewable energy production \hat{y} after T are based on the realized \tilde{V} , not v . From the current social planner's viewpoint the future extraction path is therefore not optimal (except in the case $V = v$).

3.2 Analysis of political uncertainty

The social planner incurs a loss from not choosing extraction and investment in the second period $t > T$. Denote this loss by

$$L(S, b(I), V, v) = G(S, b(I), v) - G^1(S, b(I), V, v) \geq 0 .$$

Equation (17) now reads

$$\max_{S, I} g(S) - I + G(S, b(I), v) - EL(S, b(I), \tilde{V}, v) .$$

The first order conditions are

$$g_s + G_S(S, b(I), v) - EL_S(S, b(I), \tilde{V}, v) = 0 \quad (21)$$

$$-1 + G_b(S, b(I), v)b' - EL_b(S, b(I), \tilde{V}, v)b' = 0 . \quad (22)$$

Under certainty, there is no loss and the last terms in (21) and (22) disappear. So the effect of introducing uncertainty takes the form

$$dS = \frac{1}{|\mathcal{M}|} \overbrace{(G_{bb}(b')^2 + G_b(b''))}^{-} \overbrace{(EL_S)}^{+} - \overbrace{(G_{Sb}b')}^{-} \overbrace{(b'EL_b)}^{+} \quad (23)$$

$$dI = \frac{1}{|\mathcal{M}|} \overbrace{(g_{SS} + G_{SS})}^{-} \overbrace{(b'EL_b)}^{+} - \overbrace{(EL_S)}^{+} \overbrace{(G_{Sb}b')}^{-} \quad (24)$$

The effect of political uncertainty depends on the signs of EL_S and EL_b . Those signs cannot be determined analytically. I therefore simulate the loss function derivatives to find a robust best guess. The simulations suggest that $EL_b < 0$ and $EL_S > 0$.⁵ As in the case of scientific uncertainty the signs of dS and dI are ambiguous.

There is one important difference between political and scientific uncertainty however. Comparing the first order conditions in (11) and (12) with (23) and (24), we see

⁵The details are laid out in the Appendix B. Note that while this is by far the most frequent result, parametrizations can be found that yield different outcomes, in particular for the quadratic utility function.

that the two uncertainty types have opposite effects on the first order conditions:

$$\begin{aligned}
 g_s + G_S(S, b(I), v) \overbrace{-EL_S}^{-} &= 0 && \text{(political uncertainty)} \\
 g_s + G_S(S, b(I), v) \overbrace{+\omega}^{+} &= 0 && \text{(scientific uncertainty)} \\
 -1 + G_b(S, b(I), v)b' \overbrace{-EL_b b'}^{-} &= 0 && \text{(political uncertainty)} \\
 -1 + G_b(S, b(I), v)b' \overbrace{-\xi b'}^{+} &= 0 && \text{(scientific uncertainty)}
 \end{aligned}$$

However, that does not translate into effects on dS and dI going opposite directions. Without further assumptions the relative magnitudes of EL_S versus ω and EL_b versus ξ cannot be determined. If they had similar magnitudes, we could conclude that the two types of uncertainty has opposing effects on investment and early extraction, all else equal. But they are two different concepts and hence cannot easily be compared.

3.3 Special cases: Exogenous investment and extraction

To gain intuition about the result above, suppose investment is exogenous. Then equation (21) alone determines the impact of uncertainty on S . For the case that $EL_S > 0$ more is extracted earlier:

$$dS = \frac{EL_S}{g_{SS}(S) + G_{SS}(S, b(\bar{I}))} < 0 .$$

Secondly, when extraction is exogenous, less is invested in renewable energy technology (from equation 22):

$$dI = \frac{EL_b b'}{G_{bb}(b')^2 + G_b(b'')} < 0$$

Intuitively, the current social planner wants to reduce the future social planner's possibility to deviate from the choices she would have made in the second period. She cannot control how her investments in renewable energy technology and the fossil resource⁶ are used in the future, so she invests less. If the extraction and the investment decision are made jointly, the same type of interaction effects described in section 2.3 are present: Political uncertainty makes investments less attractive. That increases the incentive to save fossil resources for the future, counteracting the effect uncertainty has on the extraction decision.

Comparing the present result to those in section 2.3, I find that scientific and political uncertainty have opposing effects on investment in renewable energy technology and

⁶We can think of the decision to “not extract” as “investment” in the resource stock.

extraction of fossil fuels when either of them is exogenous.

4 Market solution

So far, the problem was cast in terms of one or two social planners. Now I reformulate it as a decentralized market problem. A regulator sets emission taxes and chooses an R&D policy. I assume that first best R&D policy is possible. Such a policy optimally compensates for the common market failures in R&D markets. That is equivalent to the government directly investing and subsequently making the technology freely available. The owners of the non-renewable resource extract $x(t)$ in $[0, T]$ knowing the emission tax $q(t)$. They plan given their expectations about the carbon tax $\tilde{Q}(t)$ for $[T, \infty]$. The source of uncertainty is irrelevant for them. From T onwards, they optimize extraction for known $Q(t)$ and $b(I)$. Hence the optimization problem may be formulated as

$$\max_S h(S, q) + EH(S, \tilde{Q}) \quad (25)$$

where

$$h(S, q) = \max_{\{x\}_0^T} \int_0^T (p(t) - q)x(t)e^{-rt} dt \quad (26)$$

$$\text{s.t. } \dot{R}(t) = -x(t), R(0) \text{ given}, R(T) = S$$

$$H(S, Q) = \max_{\{x\}_T^\infty} \int_T^\infty (p(t) - Q)x(t)e^{-rt} dt \quad (27)$$

$$\text{s.t. } \dot{R}(t) = -x(t), R(T) = S$$

From T on, the renewable substitute producers decide how much energy $y(t)$ to provide. They will supply nothing if the price is below $b(I)$ and offer renewable energy competitively as soon as it reaches their marginal cost $p(t) = b(I)$. Demand is derived from \mathcal{U} and market clearing requires that $\mathcal{U}'(\cdot) = p(t) \forall t$.

4.1 Scientific uncertainty

In the period $[0, T]$, the social welfare maximizing regulator set the carbon tax $q(t)$ and chooses how much to invest I , given her expectations about climate damages $E\tilde{v}$. At T , she re-optimizes after learning her true evaluation v and sets the tax $Q(t)$. Technically, the regulator can implement the social planner solution if she can imitate the first order conditions for the social planner optimum for some choice of taxes $q(t), Q(t)$. In particular, the taxes must equalize the shadow price of the fossil resource for the market and the social planner such that the extraction paths both before and after T coincide. For $t < T$, $q = E\tilde{v}$ equalizes the first order conditions (for a given S). The complete

equations can be found in Appendix A. Similarly, for $t > T$ setting $Q = v$ aligns market and social planner first order conditions (for given S and I). As the derivative of the value function equals the shadow price of the resource we get that also the first order condition for the choice of S is the same (from equation 11):

$$\begin{aligned} h_S(S, q) &= g_S(S) = -\lambda(T) \\ H_S(S, Q) &= G_S(S, b(I), v) = \mu(T) \\ \lambda(T) &= E\mu(T) = EH_S(S, Q) = G_S(S, b(I), E\tilde{v}) + \omega . \end{aligned} \quad (28)$$

Hence S is chosen socially optimal by the market too. Finally, given socially optimal extraction is replicated, it is also optimal for the regulator to invest the same amount I in green technology as the social planner. Thus, not surprisingly, the social planner optimum is achieved by setting the carbon taxes equal to the (expected) marginal damages.

4.2 Political uncertainty

The second regulator sets the carbon tax $Q(t)$ for $t > T$ after learning his valuation V . The problem of the second regulator is identical to the problem for the regulator under scientific uncertainty for $t > T$ (equation 27). Hence analogously to the just described result in 4.1, he sets $Q = V$ for any given S and I .

The regulator prior to T then chooses investment I and sets the carbon tax $q(t)$. She expects the future carbon tax to be $EQ = EV = v$. Can the first social planner's solution be implemented by a fixed q ? Under scientific uncertainty it is possible to align the market to the social optimum by setting $q = E\tilde{v}$. Also for political uncertainty, this tax level is the only candidate, because both in the market and in social optimum the Hotelling rule must hold:

$$p(t) = E\tilde{v} + (p(0) - v)e^{rt} .$$

To achieve the social optimal price path *slope* in a market setting by a fixed tax, the tax must hence equal the marginal damage, $q = v \forall t$.

Under scientific uncertainty, at time T the market and social planner shadow price both are $G_S(S, b(I), E\tilde{v}) + \omega$ (equation 28). Under political uncertainty the resource owner also has this shadow price because the second regulator behaves just like the regulator under scientific uncertainty. But the first period social planner under political uncertainty has a different shadow price at time T :

$$\lambda(T) = EG_S(S, b(I), v) = G_S(S, b(I), E\tilde{v}) - EL_S(S, b(I), \tilde{V}, v).$$

We know that $\omega > 0 > -EL_S$, which is the central difference between scientific and analytical uncertainty (see section 3.2). Hence at $t = T$, $q = v$ is not possible and a fixed carbon tax is not sufficient to achieve the socially desirable outcome.

The regulator however can implement the social planner solution by a variable carbon tax $q(t)$. The social planner price path for $t < T$ is given by

$$p^S(t) = v + \lambda^S(T)e^{r(t-T)} ,$$

while the market price is given by

$$p^M(t) = q(t) + \lambda^M(T)e^{r(t-T)} .$$

The regulator sets the tax $q(t)$ to recreate the social planner price path:

$$\begin{aligned} p^M(t) &= p^S(t) && \text{for } t \in [0, T] \\ q(t) &= v - (\lambda^M(T) - \lambda^S(T))e^{r(t-T)} . \end{aligned} \tag{29}$$

The shadow prices of the resource in T differ:

$$\begin{aligned} \lambda^S(T) &= G_S(S, b(I), E\tilde{v}) - EL_S(S, b(I), \tilde{V}, v) \\ \lambda^M(T) &= G_S(S, b(I), E\tilde{v}) + \omega . \end{aligned}$$

Given that $\omega > 0 > EL_S$, the resource owners have a higher shadow value than the social planner, $\lambda^S(T) < \lambda^M(T)$. This is intuitive: The market actor is indifferent to the source of uncertainty, whereas the social planner puts a negative value on losing control over future taxes. It follows directly from equation (29) and the observation that $(\lambda^M(T) - \lambda^S(T))e^{r(t-T)}$ increases in time that

$$\begin{aligned} q(t) &< v \quad \forall t \in [0, T] \\ \dot{q}(t) &< 0 . \end{aligned}$$

The optimal carbon tax under political uncertainty is lower than the marginal climate damage, and it decreases over time.

5 Conclusion

I use a dynamic model of a stylized energy market to investigate how uncertainty about future carbon taxes influences investment in renewable energy technology and extraction of exhaustible fossil fuels. I distinguish between two types of uncertainty: Scientific uncertainty, caused by a lack of knowledge about the climate system, and uncertainty

about political decision making in the future.

When extraction of fossil fuels and investment in renewable energy technology are investigated separately, scientific uncertainty leads to more cautious behavior in the common sense: less is extracted awaiting the resolution of uncertainty, and more is invested in renewable energy technology. For political uncertainty, I get the opposite result: more is extracted and less invested. An intuitive explanation of this result is that these decisions give the future decision maker less room to maneuver. The main result of this study is that the effects are inconclusive when extraction and investment are treated jointly. The interaction effects counteract the direct effects observed in the analysis with exogenous investment or extraction. Increasing investment in the future *ceteris paribus* leads to an incentive to extract more of the resource today. So when scientific uncertainty increases the incentive to invest, this counteracts the incentive to extract less. The total effect depends on the specific situation.

In the case of scientific uncertainty a constant carbon tax equaling the marginal damages suffices to implement the social optimum. With political uncertainty, such a tax does not lead to the outcome desired by the social planner in the first period. Instead a carbon tax lower than the marginal damages and decreasing over time aligns the resource owners profit motive with the social objective of the social planner.

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Appendix

A Additional calculations

A.1 Optimality conditions social planner

Below I present the optimality conditions used in the main text from the social planner problem. For g (equation (2)), the Hamiltonian and the optimality conditions from the maximum principle are

$$\begin{aligned}\mathcal{H}^{t < T} &= \mathcal{U}(x_t) - vx_t - \lambda_t x_t \\ \mathcal{U}'(x_t) - E\bar{v} &= \lambda_t \\ \dot{\lambda}_t - r\lambda_t &= 0\end{aligned}$$

As we have a fixed terminal state, we have $R(t) = S$ and no additional transversality condition. Similarly, for G (equation (6)) we have

$$\begin{aligned}\mathcal{H}^{t > T} &= \mathcal{U}(x_t + y_t) - b(I)y_t - Qx_t - \mu_t x_t \\ \mathcal{U}'(x_t + y_t) &= V + \mu_t \\ \mathcal{U}'(x_t + y_t) &= b(I) \\ \dot{\mu}_t - r\mu_t &= 0\end{aligned}$$

So x_t and y_t are never supplied at the same time (as $b(I)$ constant). If $Q > b$ it must be the case that $S = 0$, and no extraction takes place after T . Otherwise, S is extracted completely before the substitute is produced. The maximum price to be achieved from the renewable resource is $b(I)$. The maximum shadow value of the resource is therefore $\mu_\tau = b(I) - Q$. The shadow value increases at the discount rate

$$\mu_t = (b(I) - Q)e^{-r(\tau-t)}$$

For the value function being differentiable, the adjoint variable serves as a shadow value

$$\begin{aligned}g_S &= -\lambda(T) \\ G_S = \mu(T) &= (b(I) - Q)e^{-r(\tau-T)} > 0\end{aligned}$$

For scientific uncertainty, the first order condition (4) implies

$$\begin{aligned}g_S + EG_S &= 0 \\ \lambda(T) &= E\mu(T)\end{aligned}$$

This describes the optimal solution under scientific uncertainty. For political uncertainty, the first social planner's extraction problem is solved like $\mathcal{H}^{t < T}$ and the second social planner's problem like $\mathcal{H}^{T > t}$.

To find G_b , first note that renewable energy y_t will only be supplied from τ onwards, when fossil fuel is exhausted. Rewriting the optimization slightly and making use of the envelope theorem G_b is

$$\begin{aligned} G(S, b(I), V) &= \max_{x_t, y_t} \int_T^\tau e^{-rt} [\mathcal{U}(x_t) - Vx_t] dt + \int_\tau^\infty e^{-rt} [\mathcal{U}(y_t) - b(I)y_t] dt \\ &= \max_{x_t} \left\{ \int_T^\tau e^{-rt} [\mathcal{U}(x_t) - Vx_t] dt + e^{-r\tau} \frac{\max_{y_t} [\mathcal{U}(y_t) - b(I)y_t]}{r} \right\} \\ G_b &= -e^{-r\tau} \frac{y}{r} < 0 \end{aligned}$$

To confirm that $G_{Sb} > 0$, I take the derivative with respect to b

$$G_{Sb} = \frac{\partial \mu_T}{\partial b} = b(I)' e^{-r(\tau-T)} - r(b(I) - V) e^{-r(\tau-T)} \tau_b$$

To determine τ_b , replace μ_t in the first order conditions so that

$$U'(x^*(t)) - V = (b(I) - V) e^{-r(\tau-t)}$$

This equality together with

$$\int_T^\tau x^*(t) dt = S$$

implicitly defines $x^*(S, I, V, t)$ and $\tau(S, I, V)$. Differentiating both equations yields

$$\begin{aligned} x_b^*(t) &= \frac{1}{U''} (1 - r(b(I) - V) \tau_b) e^{-r(\tau-t)} \\ \int_T^\tau x_b^*(t) dt + x^*(\tau) \tau_b &= 0 \end{aligned}$$

Inserting for x_b^*

$$\begin{aligned} \int_T^\tau \frac{1}{U''} (1 - r(b(I) - V) \tau_b) e^{-r(\tau-t)} dt + x^*(\tau) \tau_b &= 0 \\ \int_T^\tau \frac{1}{U''} e^{-r(\tau-t)} dt - r(b(I) - V) \tau_b \int_T^\tau \frac{1}{U''} e^{-r(\tau-t)} dt + x^*(\tau) \tau_b &= 0 \\ \tau_b = \frac{\int_T^\tau \frac{1}{U''} e^{-r(\tau-t)} dt}{r(b(I) - V) \int_T^\tau \frac{1}{U''} e^{-r(\tau-t)} dt - x^*(\tau)} &> 0 \end{aligned}$$

So G_{Sb} is positive.

A.2 Optimality conditions market

For the fossil fuel owners in the market, the optimality conditions for $t < T$ are

$$\begin{aligned} h_S(S, q) &= -EH_S(S, \tilde{Q}) \\ p(t) &= q + \lambda(t) \quad \forall \quad t < T \\ \lambda(t) &= \lambda(0)e^{rt} \\ \int_0^T x(t) dt &= R(0) - S \end{aligned}$$

And for $T > t$

$$\begin{aligned} p(t) &= Q + \mu(t) \quad \forall \quad t \geq T \\ \mu(t) &= \mu(0)e^{rt} \\ \int_T^\tau x(t) dt &= S \end{aligned}$$

By visual inspection one can verify the results presented in the main text: Constant carbon taxes $q = E\tilde{v}$ and $Q = v$ align the market with the social optimum under scientific uncertainty. The main text shows that political uncertainty requires a time varied carbon tax.

B Simulations

I employ three common functional forms for utility (social welfare) functions: quadratic, logarithmic and constant relative risk aversion (CRRA).

B.1 Analytical results

For given S and I , the optimal solution G for $[T, \infty]$ is found from the first order conditions. Generally τ is determined by the equation

$$\int_T^\tau [\mathcal{U}']^{-1} (v + (b - v)e^{-r(\tau-t)}) dt = S .$$

Quadratic utility

For a quadratic utility function $\mathcal{U}(x) = x - \frac{\alpha}{2}x^2$, I have

$$\begin{aligned} \int_T^\tau \frac{1}{\alpha} (v + (b - v)e^{-r(\tau-t)}) dt &= S \\ (b - v) (1 - e^{-r(\tau-T)}) + r(v - 1)(\tau - T) &= -r\alpha S . \end{aligned}$$

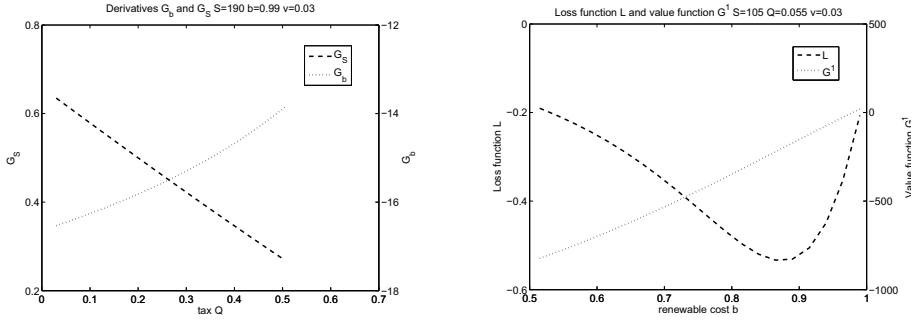


Figure 1: Quadratic utility function. Graphs for G_b and G_S that show convexity/concavity in left panel. Loss function over b in right panel, showing both $L_b > 0$ and $L_b < 0$ for same parameter values for S, v, V .

This equation cannot be solved for τ by standard algebra.

Logarithmic utility

For a logarithmic utility function $\mathcal{U}(x) = \log(x)$, τ is to be found from

$$\int_T^\tau (v + (b - v)e^{-r(\tau-t)})^{-1} dt = S$$

$$\frac{\tau - T}{v} - \frac{1}{rv} (\log b - \log [e^{-r(\tau-T)}(b - v) + v]) = S$$

$$\tau = \frac{\log \left[e^{rT} - \frac{be^{rT}}{v} + \frac{be^{rT+rSv}}{v} \right]}{r}$$

Thus the derivatives are

$$G_{bvv} = \frac{e^{-rT} (-2 + e^{rSv} (2 + rSv(-2 + rSv) - brS (2 + rSv + e^{rSv}(-2 + rSv))))}{r (b(-1 + e^{rSv}) + v)^3}$$

$$G_{Svv} = \frac{1}{(b(-1 + e^{rSv}) + v)^3} \times$$

$$be^{r(-T+Sv)} \{ r^2 S^2 v^3 + b^2 r S (2 + rSv + e^{rSv}(-2 + rSv))$$

$$- b (e^{rSv} (2 + rSv(-4 + rSv)) + 2(-1 + rSv(1 + rSv))) \}$$

CRRA utility

For a CRRA utility function $\mathcal{U}(x) = \frac{x^{1-\rho}}{1-\rho}$

$$\int_T^\tau (v + (b - v)e^{-r(\tau-t)})^{-\frac{1}{\rho}} dt = S .$$

This integral has no analytic solution.

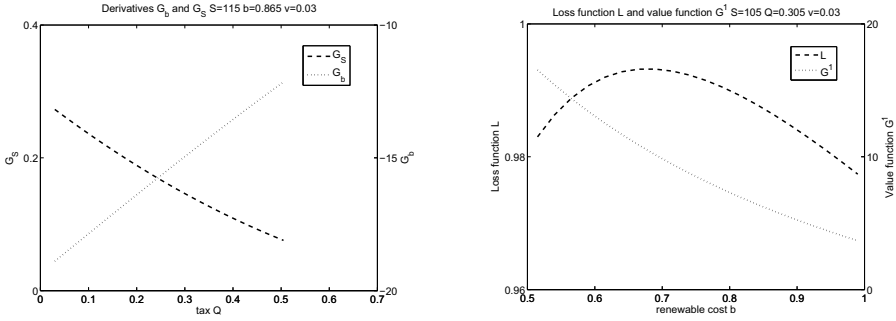


Figure 2: Logarithmic utility function, derivatives G_b and G_S in left panel. Loss function over b in right panel exhibits both $L_b > 0$ and $L_b < 0$ for one set of values S, v, V .

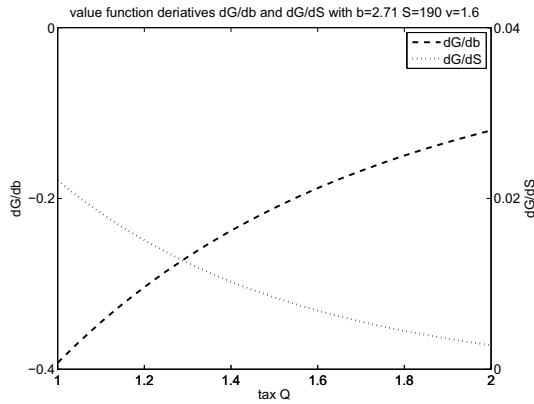


Figure 3: Derivatives of dG/db and dG/dS for a CRRA social welfare function with a risk aversion parameter of $\rho = 2$ and values $S = 190, b = 2.71$.

B.2 Numerical results

I proceed by numerically solving for G_{bvv} , G_{Svv} , L_b and L_S .⁷ I use a grid of twenty different values for each of b, S, v, V . The derivatives of interest are analyzed in two ways: by visual inspection of the plots of G_S and G_b over v and L over S and b , and by rough numerical approximations of the derivatives by the formulas⁸

$$f'(z) \approx \frac{f(z) - f(z_{-1})}{z - z_{-1}}$$

$$f''(z) \approx \frac{f(z_{+1}) - 2f(z) + f(z_{-1}))}{(.5(z - z_{-1}))^2}.$$

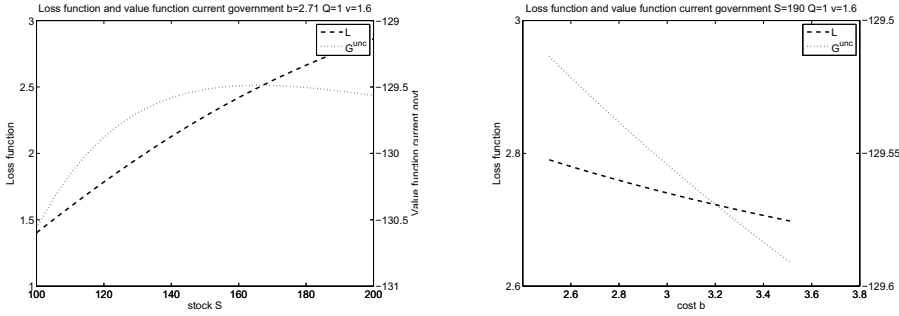


Figure 4: Loss function and current government value function under uncertainty for a CRRA utility function with $\rho = 2$ over cost b and stock S respectively.

Table 1:

	min	max
S	105	200
b	.515	.99
v, V	.03	.505

Quadratic

I use three values for α : .005, .02 and .1. The values for the other parameters are found in Table 1. Note that to have a well formulated problem, I need $\alpha < 1$ and $b < 1$. The results are inconclusive. For $\alpha = .02$, the numerical derivations suggest that $G_{bQQ} > 0$ and $G_{SQQ} > 0$. The derivatives of the loss function L_b and L_S take both positive and negative values, for constant S , b and v . The same qualitative results are achieved for $\alpha = .005$.

Log

I use the parameter values as for the quadratic utility function (Table 1). The numerical derivatives indicate $G_{Svv} > 0$, $G_{bvv} < 0$. There are some exemptions for G_{bvv} , but those are small values and a look at the graph indicates that those may be numerical errors, as the graph looks like a straight line, suggesting $G_{bvv} = 0$. The loss function derivative L_S is positive, while L_b switches sign.

CRRA

I solve the model for the standard consumption smoothing value in the literature $\rho = 2$. I also employ $\rho = .5$ and $\rho = 5$. The range of values for the other parameters is found

⁷The numerical work is undertaken in MATLAB. The code is made available upon request.

⁸Note that the grid points are evenly spaced.

Table 2:

	min	max
S	100	200
b	2.51	3.51
v, V	1	2

in Table 2. For all three values of ρ , and all possible combinations of parameter values in Table 2, I get that $G_{Svv} > 0$, $G_{Bvv} < 0$, $L_S > 0$, $L_b < 0$.

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Chapter III

Growth Uncertainty in the Integrated Assessment of Climate Change (w. Christian Traeger)

Abstract

Integrated assessments of climate change commonly rely on the assumption that technological progress outgrows climate change damages by an order of magnitude, even without any climate policy. Then, mitigating greenhouse gases is a redistribution from the poor present to a rich future. While we have experienced enormous growth over the last century, sustaining such growth over several more centuries is by no means a sure thing. We analyze the consequences of growth uncertainty on optimal abatement policies in an integrated assessment model (IAM) that was recently employed to determine the US federal social cost of carbon (SCC). For this purpose, we rebuild the IAM as a recursive-dynamic programming model and solve the non-linear, out-of-steady state problem. This approach differs largely from current state of the art Monte-Carlo simulations. We expose the rate of technological progress to iid and persistent shocks, both of which have permanent effects on the technology level in the economy. As is well known, the standard economic model fails to capture risk premia and discount rates correctly at the same time (equity premium and risk free-rate puzzle). We therefore analyze the problem as well for recursive preferences that resolve the equity premium and the risk-free rate puzzle by disentangling risk aversion from the intertemporal elasticity of substitution. We find that the impact of risk becomes significant when modeling comprehensive risk preference and/or persistent growth shocks. The sign of the effect of growth uncertainty on mitigation depends on the intertemporal elasticity of substitution. The analysis also yields an interesting insight into precautionary savings with two capital stocks, man-made and environmental.

Keywords

climate change, uncertainty, integrated assessment, growth, risk aversion, intertemporal substitution, recursive utility, dynamic programming, DICE

1 Introduction

Future economic growth is of first order importance for climate change evaluation. Extrapolating economic growth from the past century to the coming centuries makes greenhouse gas mitigation a redistribution from the present poor to the future rich. For example, even in the absence of any climate change policy, Nordhaus's (2008) widespread DICE-2007 model implies that generations living 100 years from now are five times richer than today's generation. We analyze how uncertainty about economic growth affects optimal climate policy. We model fundamental uncertainty about technological progress that is independent of climatic change. Alternatively, the growth uncertainty can also be interpreted as a consequence of economic crises, social unrest, or diverging global growth rates, as long as these sources of uncertainty are exogenous to climatic change. We do not model a direct impact of climate change on economic growth. While such a direct link would have a major impact on economic policy, this direct link is empirically more controversial than the fundamental growth uncertainty we depict. Our paper is the first to consistently analyze how growth uncertainty impacts optimal climate policies in the integrated assessment of climate change. We focus on optimal abatement effort and the optimal carbon tax. We employ a recursive dynamic programming version of the DICE-2007 model by Nordhaus (2008). The model is the most widespread integrated assessment model and was recently used as one of three models determining the US federal social cost of carbon.

It is widely known that the standard economic model is not able to simultaneously capture observed risk premia and discount rates. Agents tend to have a much higher willingness to pay for risk avoidance than the usual parameterizations of the standard model suggest (equity premium puzzle). If we increase risk aversion in the standard model we simultaneously increase aversion to intertemporal substitution. In consequence, the risk-free discount rate takes on unreasonably high values. An important branch of the finance literature resolves this puzzle by introducing Epstein-Zin-Weil preferences in combination with persistent shocks (Epstein and Zin, 1991; Weil, 1990; Vissing-Jørgensen and Attanasio, 2003; Bansal and Yaron, 2004; Bansal et al., 2010; Nakamura et al., 2010). Epstein-Zin-Weil preferences disentangle risk attitude from the propensity to smooth consumption over time. Indeed, there is no a priori reason why these quite different preferences should coincide. Epstein-Zin-Weil preferences also satisfy typical normative desiderata including time consistency and the von Neumann-Morgenstern axioms (Traeger, 2010). We therefore analyze the implication of uncertainty under standard preferences as well as under general estimates taken from the finance literature that suggest a higher coefficient of risk aversion and a lower propensity to smooth consumption over time.

In an analytic model, Traeger (2010) shows that growth uncertainty can have a

major impact on the social discount rate under Epstein-Zin-Weil preferences. He also points out the relation of the general question to the precautionary savings literature. His paper does not explicitly distinguish between capital and environmental investment (greenhouse gas mitigation). As we show, these two investment possibilities can react to uncertainty in opposite directions. Hence, our paper is also a contribution to the precautionary savings literature, analyzing two assets that differ in their depreciation rate and their consumption impact. In a semi-analytic paper, with a four period numeric example Ha-Duong and Treich (2004) analyze Epstein-Zin-Weil preferences in the case of damage uncertainty with two states of the world. Crost and Traeger (2010) employ Epstein-Zin-Weil preferences in a recursive version of DICE to evaluate damages. They point out that disentanglement is of major importance for long-term evaluation because of getting the risk-free discount rate right. However, in the damage context, they show that risk aversion itself plays no role for evaluation. We show that for growth uncertainty, risk aversion is a major determinant of optimal climate policy. Moreover, under growth uncertainty, the sign of the risk effect depends on the estimate for propensity to smooth consumption over time. Recursive implementations of DICE include the work of Kelly and Kolstad (1999) and Leach (2007) who analyze learning about climate sensitivity, and Lemoine and Traeger (2010) who analyze the policy impact of tipping points in the climate system. Karp and Zhang (2006) discuss learning about climate sensitivity and marginal abatement cost in a stylized linear quadratic model. More remotely related are the non-recursive models that analyze uncertainty or Monte-Carlo approximations to uncertainty for DICE (Nordhaus, 2008; Ackerman et al., 2010), for FUND (Anthoff and Tol, 2010), for MERGE (Richels et al., 2004), for WITCH Cian and Tavoni (2011), and for PAGE (Hope, 2006). The drawback of simultaneous methods or forward control models is that the uncertainty we are modeling in this paper would be infeasible to handle because the uncertainty tree would explode quickly (even with a finite time horizon). Monte-Carlo simulation can take up more uncertainty, but cannot properly model optimal decision making under uncertainty and are strictly speaking closer to a sensitivity analysis. Baker and Shittu (2007) give a survey of literature that incorporates uncertainty into the analysis of technical change in the climate change context.

2 Model and welfare specification

Integrated assessment models embed a model of the world economy in a model of the climate system to investigate their interactions. We build a recursive version of the DICE-2007 model, with some minor simplifications.¹ Our model is summarized graphi-

¹In order to avoid the “curse of dimensionality” in our infinite horizon dynamic programming version of DICE, we replace the three carbon sinks in DICE by single decay rate fit, and we simplify the equation of motions for temperatures, see Appendix A). The simplified model is calibrated to perfectly fit the baseline policies in DICE, but temperatures are slightly lower than in the original model.

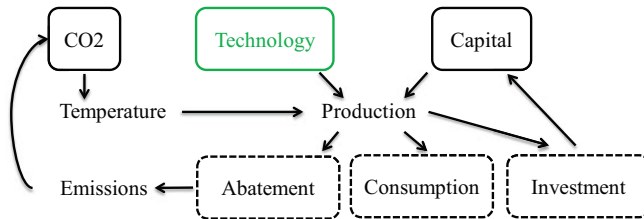


Figure 1: is an abstract representation of the climate-enriched economy model. The control variables consumption and abatement as well as the ‘residual’ investment are represented by dashed rectangles. The main state variables are depicted by solid rectangles. The green color indicates that the technology level is uncertain.

cally in Figure 1. The world economy is described by a classical Ramsey growth model. Capital accumulation is endogenous, while labor and technological growth are exogenous. Production of an aggregate commodity causes emissions that accumulate in the atmosphere. The social planner can spend part of the production on emission reductions (abatement). The emission stock in the atmosphere causes global warming and this warming is subject to exogenously parameterized feedback processes. An increase of global average temperature above pre-industrial levels causes damages that reduce world output. We solve for the optimal investment and abatement decisions.

2.1 Growth Uncertainty

Uncertainty impacts the exogenous rate of technological progress. The technology level in the economy enters the Cobb-Douglas production function and determines the overall productivity of the economy.² A shock in the growth rate has a permanent effect on the technology level in the economy. This assumption differs from the most widespread shocks in the real business cycle literature that simply affect the technology level within a period and have no long lasting effects. Such non-persistent shocks of the technology level are of little interest to our research question that is concerned with uncertainty about the long-term productivity of the economy. The technology level A_t in the economy follows the equation of motion

$$\tilde{A}_{t+1} = A_t \exp [\tilde{g}_{A,t}] \quad \text{with} \quad \tilde{g}_{A,t} = g_{A,0} * \exp [-\delta_A t] + \tilde{z}_t . \quad (1)$$

The deterministic part of the stochastic growth rate $\tilde{g}_{A,t}$ decreases over time at rate δ_A as in the original DICE-2007 model. We add a stochastic shock \tilde{z} , which is either iid or persistent. Figure 2 shows the growth under certainty ($\tilde{z}_t = 0$) in solid. Then, productivity increases roughly threefold over the 100 year time horizon.

²Given the Cobb Douglas production function the model is independent of whether technological progress affects labor productivity, capital productivity, or, as modeled by Nordhaus, overall productivity. We use labor augmenting technological progress, which seems to be the more widespread notation because for general production function only labor augmenting technological progress leads to a balanced growth path.

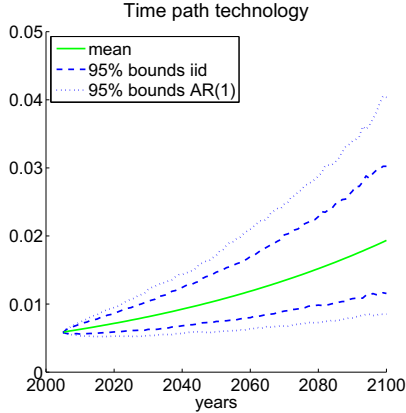


Figure 2: shows the expected draw and the 95% confidence intervals for technology time paths based on 1000 random draws of technology shock \tilde{z} time paths with $\sigma_{\tilde{z}} = 2 * g_{A,0}$. The black dotted lines correspond to iid shocks while the dashed blue lines give the confidence interval in the case where the shock has a persistent component.

Our first set of simulations analyzes the consequences of a shock that is identically and independently distributed with

$$\tilde{z}_t \sim \mathcal{N}(\mu_z, \sigma_z^2) .$$

We set the standard deviation at twice the initial growth rate ($\sigma_z = 2 * g_{A,0} \approx 0.026$). We base this value on Kocherlakota's (1996) observation for the last century of US data that the standard deviation of consumption growth is about twice its expected value. The rate of technological progress drives consumption growth in the Ramsey-Cass-Koopmans economy and, hence, we take a standard deviation of twice initial technology growth as a good proxy for a reasonable order of magnitude.³ We fix the mean of the growth shock so that $t+1$ expectations for the technology level coincide with those under certainty.⁴ The dotted lines in Figure 2 give the 95% (simulated) confidence interval for the technology levels over the next 100 years under our assumptions about the growth shocks. We emphasize that we model a non-mean reverting random walk. This specification makes the numerical implication significantly more challenging. However, given our concern is uncertainty about long-run productivity we avoid the assumption of mean-reversion.

Our second set of simulation analyzes the consequences of a shock that has a persistent component. Persistent shocks are usually part of the finance literature explaining

³Our decision maker can smooth the effect of technology shocks using capital to smooth consumption. Moreover the steady state consumption growth rate also depends on deterministic population growth. Thus, our model is not build to reproduce or calibrate consumption fluctuations. We merely take the above reasoning as a proxy for a relevant order of magnitude.

⁴A mean zero shock of the growth rate would, by Jensen's inequality, imply an increase in the expected next period technology level. The technology level in period $t+1$ is determined by the random variable $\exp[\tilde{z}]$ that is lognormally distributed. Setting $E[\tilde{z}] = -\sigma^2(\tilde{z})/2$ implies $E \exp[\tilde{z}] = 1$ and that the expected technology level equals its deterministic part.

the equity premium and the risk-free rate puzzle (Bansal and Yaron, 2004). Here, we think of the persistent shock as a more fundamental uncertain change in technological progress and economic productivity. The theoretical literature has established that persistent shocks imply decreasing social discount rates over time (Weitzman, 2009) We model persistence in form of an AR(1) process

$$\begin{aligned} \tilde{z}_t &= \tilde{x}_t + \tilde{y}_t \quad \text{where} \\ \tilde{x}_t &\sim \mathcal{N}(\mu_x, \sigma_x^2) \quad \text{and} \\ \tilde{y}_t &= \zeta y_{t-1} + \tilde{\epsilon}_t \quad \text{with } \tilde{\epsilon}_t \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2). \end{aligned} \tag{2}$$

We choose the standard deviations to $\sigma_x = \sigma_\epsilon = \sqrt{2} * g_{A,0}$, which again results in a standard deviation of the overall shock \tilde{z}_t determining the next period technology level of twice the initial growth rate: $\sigma_z = 2 * g_{A,0}$. Our second specification coincides with the first in the case of vanishing persistence $\zeta = 0$. A higher persistence increases long-run uncertainty in the second scenario. The mean values are chosen such that at the beginning of the planning horizon the expected path for the technology level equals the certain path for $y_0 = 0$.⁵ Our simulations use a persistence of $\zeta = 0.5$, implying that 50% of the ϵ -shock carries over to the growth rate in the next year. While modeling an even higher persistence would be desirable, modeling a random walk in the growth rate (instead of a mean reverting process) is a serious numerical challenge in an infinite horizon dynamic programming problem. Persistence of the shock adds significantly to this challenge. We will show that even the rather moderate persistence has strong implications for optimal climate policy.

2.2 Welfare and Bellman equation

The decision maker maximizes her value function subject to the constraints imposed by the climate-enriched economy. We formulate the decision problem recursively using the Bellman equation. This recursive structure facilitates the proper treatment of uncertainty and the incorporation of comprehensive risk preferences. The relevant physical state variables describing the system are capital K_t , atmospheric carbon M_t , and technology level A_t . In addition time t is a state variable that captures exogenous processes including population growth, changes in abatement costs, non-industrial GHG emissions, and temperature feedback processes. Finally, in the case of persistent shocks, the state d_t captures the persistent part of last period's shock that carries over to the current period. We first state the Bellman equation for standard preferences, i.e., the

⁵A short calculation shows that we achieve this equivalence by setting $E[\tilde{x}] = E[\tilde{\epsilon}] = -\sigma^2(\tilde{x})/2$.

time additive expected utility model:

$$\begin{aligned}
 V(K_t, M_t, A_t, t, d_t) = & \max_{C_t, \mu_t} \frac{L_t \left(\frac{C_t}{L_t}\right)^{1-\hat{\eta}}}{1-\hat{\eta}} \\
 & + \exp[-\delta_u] \mathbb{E} \left[V(K_{t+1}, M_{t+1}, \tilde{A}_{t+1}, t+1, \tilde{d}_{t+1}) \right].
 \end{aligned} \tag{3}$$

The value function V represents the maximal welfare that can be obtained given the current state of the system. Utility within a period corresponds to the first term on the right hand side of the dynamic programming equation (3). It is a population (L_t) weighted power function of global per capita consumption (C_t/L_t). The parameter $\hat{\eta}$ captures two preferences: the desire to smooth consumption over time and Arrow-Pratt relative risk aversion. Following Nordhaus (2008), we set $\hat{\eta} = 2$. The second term on the right hand side of equation (3) represents the maximally achievable welfare from period $t + 1$ on, given the new states of the system in period $t + 1$, which follow from the equations of motion summarized in Appendix A. The planner discounts next period welfare at the rate of pure time preference $\delta_u 1.5\%$ (also utility discount rate), where the value is again chosen to coincide with Nordhaus's (2008) DICE-2007 model. In period t , uncertainty governs the realization of next period's technology level \tilde{A}_{t+1} and, thus, gross production. Therefore, the decision maker takes expectations when he chooses the optimal control variables consumption C_t and abatement (emission control rate) μ_t . Equation (3) states that the value of an optimal consumption path starting in period t should be the maximal sum of the instantaneous utility gained in that period and the welfare gained from the continuation path. The control C_t balances immediate consumption gratification with the value of future capital stock. The control μ_t balances immediate consumption (given up for abatement) against the future stock of carbon.

Next we enrich the model to capture a comprehensive risk attitude that can simultaneously capture observed risk-free discount rates and equity premia. Hereto, we exploit the recursive structure to disentangle risk and time preferences. The standard model forces these two a priori distinct attitudes to coincide. Epstein and Zin (1989) and Weil (1990) show how to disentangle the two and Bansal and Yaron (2004) showed how this disentangled approach resolves the risk-free rate and the equity premium puzzles. We emphasize that the model satisfies time consistency and the von Neumann and Morgenstern (1944) axioms and is normatively no less desirable than the standard discounted expected utility model (Traeger, 2010). The latter paper also shows how to shift the non-linearity from the time-step as in Epstein and Zin (1989) to uncertainty aggregation, resulting in

the Bellman equation

$$V(K_t, M_t, A_t, t, d_t) = \max_{C_t, \mu_t} \frac{L_t \left(\frac{C_t}{L_t}\right)^{1-\eta}}{1-\eta} \quad (4)$$

$$+ \frac{\exp[-\delta u]}{1-\eta} \left(\mathbb{E} \left[(1-\eta)V(K_{t+1}, M_{t+1}, \tilde{A}_{t+1}, t+1, \tilde{d}_{t+1}) \right]^{\frac{1-\text{RRA}}{1-\eta}} \right)^{\frac{1-\eta}{1-\text{RRA}}}.$$

The parameter η captures the desire to smooth consumption over time (aversion to intertemporal substitution). It is the inverse of the intertemporal elasticity of substitution. The parameter RRA depicts the Arrow-Pratt measure of relative risk aversion. In the case $\eta = \text{RRA}$ we are back in the standard model and equation (4) collapse to equation (3). For a detailed analysis of the interpretation of the parameters RRA and ρ we refer to Epstein and Zin (1989) and to Traeger (2010). We base our choices of values for the disentangled preference on estimates by Vissing-Jørgensen and Attanasio (2003), Bansal and Yaron (2004), and Bansal et al. (2010). These papers suggest a best guesses of $\eta = \frac{2}{3}$ and of relative risk aversion in the proximity of the value $\text{RRA} = 10$ that we adopt. The social cost of carbon in current value units of the consumption-capital good as the ratio of the marginal value of a ton of carbon and the marginal value of a unit of the consumption good $SCC_t = \frac{\partial M_t V}{\partial K_t V}$.

2.3 Numerical Implementation

We give a short summary of the numeric implementation, discussing details of Appendix B. We approximate the value function by Chebychev polynomials and solve the Bellman equation by value function iteration. We represent the continuous distribution capturing technological progress by Gauss-Legendre quadrature nodes. The Bellman equations (3) and (4) are not convenient for a numerical implementation for two reasons. First, capital and technology are subject to enormous growth and any value function approximation with a reasonable number of nodes would be very coarse on the space.⁶ Second, modeling a random walk without mean reversion is a major challenge and the Bellman equation as cited above would not convergence with the amount of uncertainty we are capturing. Therefore, we renormalize consumption and capital in per effective labor units. For the technology level, our state variable captures the deviation from the deterministic evolution of technology. Finally, we map the infinite time horizon on a $[0, 1]$ interval. We adjust the Bellman equation conveniently to these changes in the state variables and control variables obtaining equation (6) in Appendix B.

⁶More precisely, the relevant part of the state space at different times would be disconnected. Our renormalization achieves that the relevant values lie in the same reduced region of the state space at all times. That allows us to obtain a much better approximation of the value function with less nodes.

3 Results

We first discuss results for standard preferences and an iid shock. Then we increase the disentangled coefficient of relative risk aversion to the value suggested in the finance literature and, in a second step, introduce persistence in the growth shock. Finally, we reduce propensity to smooth consumption over time to the degree suggested in the finance literature for disentangled preference specifications.

3.1 Entangled standard preferences ($\eta = \text{RRA} = 2$)

Figure 3 compares the deterministically optimal climate policy with the case of an iid shock on the rate of technological project, inducing a random walk of the technology level. The shock in the growth rate is normally distributed with a standard deviation of twice the initial growth rate ($\sigma_z \approx 0.026$). With isoelastic entangled preferences ($\text{RRA} = \eta = 2$), the iid shock has a very minor effect on the optimal policies. For the current century, the optimal abatement is .2-.6 percentage points higher under uncertain than under certain growth. In addition, current investment goes up by .35% percentage points. Hence, we find a small precautionary savings effect in both capital dimensions: produced productive capital and natural capital in terms of a clean atmosphere.

The social discounting literature offers an explanation for the economically insignificant magnitude of the impact. Traeger (2012) points out that uncertainty in the Ramsey discounting equation stemming from uncertain growth has a negligible impact on intertemporal trade-offs under the assumption of entangled preferences. He explains that when the desire to smooth consumption coincides with the aversion to risk, the decision maker is what he calls “intertemporal risk neutral”: Suppose the decision maker is indifferent between two alternative consumption paths fluctuating over time. From these two paths, construct a “high” consumption path by picking the higher consumption outcome in each period, and a “low” consumption path by picking the lower consumption outcome in each period. The decision maker is intertemporal risk neutral if she is indifferent between receiving either of the two original paths with certainty and receiving a lottery with a 50/50 chance over the “high” and the “low” path (Traeger, 2012).

3.2 Increasing risk aversion to $\text{RRA} = 10$

The standard model does not accurately capture equity premia. The premia actual agents pay for risk-reductions are higher than we can explain in a model where both relative risk aversion and aversion to intertemporal substitution equal to 2. Increasing the coefficient of relative risk aversion to $\text{RAA} = 10$ is a major step towards resolving the equity premium puzzle. Increasing aversion to intertemporal substitution to the same degree would result in ridiculously high consumption discount rates, defying all

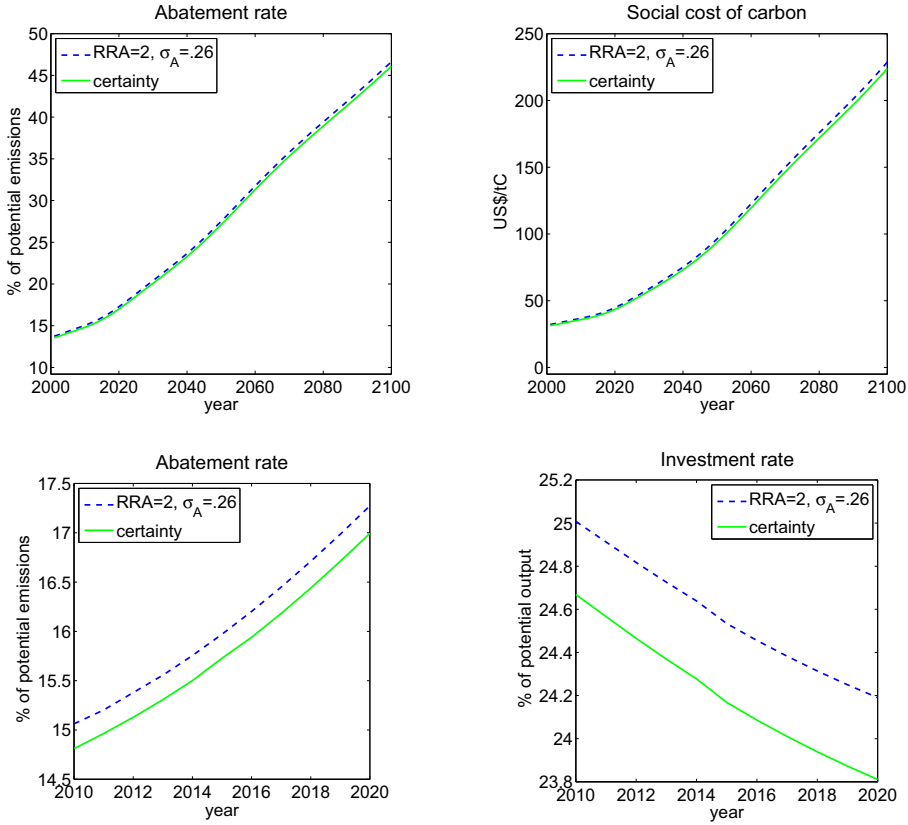


Figure 3: compares the optimal abatement rate and social cost of carbon under certainty and iid uncertainty with standard preferences and $RRA = \eta = 2$.

empirical evidence. Hence, we have to employ the disentangling Bellman equation (4) in order to capture the higher observed risk aversion.

Figure 4 shows the optimal climate policy under Epstein-Zin preferences that keep $\eta = 2$ and increase Arrow-Pratt risk aversion to $RRA = 10$. We observe a modest increase in abatement under uncertainty. Optimal abatement and the optimal social cost of carbon are approximately 10 percent higher over the first 100 years. The more risk averse decision maker is more cautious, abating and investing more and consuming less. Robustness checks (not shown) confirm that these findings increase in the variance of the stochastic shock. With Arrow-Pratt risk aversion exceeding the consumption smoothing parameter ($RRA = 10 > \eta = 2$), the decision maker is now intertemporally risk averse. This distaste of intertemporal uncertainty provokes precautionary savings. To insure against falling short of expected growth, she invests more in manmade capital and abates more to protect environmental capital. She raises both in roughly the same proportions.

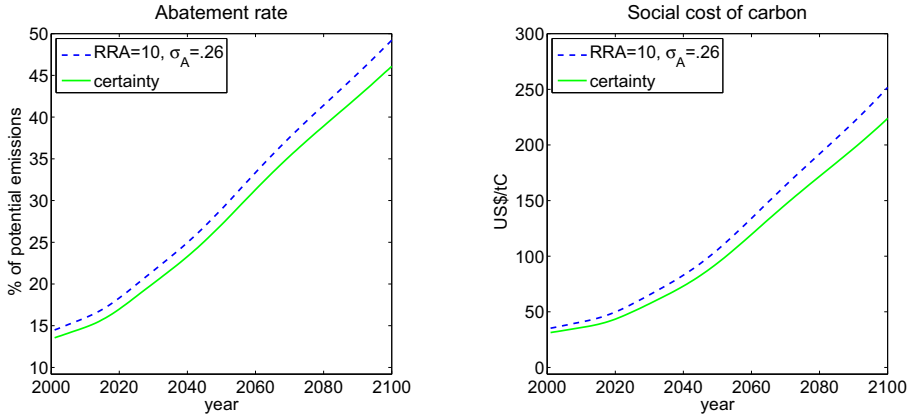


Figure 4: compares the optimal abatement rate and social cost of carbon under certainty and uncertainty with Epstein-Zin preferences, a coefficient of relative risk aversion of $RRA = 10$ and a coefficient of aversion to intertemporal substitution of $\eta = 2$.

3.3 Persistence in growth shocks

The iid shock on technological growth makes the technology level essentially a random walk. However, technological progress over the time span relevant to climate change evaluation is intertemporally correlated. If the future will show that there are periods where our current growth cannot be sustained, then the progress most likely does not just fall behind for a single period. Similarly, if we are in a time of prosperous economic and research progress, surrounding conditions and discoveries will most likely have lasting effects on growth. Here, we model a relatively moderate persistence of growth shocks according to equation (2). In addition to an iid shock component, the rate of technological growth experiences a persistent shock whose impact on technological growth decays by 50% per year.

The dotted lines in Figure 5 show the optimal climate policy under persistent growth shocks. The dashed lines represent optimal policy in the setting without persistence, but with the same growth uncertainty from one period to the next (which is slightly higher here than in the previous section). Introducing persistence amplifies the long-run uncertainty, while keeping immediate uncertainty unchanged. The modeled persistence approximately doubles the impact of uncertainty on optimal climate policy.

3.4 Decreasing consumption smoothing to $\eta = 2/3$

In the standard model $\hat{\eta}$ captures both relative risk aversion and aversion to intertemporal consumption smoothing. Estimating both parameters separately in an Epstein-Zin framework not only leads to a higher risk aversion parameter, but also to a lower aversion to intertemporal consumption fluctuation. We follow the empirical finance literature

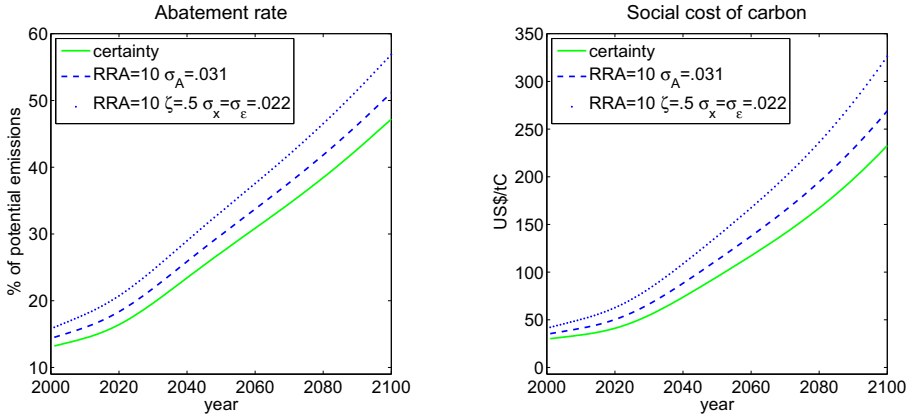


Figure 5: compares the optimal abatement rate and the social cost of carbon under certainty, iid uncertainty and persistent uncertainty with persistence $\zeta = 0.5$ for $RRA = 10$ and $\eta = 2$.

suggesting a best estimate of the consumption smoothing parameter of $\eta = 2/3$. Note that a reduction of η immediately decreases the consumption discount rate, making investment into the future more rewarding. This finding immediately relates to the observed risk-free rate being significantly lower than explained by the standard model with $\hat{\eta} = 2$. A reasoning by Nordhaus (2007) suggests that whenever we decrease η we should increase the pure rate of time preference in order to keep the overall consumption discount rate fix. We emphasize that this reasoning would be wrong in the current setting. Lowering η implies that we match the observed risk-free rate much better than the standard model. On the other hand, the higher risk aversion parameter explains the higher interest on risky assets, again better than in the standard model. In fact, the empirical literature calibrating the Epstein-Zin model generally finds a lower pure time preference than Nordhaus’s (2008) and our $\delta_u = 1.5\%$ along the $\eta = 2/3$ and $RAA = 10$. Given our focus on the effects of uncertainty, however, we decided not to change pure time preference with respect to DICE-2007 in this paper.

The solid lines in Figure 6 display the effect of lowering η from 2 to $2/3$ under certainty. The reduction in the parameter and, thus, the risk-free discount rate increases optimal mitigation significantly. The optimal carbon tax doubles and the optimal abatement rate close to doubles. The decision maker is now less averse to shifting consumption over time to increase aggregate welfare. Hence she evaluates the prospect of additional welfare for the relatively affluent generations in the future more positively than a decision maker with a higher aversion of $\eta = 2$. Introducing uncertainty into the model with $\eta = 2/3$ and $RRA = 10$ decreases the optimal abatement and the social cost of carbon. This finding is opposite to the effects of uncertainty observed in the earlier settings. Investment in manmade capital still increases (not shown).

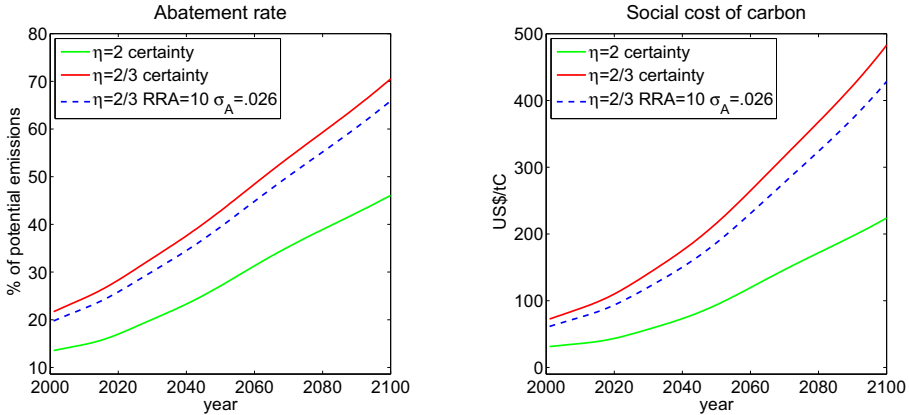


Figure 6: compares the optimal abatement rate and social cost of carbon under certainty with two different values for the consumption smoothing coefficient, $\eta = 2/3$ and $\eta = 2$, and uncertainty with Epstein-Zin preferences with $RRA = 10$ and $\eta = 2/3$.

The decision maker with $\eta = 2/3$ is relatively more willing to make up for consumption losses due to high temperatures in times with lower temperatures. Introducing uncertainty has two effects. First, future income becomes uncertain reducing expected future welfare. In general, this effect induces a precautionary savings effect under isoelastic preferences (which satisfy decreasing absolute risk aversion). However, because of uncertainty in technological progress, also the future productivity of a saved consumption unit becomes uncertain, making current consumption relatively more attractive. This second effect reduces the savings motive. Our numerical simulation shows that overall investment into manmade goods still goes up under uncertainty, while consumption stays almost constant and investment into the clean atmosphere decreases. Appendix C shows that, once more, persistence in the growth shock increases the growth uncertainty effect, further reducing optimal policy.

4 Conclusions

Extrapolating current growth into the future implies that climate policy is a redistribution from a relatively poor present generation to far richer future generations. While extrapolating recent growth might be the best guess, it is certainly not a sure prediction. We analyze the implication of growth uncertainty on optimal climate policy. We translate the DICE-2007 model of Nordhaus (2008) into a recursive dynamic programming framework to consistently model stochastic growth. Our shocks on the rate of technological progress make the economy's technology level a random walk. We find that a normally distributed shock in the growth rate has a rather small effect on optimal greenhouse gas abatement (fraction of a percentage point) and the optimal carbon tax.

This insensitivity of the optimal policy to growth uncertainty results from the standard model's insensitivity to risk. The same phenomenon gives rise to the equity premium (too low a risk premium) and the risk-free rate puzzle (too high a discount rate) in the finance literature. To evaluate climate change under uncertainty, we acknowledge the primordial importance of getting the discount rate and the risk premium right: we follow the approach suggested in the finance literature resolving the mentioned puzzles by disentangling risk aversion from a decision maker's propensity to smooth consumption over time. The resulting model satisfies the same rationality constraints as the standard discounted expected utility model, including time consistency.

Increasing relative risk aversion to the degrees measured in finance significantly increases optimal mitigation policies under uncertainty. Our iid shock on the rate of technological growth increases optimal mitigation and the optimal carbon tax notably. Introducing a moderate persistence to the shock doubles the uncertainty effect on both policy measures. However, the empirical findings in the finance literature using disentangled Epstein-Zin preferences also suggest that the propensity to smooth consumption over time is lower than the value in DICE-2007. Reducing this aversion to intertemporal substitution turns the effect of uncertainty on optimal climate policy on its head. Abatement now decreases in response to uncertainty. However, it does so from an overall much higher abatement level, because a lower aversion to intertemporal substitution decreases the consumption discount rate and the overall mitigation effort (under certainty) significantly. Thus, the fully disentangled model still results in a highest abatement rate, but not because of uncertainty. It is merely a consequence of better capturing the low risk-free discount rate.

The precautionary savings literature is well aware that a low intertemporal elasticity of substitution can result in a decrease in savings under uncertainty. Our model features two different investment possibilities. The natural capital has a somewhat complicated intertemporal payoff structure and investment is capped at the point of full abatement. Our simulation show that investment into manmade capital increases under uncertainty in all preference specifications. Only the effect of uncertainty on investment into the natural capital "clean atmosphere" turns around for the low aversion to intertemporal substitution. Our paper employs observed preference specifications that are fully rational. In the context of climate change, future wealth is the wealth consumed by future generations not currently alive. Instead of employing observed preferences, we could argue for the use of normative evaluation criteria. Then, equality of generation over time would most likely play a prominent role. Our simulation, as well as straight forward social discounting arguments, show how a low intergenerational substitutability over time (high aversion) implies higher emissions under certainty. In this scenario, uncertainty aversion has again a strong enhancing effect on optimal mitigation efforts.

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Appendix

A The climate enriched economy model

The following model is largely a reproduction of DICE-2007. The three most notable differences are the annual time step (DICE-2007 features ten year time periods), the infinite time horizon, and the replacement of the carbon sink structure by a decay rate. This simplification is necessary because each carbon sink would require an own state variable in a recursive framework, which is computationally too costly. For a detailed description of the procedure, see Lemoine and Traeger (2010). All parameters are characterized and quantified in Table B on page 110.

Carbon in the atmosphere is accumulated according to

$$M_{t+1} = M_{pre} + (M_t - M_{pre})(1 - \delta_M(M, t)) + E_t \quad \text{with}$$

$$\delta_{M,t} = \delta_{M,\infty} + (\delta_{M,0} - \delta_{M,\infty}) \exp[-\delta_M^* t] .$$

The stock of CO₂ (M_t) exceeding preindustrial levels (M_{pre}) decays exponentially at the rate $\delta_M(M, t)$. This decay rate falls exogenously over time to replicate the carbon cycle in DICE-2007, mimicking that the ocean reservoirs reduce their uptake rate as they fill up (see Lemoine and Traeger, 2010). The variable E_t characterizes yearly CO₂ emissions, consisting of industrial emissions and emissions from land use change and forestry B_t

$$E_t = (1 - \mu_t) \sigma_t A_t L_t k_t^\kappa + B_t .$$

Emissions from land use change and forestry fall exponentially over time

$$B_t = B_0 \exp[g_B t] .$$

Industrial emissions are proportional to gross production $A_t L_t k_t^\kappa$. They can be reduced by abatement. As in the DICE model, we in addition include an exogenously falling rate of decarbonization of production σ_t

$$\sigma_t = \sigma_{t-1} \exp[g_{\sigma,t}] \quad \text{with} \quad g_{\sigma,t} = g_{\sigma,0} \exp[-\delta_\sigma t] .$$

The economy accumulates capital according to

$$k_{t+1} = [(1 - \delta_k) k_t + y_t - c_t] \exp[-(g_{A,t} + g_{L,t})] ,$$

where δ_K denotes the depreciation rate, $y_t = \frac{Y_t}{A_t L_t}$ denotes production net of abatement costs and climate damage per effective labor, and c_t denotes aggregate global consumption of produced commodities per effective unit of labor. Population grows exogenously by

$$L_{t+1} = \exp[g_{L,t}] L_t \quad \text{with} \quad g_{L,t} = \frac{g_L^*}{\frac{L_\infty}{L_0} \exp[g_L^* t] - 1} .$$

Here L_0 denotes the initial and L_∞ the asymptotic population. The parameter g_L^* characterizes the convergence from initial to asymptotic population. Technological progress is exogenously given by equation (4) in section 2.1.

Net global GDP per effective unit of labor is obtained from the gross product per effective unit of labor as follows

$$y_\tau = \frac{1 - \Lambda(\mu_t)}{1 + D(T_t)} k_t^\kappa$$

where

$$\Lambda(\mu_t) = \Psi_t \mu_t^{a_2} \tag{5}$$

characterizes abatement costs as percent of GDP depending on the emission control rate $\mu_t \in [0, 1]$. The coefficient of the abatement cost function Ψ_τ follows

$$\Psi_t = \frac{\sigma_t}{a_2} a_0 \left(1 - \frac{(1 - \exp[g_\Psi t])}{a_1} \right)$$

with a_0 denoting the initial cost of the backstop, a_1 denoting the ratio of initial over final backstop, and a_2 denoting the cost exponent. The rate g_Ψ describes the convergence from the initial to the final cost of the backstop.

Climate damage as percent of world GDP depends on the temperature difference T_t of current to preindustrial temperatures and is characterized by

$$D(T_t) = b_1 T_t^{b_2} .$$

Nordhaus (2008) estimates $b_1 = 0.0028$ and $b_2 = 2$, implying a quadratic damage function with a loss of 0.28% of global GDP at a 1 degree Celsius warming.

Temperature change T_t relative to pre-industrial levels is determined by a measure for the CO₂ equivalent greenhouse gas increase Φ_t , climate sensitivity s , and transient feedback adjustments χ_t

$$T_t = s \Phi_t \chi_t .$$

In detail, climate sensitivity is

$$s = \frac{\lambda_1 \lambda_2 \ln 2}{1 - f_{eqt}} ,$$

the measure of equivalent CO₂ increase is

$$\Phi_t = \frac{\ln(M_t/M_{pre}) + EF_t/\lambda_1}{\ln 2} ,$$

where exogenous forcing EF_t from non-CO₂ greenhouse gases, aerosols and other processes is assumed to follow the process

$$EF_t = EF_0 + 0.01(EF_{100} - EF_0) \times \max\{t, 100\} .$$

Note that it starts out slightly negatively. Our transient feedback adjustment is given by

$$\chi_t = \frac{1 - f_{eqt}}{1 - (f_{eqt} + f_t)} .$$

For more details, see Lemoine and Traeger (2010).

B Numerical method and implementation

We approximate the value function by the collocation method, employing Chebychev polynomials. We solve the Bellman equation for its fixed point by function iteration. For all models we use seven collocation nodes for each of the state variables capital, carbon dioxide, technology level and the persistent shock. Along the time dimension, we fit the function over ten nodes for the model without, and seven nodes for the model with persistence in the shock. The function iteration is carried out in MATLAB. We utilize the third party solver KNITRO to carry out the optimization and make use of the COMPECON toolbox by Miranda and Fackler (2002) in approximating the value function.

To accomodate the infinite time horizon of our model, we map real time into artificial time by the following transformation:

$$\tau = 1 - \exp[-\iota t] \in [0, 1] .$$

This transformation also concentrates the Chebychev nodes at which we evaluate our Chebychev polynomials in the close future in real time, where most of the exogenously driven changes take place.

Further, we improve the performance of the recursive numerical model significantly by expressing the relevant variables in effective labor terms. Due to the uncertainty in the level of technology, we normalize by the deterministic technology level A^{det} . This is the level of technology under certainty (with all shocks equal zero, $z_t = 0 \forall t$)

$$A_{t+1}^{det} = A_t^{det} \exp [g_{A,t}]$$

Expressing consumption and capital in effective labor terms results in the definitions $c_t = \frac{C_t}{A_t^{det} L_t}$ and $k_t = \frac{K_t}{A_t^{det} L_t}$. Moreover, we also define $a_t = \frac{A_t}{A_t^{det}}$. The normalized productivity one period ahead is then defined as

$$\tilde{a}_{t+1} = \frac{\tilde{A}_{t+1}}{A_{t+1}^{det}} = \frac{\exp [\tilde{g}_{A,t}] A_t}{\exp [g_{A,t}] A_t^{det}} = \exp [\tilde{z}] a_t .$$

Using all of those new variables we can transform the Bellman equation (4) and define

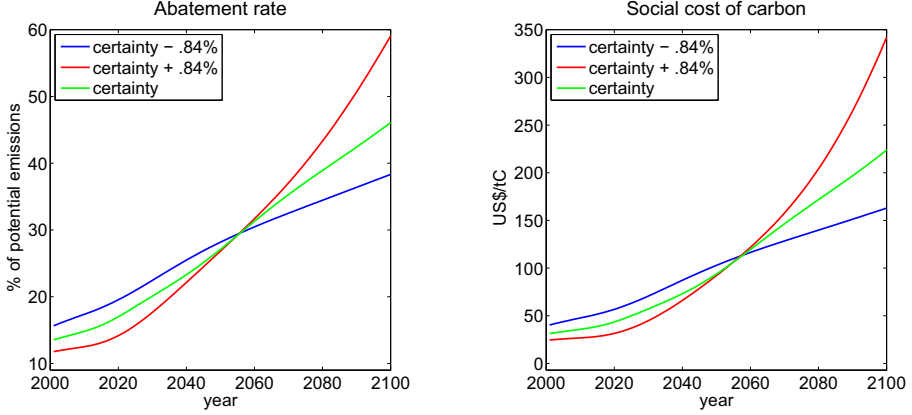


Figure 7: compares the optimal abatement rate and social cost of carbon under certainty with the DICE growth rate $g_{DICE,t}$, a high and a low growth rate ($g_{DICE,t} \pm 0.84\%$).

the new Bellman equation

$$V^*(k_\tau, M_\tau, a_\tau, \tau, d_\tau) = \max_{c_\tau, \mu_\tau} \frac{c_\tau^{1-\eta}}{1-\eta} + \frac{\exp[-\delta_u + g_{A,\tau}1 - \eta + g_{L,\tau}]}{1-\eta} \times \quad (6)$$

$$\left(E[1 - \eta V^*(k_{\tau+\Delta\tau}, M_{\tau+\Delta\tau}, a_{\tau+\Delta\tau}, \tau + \Delta\tau, d_{\tau+\Delta\tau})]^{\frac{1-RRR}{1-\eta}} \right)^{\frac{1-\eta}{1-RRR}}$$

For details on the transformations and how to regain the original Bellman equation from the transformed one, see Crost and Traeger (2010).

In the numerical implementation of the model it turns out useful to maximize over the abatement cost Λ_t , which is a strictly monotonic transformation of μ_t (see equation 5). This switch of variables turns the constraints on the optimization problem linear.

C Further results

Figure 7 shows the impact of varying the growth rate in a deterministic environment. The three growth rates represented correspond to the original DICE-2007 growth rate, a 0.84 percent decrease, and a 0.84 increase at all times. The left panel in Figure 7 shows the optimal abatement rate and the right panel shows the optimal social cost of carbon (SCC). The differences in the three time paths reflects the importance of growth for the timing and level of abatement. The higher the deterministic growth rate, the lower the initial CO₂ abatement: Wealth is taken from rich future generations and transferred to the relatively poorer current generations by depreciating environmental capital. In the lowest growth scenario the optimal policy never reaches full abatement (not shown). With relatively high growth, abatement increases steeply, is between 12 and 13 percent higher after 100 years, and reaches full abatement more than 50 years earlier as compared to the DICE-2007 baseline. Observe that the deterministic growth

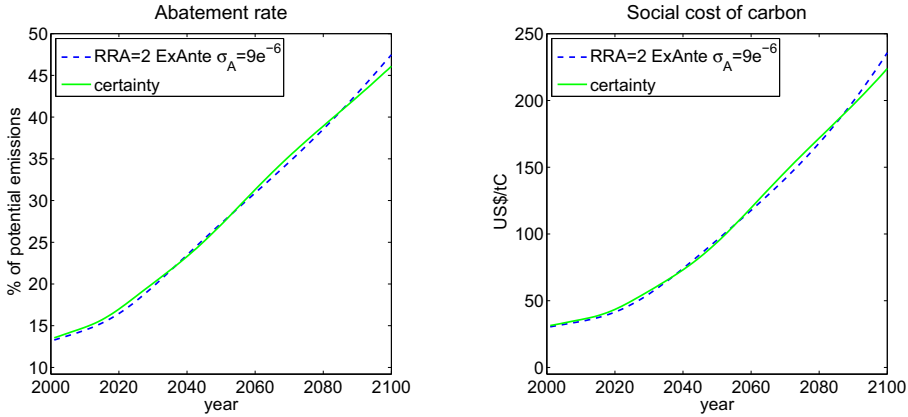


Figure 8: compares the optimal abatement rate and social cost of carbon under certainty and ex ante uncertainty with $\sigma(x) = g_{A,0}/\sqrt{20}$.

rate changes all imply a non-monotonic change of the abatement rate with respect to the original deterministic DICE-2007 baseline. In contrast, our uncertainty simulation all change the optimal climate policy into a single direction, increasing abatement and SCC for $\eta = 2$ and decreasing abatement and SCC for $\eta = 2/3$. Figure 8 shows that a probability weighted averaging of the deterministic runs has almost no effect on optimal policy.⁷ Such probabilistic averaging, or Monte-Carlo analysis, of deterministic runs is sometimes performed as a first approximation to modeling uncertainty.

Figure 9 shows that persistence in the growth shock also increases the negative effect of uncertainty on mitigation in the setting with a low propensity to smooth consumption over time, where $\eta = 2/3$ (and $RRA = 10$). Numerically the case of $\eta = 2/3$ is harder than the case where $\eta = 2$ because the parameter choice effectively reduces the contraction of the Bellman equation (6). Thus, we had to settle for a considerably lower levels of uncertainty, still showing how persistence increases the negative effect of uncertainty on mitigation.

⁷The figure averages five runs corresponding to Gaussian quadrature nodes in a normal distribution over the permanent growth ‘shock’, where $\sigma(\hat{z}) = g_{A,0}/\sqrt{20}$, $E[\hat{z}] = -\sigma^2(\hat{z})/2$. Three of these runs are the ones depicted in Figure 7. The permanent shocks imply major changes to the growth dynamics, including destabilizing the numerical model. Thus, we chose a relatively smaller variance to illustrate the effect of Monte-Carlo averaging as opposed to the one chosen in the truly stochastic model.

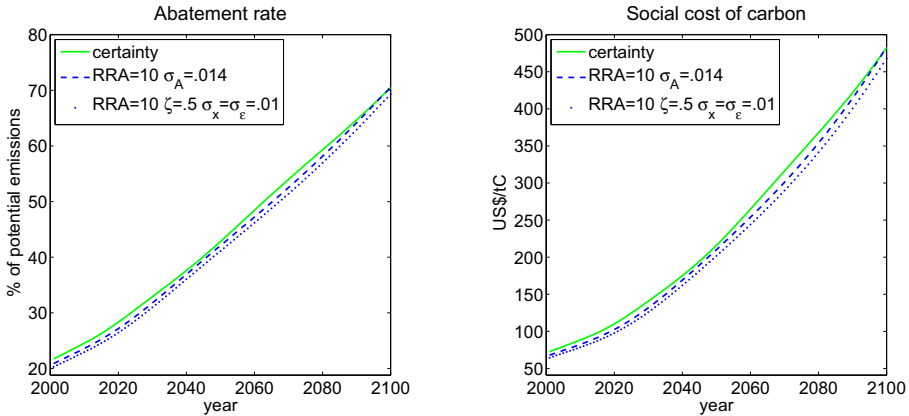


Figure 9: compares the optimal abatement rate and the social cost of carbon under certainty, iid uncertainty and persistent uncertainty with persistence $\zeta = 0.5$ for $RRA = 10$ and $\eta = 2/3$.

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Table 1: Parameters of the model

Economic Parameters		
η	$\frac{2}{3}, 2$	intertemporal consumption smoothing preference
RRA	0, 2, 9.5, 50	coefficient of relative Arrow-Pratt risk aversion
b_1	0.00284	damage coefficient; for uncertain scenario normally distributed with standard deviation 0.0013 (low) and 0.0025 (high)
b_2	2	damage exponent; for uncertain scenario normally distributed with standard deviation 0.35 (low) and 0.5 (high)
δ_u	1.5%	pure rate of time preference
L_0	6514	in millions, population in 2005
L_∞	8600	in millions, asymptotic population
g_L^*	0.035	rate of convergence to asymptotic population
K_0	137	in trillion 2005-USD, initial global capital stock
δ_K	10%	depreciation rate of capital
κ	0.3	capital elasticity in production
A_0	0.0058	initial labor productivity; corresponds to total factor productivity of 0.02722 used in DICE
$g_{A,0}$	1.31%	initial growth rate of labor productivity; corresponds to total factor productivity of 0.9% used in DICE
δ_A	0.1%	rate of decline of productivity growth rate
σ_0	0.1342	CO ₂ emission per unit of GDP in 2005
$g_{\sigma,0}$	-0.73%	initial rate of decarbonization
δ_σ	0.3%	rate of decline of the rate of decarbonization
a_0	1.17	cost of backstop 2005
a_1	2	ratio of initial over final backstop cost
a_2	2.8	cost exponent
g_Ψ	-0.5%	rate of convergence from initial to final backstop cost
Climatic Parameters		
T_0	0.76	in °C, temperature increase of preindustrial in 2005
M_{preind}	596	in GtC, preindustrial stock of CO ₂ in the atmosphere
M_0	808.9	in GtC, stock of atmospheric CO ₂ in 2005
$\delta_{M,0}$	1.7%	initial rate of decay of CO ₂ in atmosphere
$\delta_{M,\infty}$	0.25%	asymptotic rate of decay of CO ₂ in atmosphere
δ_M^*	3%	rate of convergence to asymptotic decay rate of CO ₂
B_0	1.1	in GtC, initial CO ₂ emissions from LUCF
g_B	-1%	growth rate of CO ₂ emission from LUCF
s	3.08	climate sensitivity, i.e. equilibrium temperature response to doubling of atmospheric CO ₂ concentration with respect to preindustrial concentrations
EF_0	-0.06	external forcing in year 2000
EF_{100}	.3	external forcing in year 2100 and beyond
σ_{forc}	3.2%	warming delay, heat capacity atmosphere
σ_{ocean}	0.7%	warming delay, ocean related

Chapter IV

Optimally Climate Sensitive Policy: A Comprehensive Evaluation of Uncertainty & Learning (w. Christian Traeger)

Abstract

The long-run relation between greenhouse gas concentrations and temperatures is currently unknown. We learn this climate sensitivity over the next decades and centuries by observing stochastic global temperatures. This paper analyzes the effects of stochastic temperatures and uncertain climate sensitivity on optimal mitigation and investment policy in a Bayesian learning model. We find that stochasticity of temperature increases optimal capital investment, while uncertainty about climate sensitivity increases optimal greenhouse gas mitigation. The scientific community has not reached a consensus about the Bayesian prior governing climate sensitivity. We address this lack of confidence into the Bayesian prior by modeling deep uncertainty in terms of ambiguity and smooth ambiguity aversion. We find that ambiguity aversion has a negligible effect on welfare and no effect on optimal policy.

Keywords

climate change, uncertainty, ambiguity aversion, smooth ambiguity model, Bayesian learning, recursive utility, dynamic programming, integrated assessment, DICE

1 Introduction

The scientific community has been aware of the greenhouse effect for several decades, but is still greatly uncertain about the long-term temperature change induced by a given level of greenhouse gas (GHG) emissions. Climate sensitivity characterizes how a doubling of GHG concentrations in the atmosphere affects the global mean surface temperatures in the long-run equilibrium. The climate system's complexity makes an assessment of climate sensitivity difficult. Temperatures fluctuate and feedback processes take time until they become clearly observable. Yet climate sensitivity lies at the core of the economic climate change problem: It determines the cost of GHG emissions. If the true value turns out high, temperatures will rise strongly and cause severe damages. If temperatures hardly react to emissions, then we should not cut back on economic production in order to mitigate GHGs. Our current decisions have to deal with uncertainty over the true value of climate sensitivity and stochastic global surface temperatures which will cover up the true climate sensitivity for decades if not centuries to come.

In this paper we analyze how temperature stochasticity, uncertainty about climate sensitivity, and learning affect optimal GHG mitigation policies. For that purpose, we translate the widely employed integrated assessment model DICE by Nordhaus (2008) into a recursive dynamic programming model. The stochasticity of temperatures for any given climate sensitivity determines the speed of learning. At the same time, this stochasticity increases expected damages in a world governed by damages that are convex in temperatures. We disentangle the effect temperature stochasticity has on optimal policies from the effect of uncertainty about the climate sensitivity. We find that temperature stochasticity affects investment, increasing the capital stock and, thus, future production and emissions. In contrast, uncertainty about the long-run GHG-temperature relation increases the optimal abatement rate. We show how these two effects interact under different speeds of learning.

The scientific community does not agree on a particular prior on climate sensitivity. This prior is a much more subjective distribution than e.g. the stochasticity of temperatures from one year to the next. We extend our model to explicitly distinguish between attitudes with respect to mostly objective stochasticity and subjective, or low confidence uncertainty. For this purpose, we employ Klibanoff et al. (2009)'s smooth ambiguity model and analyze the effect of ambiguity aversion on optimal climate policy. Ambiguity aversion captures the attitude of decision makers who prefer a world with well known probabilities to a world governed by subjective guesstimates. Traeger (2011) shows that the smooth ambiguity model is fully rational once a decision maker acknowledges that objective and subjective lotteries are distinct objects.¹ Like any decision

¹The model satisfies in particular time consistency and is a straightforward adaptation of the classical

maker can have different preference about apples and bananas, he can also have different risk attitudes with respect to high confidence and with respect to low confidence probabilistic estimates.

Closest to our analysis is the seminal work by Kelly and Kolstad (1999), who investigate Bayesian learning about climate sensitivity in a similar model. While Kelly and Kolstad (1999) analyze learning time in much detail, they pay relatively little attention to the precise effects of uncertainty and stochasticity on optimal policies. They compare a situation in which the initial Bayesian prior is lower than the true value of climate sensitivity to the optimal policy given this true value is known with certainty. They do the same in a case in which the prior is higher than the true value. They find that the optimal abatement rate under learning is initially closer to the case of a low true climate sensitivity value. In contrast, we find that the optimal abatement rate is closer to the high scenario, i.e. the decision maker hedges against the bad outcome. This difference results from our more symmetric comparison as well as most likely our higher numerical precision. Our focus, however, is not on the comparison of learning scenarios in which the climate sensitivity is either lower or higher than the expected value. We analyze how a mean preserving spread over the prior changes the optimal policy, i.e., we compare scenarios that differ in uncertainty and in stochasticity, but keep expected values constant. Our analysis therefore requires more numerical precision (benefiting from the evolving computational power and a more precise approximation procedure). We disentangle effects of stochasticity and deep uncertainty, and analyze how ambiguity aversion affects the optimal policies. Moreover, the early DICE model employed by Kelly and Kolstad (1999) implies extremely low optimal abatement in the range of 7.5 – 13% of total emissions in the current century, vastly different mitigation policy's than our currently used DICE-2007 version.

Leach (2007) expands the work by Kelly and Kolstad (1999) by modeling a second climate parameter, the warming delay, as uncertain. He finds that modeling more than a single parameter as uncertain may practically prohibit learning. He also considers the effect on optimal abatement in his setting, suggesting that a decision maker may lower abatement rates in order to speed up learning. Also Leach (2007) focuses on the process and speed of learning more than on the implications for optimal abatement policies. In fact, we find that in our model the speed of learning has very little influence on the currently optimal policies, much less than the level of stochasticity and the prior uncertainty.

Millner et al. (2010) and Lemoine and Traeger (2010) relate to our analysis in that they model ambiguity aversion in the context of climate sensitivity. Millner et al. (2010) assume that the decision maker has a prior over the right model governing the warming of the world. These models differ based on different climate sensitivity distributions

von Neumann-Morgenstern axioms.

taken from the scientific literature. For a given climate sensitivity distribution, the authors generate the evolution of consumption in each of these models. In every period the decision maker averages over the different models with an exogenous, ambiguous distribution, exhibiting smooth ambiguity aversion. The authors find that ambiguity aversion has small welfare effects given the standard DICE damage function and large welfare effects when employing a more convex damage function. In contrast to Millner et al. (2010) we do not analyze the welfare effect of a given policy, but derive the optimal policy under uncertainty. Moreover, our decision maker behaves as a fully consistent Bayesian learner.

Lemoine and Traeger (2010) model abrupt and irreversible changes in climate sensitivity once the climate system crosses an a priori unknown temperature threshold. The learning in their model reduces to realizing that any temperature level reached without crossing the threshold is safe. Before and after crossing the threshold the climate sensitivity is known deterministically. In contrast, our decision maker learns the climate sensitivity smoothly over the course of decades and centuries. Lemoine and Traeger (2010) capture an extreme of sudden irreversible changes due to highly non-convex feedback processes. There, learning ahead of time is impossible. In contrast, we capture a world with smooth feedbacks and continuous learning. Moreover, the decision maker in Lemoine and Traeger (2010) can reduce the ambiguous risk of crossing a threshold by reducing emissions. In our model, the decision maker can only reduce her uncertainty about the ambiguous climate sensitivity prior by increasing emissions in order to learn faster. This difference translates into a different effect of ambiguity aversion in the two models.

2 Model

We model a Ramsey growth economy that interacts with the climate system. Emissions increase the stock of greenhouse gases in the atmosphere, which heat the planet. Heating the atmosphere takes time and eventually increases the surface temperature. Climate sensitivity measures the relation between the greenhouse gas stock and the long-run-equilibrium temperature. Our social planner has an initial prior over climate sensitivity and updates this prior based on her observations (learning). She invests into capital and purchases emission reductions. Her optimal decisions anticipate learning.

We formulate our model as a discrete time, infinite horizon dynamic programming problem and introduce period by period temperature stochasticity. To model learning about climate sensitivity we employ Bayesian inference. Using the smooth ambiguity model by Klibanoff et al. (2009), we specify separate preferences for risk and ambiguity. The decision maker maximizes welfare subject to the constraints imposed by the climate system and the economy. We characterize the economy, the climate system and the

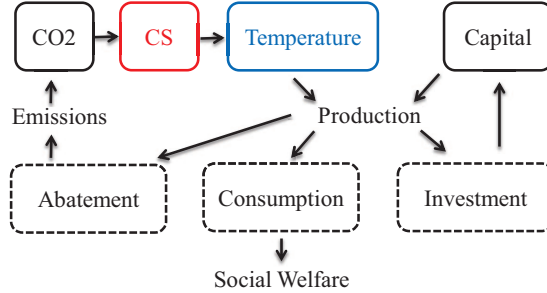


Figure 1: The main relations in the climate-enriched economy model. Control variables are represented by dashed rectangles. Main state variables are depicted by solid rectangles. Climate sensitivity (‘CS’) is uncertain. The decision maker has a prior over its value (2 state variables). Temperature is stochastic.

interactions between them in a modified version of DICE (Nordhaus, 2008). First, we reformulate it as a recursive dynamic programming problem. The recursive structure enables the analysis of stochasticity and deep uncertainty: (Stochastic) temperature and the decision maker’s prior over climate sensitivity are captured by state variables. In addition, the smooth ambiguity preferences are defined recursively and can only be employed in a dynamic programming setting. Second, we replace its climate system consisting of three equilibrating carbon sinks by an single atmospheric stock of carbon. This simplification is necessary to reduce the computational burden and circumvent the ‘curse of dimensionality’. Figure 1 depicts a stylized representation of our model, and Appendix A contains the complete mathematical representation.

2.1 Bayesian learning about climate sensitivity

The decision maker learns the value of climate sensitivity from observing the CO_2 stock M_t and temperatures T_t over time. We assume that she knows all the transient feedbacks that are not part of climate sensitivity. She believes that the following initial prior $\Pi(s)$ governs climate sensitivity

$$\tilde{s}_0 \sim \Pi(s) = \mathcal{N}(\mu_{s,0}, \sigma_{s,0}^2) \quad \text{with} \quad \mu_{s,0} = 3 \quad \sigma_{s,0}^2 = 1, 2 .$$

Most commonly, estimates of climate sensitivity take fat-tailed distributional forms such as the log-normal. To simplify the characterization of learning, we assume a normal distribution. Given this limitation, $\sigma_{s,0}^2 = 3$ is a rounded-up empirical approximation to the set of distributions found in IPCC (2007). To analyze the dynamics of learning, we vary the prior variance over the interval $[1, 2]$.²

²We also analyze $\sigma_{s,0}^2 = 3$, and the results are qualitatively the same. Given some minor remaining numerical challenges, we use $\sigma_{s,0}^2 = 2$ instead.

Every period the decision maker updates this prior. Precisely, she foresees what a future realization of the temperature teaches her about climate sensitivity distribution. Conditional on a given value of climate sensitivity s and, thus, the expected value of temperature $\mu_{T,t}$, we assume a normal temperature distribution capturing stochasticity of temperature

$$\tilde{T}_t \sim \mathcal{N}(\mu_{T,t}(s), \sigma_T^2) \quad \text{with} \quad \sigma_T^2 = 0.05, 0.2, 0.7 .$$

The variance σ_T^2 is exogenously given temperature volatility. Empirical estimates suggest annual volatility in global mean temperature in the order of magnitude of $\sigma_T^2 = 0.05$.³ For most of our analysis we will use considerably larger values. The reason is three-fold. First, this estimate measures only global averages, whereas the within-country fluctuations are significantly larger, closer to our next higher value of $\sigma_T^2 = 0.2$. The effective damage increase of stochastic temperatures is captured better by a country's temperature volatility. Second, our analysis assumes that climate sensitivity is the only uncertain parameter whilst every other climate parameter is known. Modeling multiple parameters as uncertain slows learning considerably (Leach, 2007). Higher stochasticity of the temperature also captures this reduction in the speed of learning. Third, and that motivates our value of $\sigma_T^2 = 0.7$, we use a high value of climate sensitivity to better disentangle temperature stochasticity, climate sensitivity uncertainty, and learning. In particular, the high stochasticity reduces learning significantly and allows us to isolate the effect of uncertainty about climate sensitivity.

The temperature mean depends on climate sensitivity s

$$\mu_{T,t} = s \chi_t(M_t, t) + \xi(T_t, t)$$

where

$$\begin{aligned} \chi_t(M_t, t) &= \sigma_{forc} \frac{\log \frac{M_{t+1}}{M_{pre}}}{\log 2} + \frac{EF_t}{\eta_{forc}} , \quad \text{and} \\ \xi(T_t, t) &= (1 - \sigma_{forc})T_t - \sigma_{ocean}\Delta T_t . \end{aligned} \tag{1}$$

The factors in (1) describe the global warming dynamics of our model. The multiplicative factor χ_t captures forcing given CO_2 in addition to other, exogenous radiative forcing. The additive factor ξ_t contains forcing from temperature and warming delay caused by atmospheric as well as the oceans' heat capacity. Those are so called transient feedbacks.

The decision maker's posterior in period t is the prior conditional on historic temperature realizations $\Pi(s|\hat{T}_1, \dots, \hat{T}_t)$. This posterior also depends on the historic CO_2 stock

³Kelly and Kolstad (1999) and Leach (2007) both use $\sigma_T^2 = 0.1$. Averaging temperatures over 174 countries and estimating yearly fluctuations with respect to a common trend over 109 years results instead in the lower $\sigma_T^2 = 0.04$. We thank Christian Almer from the University of Bern for this estimate.

information which we suppress for notational convenience. Given the current stock M_t , a realization of temperature \hat{T}_{t+1} in the subsequent period results in the updated posterior $\Pi(s|\hat{T}_1, \dots, \hat{T}_{t+1})$. In Appendix B we show that the updated posteriors are again normally distributed so that at all times $\Pi(s|\hat{T}_1, \dots, \hat{T}_t) = \mathcal{N}(\mu_{s,t}, \sigma_{s,t}^2)$ for some $\mu_{s,t}$ and $\sigma_{s,t}^2$. Moreover, we prove the following updating rules for the expected value

$$\mu_{s,t+1} = \frac{\chi_t^2 \sigma_{s,t}^2 \frac{\hat{T}_{t+1} - \xi_t}{\chi_t} + \sigma_T^2 \mu_{s,t}}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2}$$

and the variance

$$\sigma_{s,t+1} = \frac{\sigma_T^2 \sigma_{s,t}^2}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2}. \quad (2)$$

The new expected value of the parameter s is a weighted mean of the previous expected value and the inferred ‘‘climate sensitivity observation’’, $\frac{\hat{T}_{t+1} - \xi_t}{\chi_t}$. The weight on the new observation is proportional to the precision (the inverse of the variance) of the temperature and the magnitude of the multiplicative factor χ_t , which increases in the carbon stock. The decision maker learns faster the lower the temperature stochasticity and the larger the carbon stock. This insight follows from observing that the first summand in the bracket in equation (2) grows in $1/\sigma_T^2$ and in χ_t .

When we evaluate under ambiguity, we need to treat the two uncertainty layers separately rather than using the predictive distribution. The likelihood function capturing temperature stochasticity in equation (6) corresponds to risk and the decision maker evaluates it as usual. The uncertainty characterized by the posterior and governing the unknown climate sensitivity s corresponds to ambiguity and is evaluated using the additional aversion function.

2.2 Welfare specification and Bellman equation

The decision maker distinguishes between (objective) risk and subjective uncertainty. We model those two preferences by two different aggregator functions (Klibanoff et al., 2009). The social planner has standard constant relative risk aversion (CRRA) preferences with $\eta = 2$. This utility function describes her risk aversion as well as her desire to smooth consumption over time.⁴ In the second aggregator function $f(z) = [(1 - \eta)z]^{\frac{1-RAA}{1-\eta}}$, RAA characterizes aversion to subjective risk.⁵ Given those prefer-

⁴Those two preferences are a priori unrelated and could be disentangled as well, see Traeger (2012).

⁵ RAA stands for: Constant coefficient of **R**elative **A**mbiguity **A**version. Traeger (2012) defines the measure analogously to Arrow-Pratt relative risk aversion.

ences, the Bellman equation reads⁶

$$V(k_t, M_t, t, T_t, \mu_{s,t}, \sigma_{s,t}) = \max_{c_t, \mu_t} L_t \frac{\left(\frac{c_t}{L_t}\right)^{1-\eta}}{1-\eta} + \frac{\exp[-\delta_u]}{1-\eta} \times \quad (3)$$

$$\left\{ \int_{\Theta} \left((1-\eta) E_{\psi(s)} \left[V(k_{t+1}, M_{t+1}, t+1, \tilde{T}_{t+1}, \mu_{s,t+1}, \sigma_{s,t+1}) \right] \right)^{\frac{1-RAA}{1-\eta}} d\Pi(s) \right\}^{\frac{1-\eta}{1-RAA}}.$$

Welfare today is the maximized sum of instantaneous welfare from population (L_t) weighted per capita consumption c_t/L_t and expected future welfare. The state variables capital k , CO_2 stock M , time t , temperature T , and the climate sensitivity prior s completely describe the state of the climate and the economy. For a particular realization of climate sensitivity, temperature is stochastic and normally distributed, $\mathcal{N}(\mu_{T,t}(s), \sigma_T^2)$. The expectation operator in the inner bracket takes expected future welfare with respect to this well-known stochasticity. In addition, the decision maker is subjectively uncertain about climate sensitivity over which he has the prior $\Pi(s) \sim \mathcal{N}(\mu_{s,t}, \sigma_{s,t}^2)$. The integral with respect to the prior Π expresses this second uncertainty integration. The ambiguity aversion function $f(z) = [(1-\eta)z]^{\frac{1-RAA}{1-\eta}}$ curves the argument of this second uncertainty aggregation additionally, expressing additional aversion because of the low confidence over the prior. Observe that for $RAA = \eta$ the additional aversion vanishes and the Bellman equation collapses to its standard form.

The social planner maximizes the dynamic programming equation (3) by choosing abatement μ_t^7 and consumption c_t , subject to the set of equations characterizing the climate embedded economy.

The social cost of carbon is the welfare cost caused by the marginal emission unit. We recover the optimal social cost of carbon from the value function as the ratio of the marginal value of a ton of carbon and the marginal value of a unit of the consumption good

$$SCC_t = \frac{\partial_{M_t} V(\cdot)}{\partial_{K_t} V(\cdot)}.$$

The so called ‘‘balanced growth equivalent’’ measures welfare effects of a set of optimal abatement and consumption policies (Mirrlees and Stern, 1972; Anthoff and Tol, 2009). It is the per capita consumption \bar{c} that, growing at some fixed rate g , would yield the same welfare as the (optimal) policy \mathcal{A}

$$\bar{c}^{\mathcal{A}}(\cdot) = \left[\frac{(1-\eta)V^{\mathcal{A}}(\cdot)}{\frac{L_{\infty}}{1-\exp[(1-\eta)g-\delta_u]} - \frac{L_{\infty}-L_0}{1-\exp[(1-\eta)g-\delta_u-g_L^*]}} \right]^{\frac{1}{1-\eta}}.$$

⁶For numerical reasons we express several variables in our model in effective labor terms. In (3) c_t and k_t are normalized. For a description of the reformulation of the dynamic programming equation, see Crost and Traeger (2011).

⁷In the model we maximize with respect to abatement cost $\Lambda(\mu)$. This variable switch linearizes the constraints.

We can conveniently compare the policies under two alternative scenarios \mathcal{A} and \mathcal{B} by the percentage difference in their respective balanced growth equivalents

$$\Delta^{AB}\bar{c}(\cdot) = \frac{\bar{c}^{\mathcal{A}} - \bar{c}^{\mathcal{B}}}{\bar{c}^{\mathcal{A}}} = 1 - \left[\frac{V^{\mathcal{B}}(\cdot)}{V^{\mathcal{A}}(\cdot)} \right]^{\frac{1}{1-\eta}}.$$

2.3 Numerical implementation

We solve the dynamic programming equation (3) by function iteration, using the collocation method to approximate the value function. As basis functions we choose Chebychev polynomials with 22, 400 Chebychev nodes and coefficients⁸. The normal distributions for temperature stochasticity and the climate sensitivity prior are approximated by Gauss-Legendre quadrature with 3 nodes each⁹, resulting in a total of 9 nodes for the predictive distribution of temperature. The code is written in Matlab. We use the CompEcon toolbox by Miranda and Fackler (2002) to generate and evaluate the Chebychev polynomials, and let the solver KNITRO to carry out the optimization.

3 Stochasticity, uncertainty and learning

In this section we present the results for three different scenarios that build upon each other: Pure temperature stochasticity, climate sensitivity uncertainty and learning. We discuss ambiguity aversion separately in Section 4.

3.1 Temperature stochasticity

Figure 2 presents our results for pure temperature stochasticity. In this scenario, the decision maker knows the climate sensitivity. The four panels show the abatement rate, the social cost of carbon, the investment rate and emissions over the first 100 years. We distinguish three scenarios: deterministic temperature (‘certainty’) and three levels of stochasticity ($\sigma_T^2 = 0.05, 0.2, 0.7$). To generate the stochastic paths, we draw the expected value of temperature in each period, such that each period the shock is zero. This procedure ensures that for a given set of abatement and investment policies, temperatures coincide under certainty and stochasticity.

We find that even high temperature stochasticity has no discernible effect on the optimal abatement policy and the associated social cost of carbon. Individual shocks have no direct lasting impact on the climate system, so the decision maker sees no need to accommodate them by adjusting abatement. Investment in manmade capital however increases. All else equal, a high temperature realization causes high damages for one period. Production falls, and hence investment (in absolute terms) is lower. Thus

⁸Along each dimension of the state space: $k = 7, M = 4, t = 8, T = 4, c = 5, s = 5$.

⁹Results are unaffected by increasing the number of Gauss-Legendre nodes for temperature to 5.

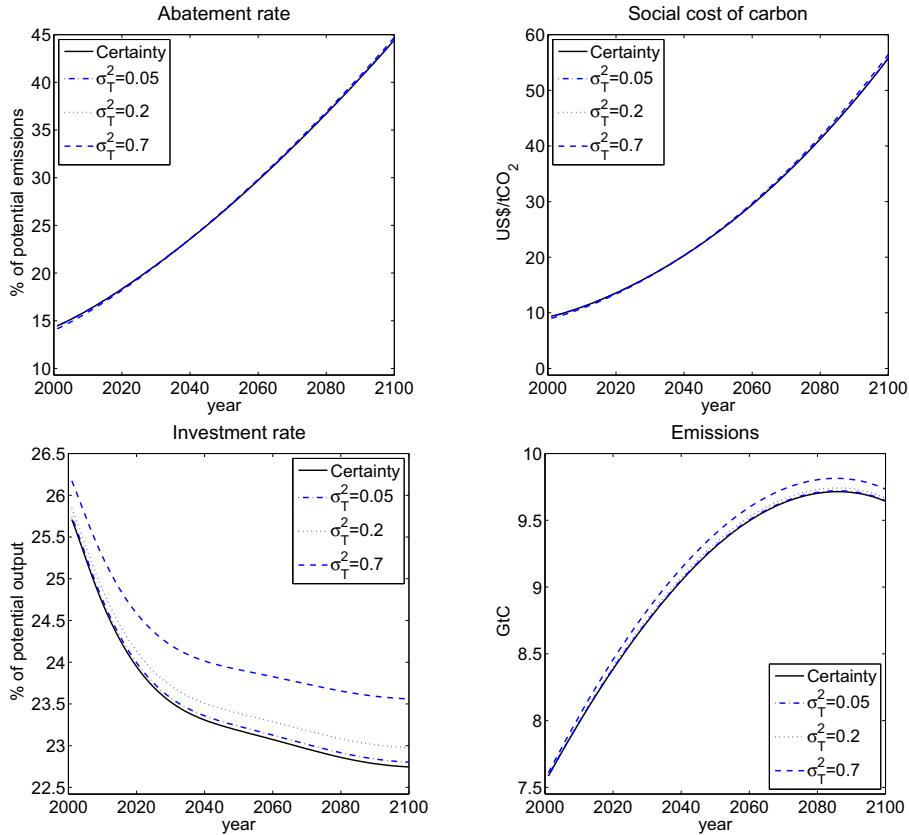


Figure 2: Abatement rate, optimal social cost of carbon, emissions and investment rate for the first 100 years with certain and stochastic temperature and 3 different temperature variances, $\sigma_T^2 = 0.05$, $\sigma_T^2 = 0.2$ and $\sigma_T^2 = 0.7$.

the single shock is propagated via the capital stock and remains in the economy for multiple time periods. To insure against this expected welfare loss, the decision maker invests more in manmade capital at any given time. The higher level of investment leads to a higher capital stock which eventually increases total emissions. In the present setting, temperature stochasticity alone hence does not influence the optimal social cost of carbon. Of course, this result crucially depends on the absence of non-linear, self-enforcing feedbacks (the melting of the Antarctic ice-sheet, or methane release from thawing permafrost, for example).

3.2 Climate sensitivity uncertainty

Figure 3 shows the same set of graphs as Figure 2. Temperature is stochastic with $\sigma_T^2 = 0.7$, whereas climate sensitivity is either known with certainty or subjectively uncertain with a climate sensitivity prior mean of $\mu_{s,0} = 3$. We distinguish two possible

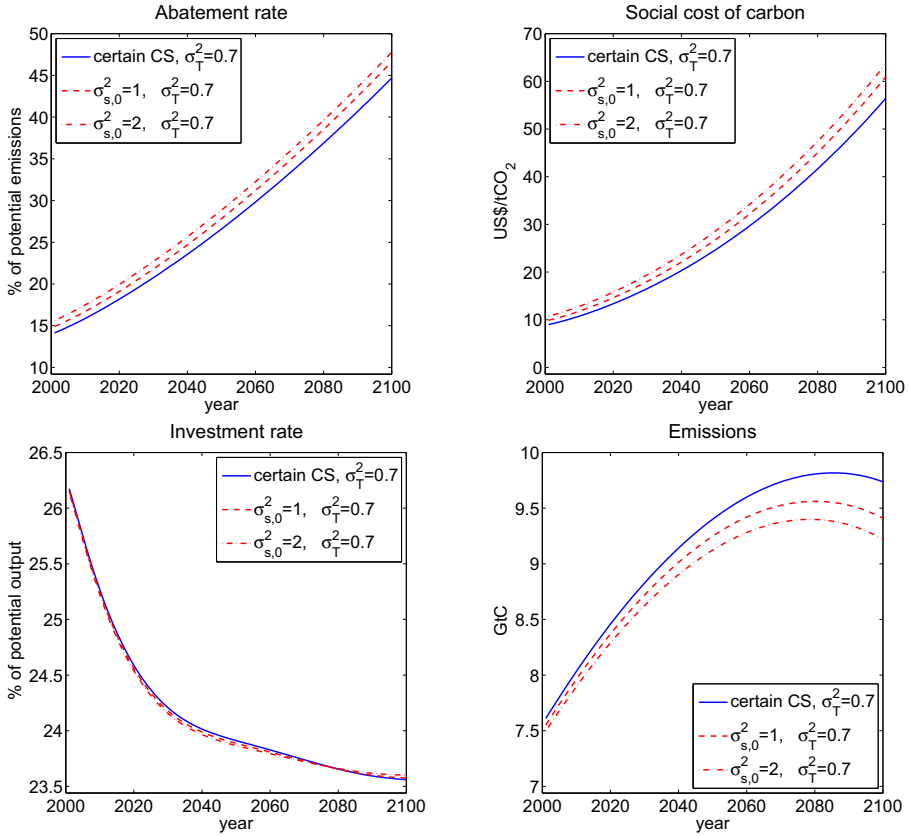


Figure 3: Abatement rate, social cost of carbon, emissions and investment rate for the first 100 years with stochastic temperature ($\sigma_T^2 = .7$) and uncertain climate sensitivity. The unbiased prior mean is $\mu_{s,0} = 3$. Initial prior variances are $\sigma_{s,0}^2 = 1$ and $\sigma_{s,0}^2 = 2$.

prior variances, $\sigma_{s,0}^2 = 1$ and $\sigma_{s,0}^2 = 2$. Again we plot the paths along the expected values for temperature stochasticity. The decision maker’s climate sensitivity prior is unbiased, so her expectation coincides with the true value.

Subjective uncertainty about the value of climate sensitivity modestly raises the abatement rate. With a temperature stochasticity of $\sigma_T^2 = 0.7$, initially 14.1% of emissions are abated. This rate increases to 44.7% after 100 years. With climate sensitivity uncertainty and an initial prior of $\mathcal{N}(3, 2)$, abatement starts out at 15.6% and rises to 47.8% over the first century. The initial rate is about 9% higher for the case with uncertainty and learning, and this difference falls over the century to approximately 6.5%.¹⁰

The decision maker acts precautiously in the face of possibly very different realities: In comparison to stochasticity, subjective uncertainty means the “realized shock” lasts

¹⁰For $\sigma_{s,0}^2 = 1$ the abatement rates are 14.9% (2000) and 46.6% (2100) respectively.

as long as the carbon stock, i.e. for several centuries. Learning is extremely slow, which we infer from the lack of convergence in the curves for uncertain and certain climate sensitivity (We can also see the learning speed directly in Figure 4). Therefore we interpret these results as caused by uncertainty, not learning. The temperature stochasticity $\sigma_T^2 = 0.7$ is so high that a single temperature observation receives very little weight when the decision maker updates her prior, and the decision maker anticipates that she will learn very little. Unlike stochastic temperature, subjective uncertainty does not affect the investment rate, such that higher abatement rates translate without moderation into lower emissions. We observe an interesting dichotomy: Temperature stochasticity affects economic policy (investment), whereas climate sensitivity uncertainty changes climate policies (abatement).

3.3 Learning about climate sensitivity

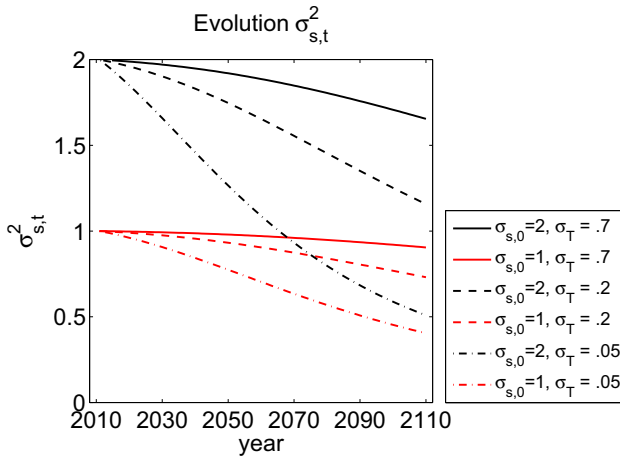


Figure 4: Climate sensitivity prior variance $\sigma_{s,t}^2$ for initial values $\sigma_{s,0}^2 = 2$ and $\sigma_{s,0}^2 = 1$ over time for three different values of temperature stochasticity ($\sigma_T^2 = 0.05, 0.2, 0.7$).

Figure 4 shows the evolution of the climate sensitivity prior variance for 100 years for two different initial priors and three different values of temperature stochasticity. Temperature is realized at its expected value, therefore the climate sensitivity prior mean remains unchanged at $\mu_{s,0} = \mu_{s,t} = 3$. The expectations of the decision maker are confirmed with every single observation, yet for $\sigma_T^2 = 0.7$ her confidence in her prior does increase only slightly over the first 100 years when temperature stochasticity is high. The two solid lines correspond to the scenarios in Figure 3. Only with lower levels of temperature volatility, in particular $\sigma_T^2 = 0.05$, meaningful learning takes place.

How does learning at different speeds translate optimal abatement policies? We compare learning for the three different values of temperature stochasticity in Figure

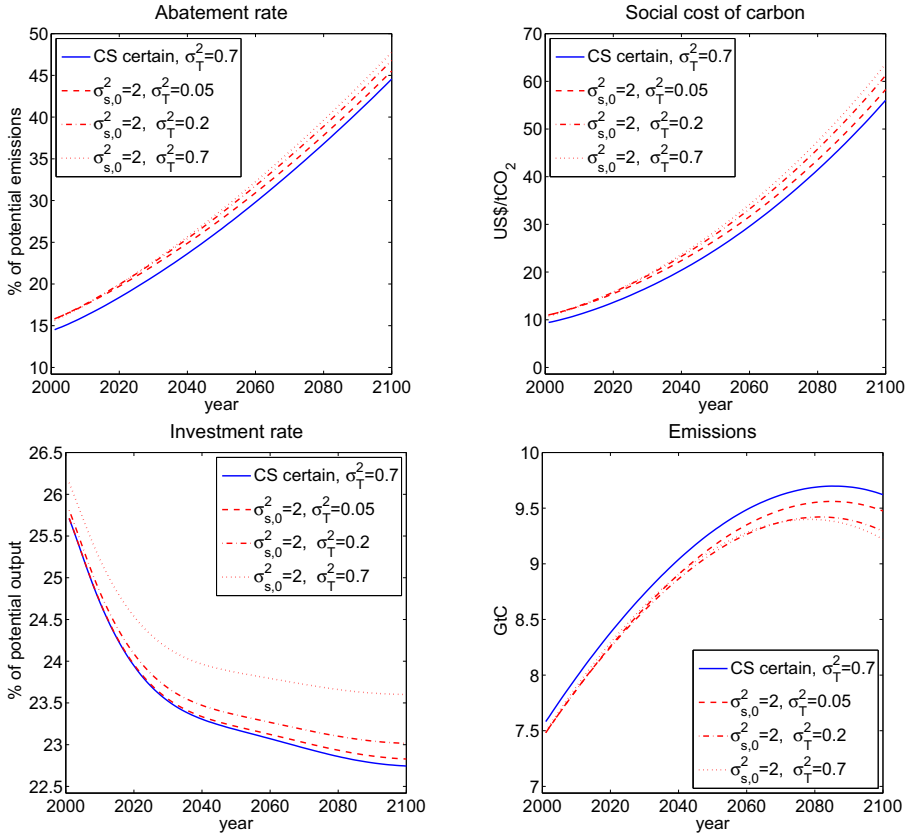


Figure 5: Abatement rate, social cost of carbon, investment rate and emissions for the first 100 years with uncertain climate sensitivity with initial prior variance $\sigma_{s,0}^2 = 2$. Three different temperature stochasticities: $\sigma_T^2 = .05$, $\sigma_T^2 = .2$ and $\sigma_T^2 = .7$.

5. Again we display the abatement rate, the social cost of carbon, investment and emissions. The paths for high temperature stochasticity ($\sigma_T^2 = .7$) and known climate sensitivity ('CS certain') are the same as in Figure 3. The new abatement paths with lower temperature volatilities start out at the same level as their high volatility counterpart, confirming that temperature stochasticity by itself has no significant effect. But rather immediately the difference in confidence in the prior becomes apparent, as the different paths approach the level of pure temperature stochasticity at different speeds. We also observe an interesting interaction between climate and economy for emissions: The emission paths for $\sigma_T^2 = .2$ and $\sigma_T^2 = .7$ cross. Investment increases permanently, as the temperature stochasticity is irreducible. For high stochasticity $\sigma_T^2 = .7$ also the impact from subjective uncertainty last for the entire century. For $\sigma_T^2 = .2$ on the contrary, the increase in abatement wears off as the decision maker becomes more confident over time. Hence emissions increase faster, and eventually overtake emissions for the

high stochasticity scenario. We do not observe active learning: As the investment rate remains the same, the decision maker does not increase emissions in order to speed up the learning process.¹¹

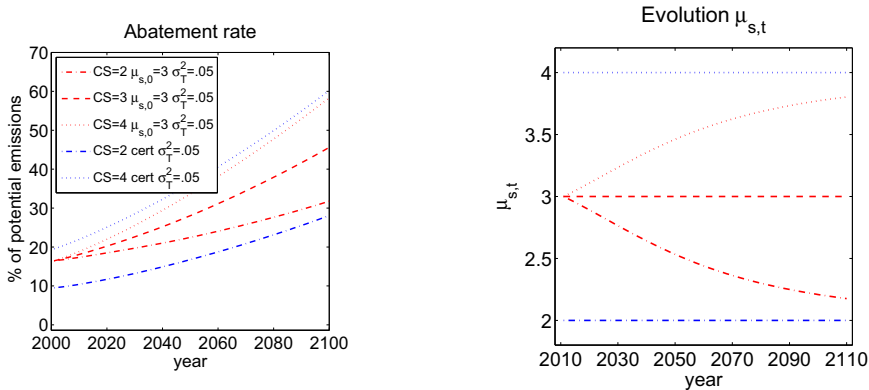


Figure 6: Abatement rate and dynamics of climate sensitivity prior mean ($\mu_{s,0} = 3$) for varying levels of true climate sensitivity ('CS'). Variance $\sigma_{s,0}^2 = 2$ and temperature stochasticity $\sigma_T^2 = 0.05$. True climate sensitivity 2, 3 and 4.

Another important aspect of the learning dynamics is the correction of wrong expectations. In Figure 6 we show how a decision maker with a wrong initial prior adjusts abatement, and how the mean of her prior evolves. Here we use the low, empirically accurate temperature volatility of $\sigma_T^2 = 0.05$. Correcting the wrong belief takes long, even with low temperature volatility. Secondly, the decision maker insures herself against a “too low” expected climate sensitivity: The initial abatement rate under uncertainty is biased towards the optimal policy under high, certain climate sensitivity.¹²

4 Ambiguity aversion

Ambiguity aversion captures the attitude of a decision maker who prefers a world with well known probabilities to a world governed by subjective guesstimates. In the case of subjectively uncertain climate sensitivity the decision maker’s ability to change or avoid subjective uncertainty is limited. She can increase her emissions in order to learn faster. However, the learning comes at the cost of being even worse-off in the situation where climate sensitivity turns out to be high. We find that overall ambiguity aversion has virtually no effect on optimal policies (Figure 7). The ambiguity averse social planner acts identically to one who evaluates risk and subjective uncertainty equally. Figure 9 shows an excerpt of the optimal abatement policy for the ambiguity averse

¹¹This contradicts the result by Leach (2007) in a setting with two uncertain parameters.

¹²This result is inconsistent with Kelly and Kolstad (1999), who however note that they face numerical difficulties.

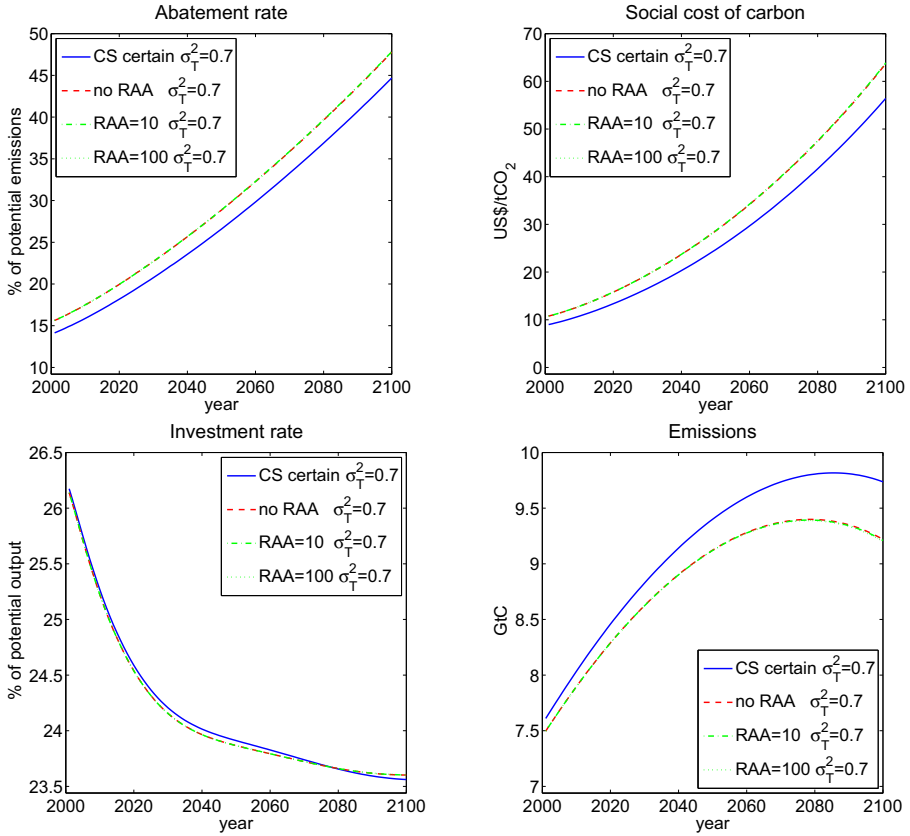


Figure 7: Abatement rate, social cost of carbon, investment rate and emissions for the first 100 years with stochastic temperature ($\sigma_T^2 = .7$), uncertain climate sensitivity with initial prior variance $\sigma_{s,0}^2 = 2$ and ambiguity aversion of $RAA = 10$ and $RAA = 100$.

social planner. It is identical to the corresponding control rule of a decision maker who has standard preferences. Abatement increases in temperature and in the the subjective uncertainty. Also, no loss in welfare is experienced even with strong aversion to subjectivity. In the same Figure, the value function is displayed. Whereas welfare is decreasing and slightly concave in temperature, reflecting the damages associated with warming, the confidence in expected climate sensitivity ($\sigma_{s,0}^2$) has no effect on welfare. Figure 8 compares the balanced growth equivalent for an ambiguity averse decision maker to a standard expected utility maximizer. The percentage difference in per capita consumption that makes them equally well off is in the order of magnitude of 10^{-5} , or numerically zero.

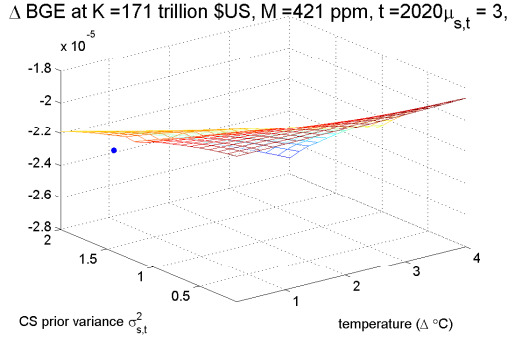
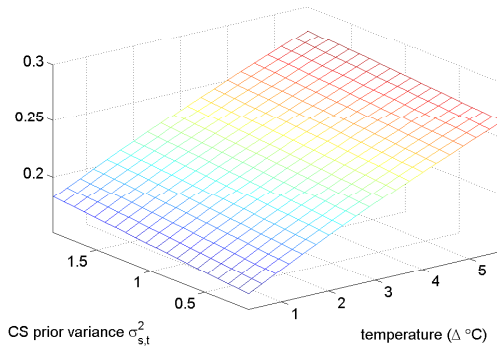


Figure 8: Difference in balanced growth equivalent between expected utility maximizer and ambiguity averse decision maker with $RAA = 10$. $\Delta RAA10\text{-learn} \bar{c} = \frac{\bar{c}^{RAA10} - \bar{c}^{learn}}{\bar{c}^{RAA10}}$. Plotted over climate sensitivity prior variance and temperature for the year 2020, a carbon stock of 896 GtC (421 ppm CO_2), a capital stock of 171 US trillion dollars and a climate sensitivity prior mean of $\mu_{s,0} = 3$.

Abatement at K=171 trillion \$US, M=421ppm, t=2020, $\mu_{s,t} = 3$



Value at K =171 trillion \$US, M =421ppm, t =2020, $\mu_{s,t} = 3$,

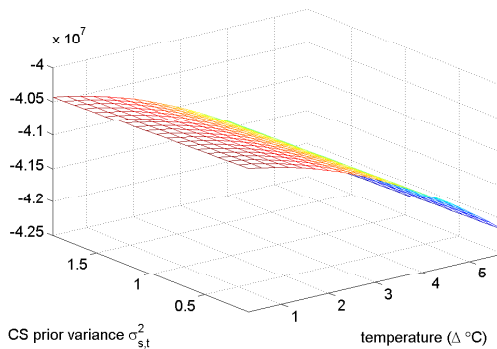


Figure 9: Value function and control rule for an ambiguity averse decision maker with $RAA = 10$ for the year 2020, a carbon stock of 896 GtC (421 ppm CO_2), a capital stock of 171 US trillion dollars and a climate sensitivity prior mean of $\mu_{s,0} = 3$.

5 Conclusions

We incorporate stochastic temperature, deep Bayesian uncertainty about climate sensitivity, and ambiguity aversion into a widely used model of integrated climate change assessment. Stochastic temperatures affect both, the speed of learning and the expected damages. Isolating the damage effect we show that it slightly increases optimal investment but leaves the optimal abatement rate and SCC largely untouched. In consequence, increased investment and growth lead to a slight increase in the optimal emission level. Uncertainty about the climate sensitivity counteracts this effect by increasing the optimal abatement rate. Over time, learning reduces uncertainty, but the uncertainty effect dominates at least for the remainder of the current century in reducing the optimal absolute level of GHG emissions.

The prior over climate sensitivity relies necessarily on incomplete climate models and the literature suggests a variety of different functional forms for the probability distribution. We choose the normal distribution because of its convenience in modeling Bayesian learning in a dynamic optimizing model. While other forms are closer to current estimates, any individual prior by itself will lack confidence by a major part of the scientific community. We therefore extend our analysis by considering the Bayesian prior as an ambiguous distribution. We analyze the policy effect of aversion against the lack of confidence into these priors and find that ambiguity aversion has virtually no effect on our optimal policies. The important consequence of this finding is that even if uncertainty about climate sensitivity is large, and even if we do not trust any given guess of the distribution, we should still simply follow the results of the Bayesian learning model.

The main message of our model is that even deep uncertainty about the relation between GHG emissions and climate change should not make us wait and see. In contrast, we find that this uncertainty increases optimal current abatement. Moreover, we find that the precise speed of learning, which has been a major focus of earlier analyses, is not of major relevance for current policies. Interesting extensions of the model include more convex damage functions and fat-tailed climate sensitivity priors. Both of these extensions would increase the effects of temperature stochasticity and uncertainty about climate sensitivity, and the main question would be whether the effects still balance in a similar way. While increased damage convexity mostly enhances the damage effects of stochasticity, fat tails can decrease the speed of learning.

Acknowledgements

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Appendix

A Details on the climate enriched economy model

The following model is largely a reproduction of DICE-2007. The three most notable differences are the annual time step (DICE-2007 features ten year time periods), the infinite time horizon, and the replacement of the carbon sink structure by a decay rate. This simplification is necessary because each carbon sink would require an own state variable in a recursive framework, which is computationally too costly. For a detailed description of the procedure, see Crost and Traeger (2011). All parameters are characterized and quantified in Table B on page 110.

Carbon in the atmosphere is accumulated according to

$$M_{t+1} = M_{pre} + (M_t - M_{pre})(1 - \delta_M(t)) + E_t \quad \text{with}$$

$$\delta_{M,t} = \delta_{M,\infty} + (\delta_{M,0} - \delta_{M,\infty}) \exp[-\delta_M^* t] .$$

The stock of CO₂ (M_t) exceeding preindustrial levels (M_{pre}) decays exponentially at the rate $\delta_M(M, t)$. This decay rate falls exogenously over time to replicate the carbon cycle in DICE-2007, mimicking that the ocean reservoirs reduce their uptake rate as they fill up (see Lemoine and Traeger, 2010). The variable E_t characterizes yearly CO₂ emissions, consisting of industrial emissions and emissions from land use change and forestry B_t

$$E_t = (1 - \mu_t) \sigma_t A_t L_t k_t^\kappa + B_t .$$

Emissions from land use change and forestry fall exponentially over time

$$B_t = B_0 \exp[g_B t] .$$

Industrial emissions are proportional to gross production $A_t L_t k_t^\kappa$. They can be reduced by abatement. As in the DICE model, we in addition include an exogenously falling rate of decarbonization of production σ_t

$$\sigma_t = \sigma_{t-1} \exp[g_{\sigma,t}] \quad \text{with} \quad g_{\sigma,t} = g_{\sigma,0} \exp[-\delta_\sigma t] .$$

The economy accumulates capital according to

$$k_{t+1} = [(1 - \delta_k) k_t + y_t - c_t] \exp[-(g_{A,t} + g_{L,t})] ,$$

where δ_K denotes the depreciation rate, $y_t = \frac{Y_t}{A_t L_t}$ denotes production net of abatement costs and climate damage per effective labor, and c_t denotes aggregate global consumption of produced commodities per effective unit of labor. Population grows exogenously by

$$L_{t+1} = \exp[g_{L,t}] L_t \quad \text{with} \quad g_{L,t} = \frac{g_L^*}{\frac{L_\infty - L_0}{L_\infty - L_0} \exp[g_L^* t] - 1} .$$

Here L_0 denotes the initial and L_∞ the asymptotic population. The parameter g_L^* characterizes the convergence from initial to asymptotic population. Technology grows exogenously

$$\tilde{A}_{t+1} = A_t \exp [\tilde{g}_{A,t}] \quad \text{with} \quad \tilde{g}_{A,t} = g_{A,0} * \exp [-\delta_A t] . \quad (4)$$

Net global GDP per effective unit of labor is obtained from the gross product per effective unit of labor as follows

$$y_t = \frac{1 - \Lambda(\mu_t)}{1 + D(T_t)} k_t^\kappa$$

where

$$\Lambda(\mu_t) = \Psi_t \mu_t^{a_2} \quad (5)$$

characterizes abatement costs as percent of GDP depending on the emission control rate $\mu_t \in [0, 1]$. The coefficient of the abatement cost function Ψ_t follows

$$\Psi_t = \frac{\sigma_t}{a_2} a_0 \left(1 - \frac{(1 - \exp[g_\Psi t])}{a_1} \right)$$

with a_0 denoting the initial cost of the backstop, a_1 denoting the ratio of initial over final backstop, and a_2 denoting the cost exponent. The rate g_Ψ describes the convergence from the initial to the final cost of the backstop.

Climate damage as percent of world GDP depends on the temperature difference T_t of current to preindustrial temperatures and is characterized by

$$D(T_t) = b_1 T_t^{b_2} .$$

Nordhaus (2008) estimates $b_1 = 0.0028$ and $b_2 = 2$, implying a quadratic damage function with a loss of 0.28% of global GDP at a 1 degree Celsius warming.

Exogenous forcing EF_t from non-CO2 greenhouse gases, aerosols and other processes is assumed to follow the process

$$EF_t = EF_0 + 0.01(EF_{100} - EF_0) \times \max\{t, 100\} .$$

Note that it starts out slightly negatively. The other relevant temperature change equations are found in section 2.1 in the main text. For more details, see Lemoine and Traeger (2010).

B Updating rules for climate sensitivity prior

This appendix derives the updating rules for the prior and the predictive distribution. Let $l_t(x_{t+1}|s) = \mathcal{N}(\mu_{x,t+1}, \sigma_T^2|s, x_t, h_t)$ denote the likelihood function in period t . Then¹³

$$\Pi(s|\hat{T}_1, \dots, \hat{T}_{t+1}) = \frac{l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, \dots, \hat{T}_t)}{\int_{-\infty}^{\infty} l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, \dots, \hat{T}_t)ds}. \quad (6)$$

We use the sign \propto to denote proportionality and suppress the normalization constants of the distributions, finding

$$\begin{aligned} l_t(x|s) \Pi(s|\hat{T}_1, \dots, \hat{T}_t) &\propto \exp\left(-\frac{(x - \mu_{x,t+1}(s))^2}{2\sigma_T^2}\right) \exp\left(-\frac{(s - \mu_{s,t})^2}{2\sigma_{s,t}^2}\right) \\ &\propto \exp\left(-\frac{(x - (s\chi_t + \xi_t))^2}{2\sigma_T^2} - \frac{(s - \mu_{s,t})^2}{2\sigma_{s,t}^2}\right) \\ &\propto \exp\left(-\frac{x^2 - 2x(s\chi_t + \xi_t) + (s\chi_t + \xi_t)^2}{2\sigma_T^2} - \frac{s^2 - 2s\mu_{s,t} + \mu_{s,t}^2}{2\sigma_{s,t}^2}\right) \\ &\propto \exp\left(-\frac{x^2 - 2xs\chi_t - 2x\xi_t + s^2\chi_t^2 + 2s\chi_t\xi_t + \xi_t^2}{2\sigma_T^2} - \frac{s^2 - 2s\mu_{s,t} + \mu_{s,t}^2}{2\sigma_{s,t}^2}\right) \\ &\propto \exp\left(-\frac{1}{2}\left[s^2\left(\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}\right) - 2s\left(\frac{(x - \xi_t)\chi_t}{\sigma_T^2} + \frac{\mu_{s,t}}{\sigma_{s,t}^2}\right) + \frac{x^2 - 2x\xi_t + \xi_t^2}{\sigma_T^2} + \frac{\mu_{s,t}^2}{\sigma_{s,t}^2}\right]\right) \\ &\propto \exp\left(-\frac{1}{2}\left[s^2\left(\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}\right) - 2s\left(\frac{(x - \xi_t)\chi_t}{\sigma_T^2} + \frac{\mu_{s,t}}{\sigma_{s,t}^2}\right) + \frac{(x - \xi_t)^2}{\sigma_T^2} + \frac{\mu_{s,t}^2}{\sigma_{s,t}^2}\right]\right) \\ &\propto \underbrace{\exp\left(-\frac{1}{2}\left(\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}\right)\left(s - \frac{\frac{(x - \xi_t)\chi_t}{\sigma_T^2} + \frac{\mu_{s,t}}{\sigma_{s,t}^2}}{\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}}\right)^2\right)}_{\equiv \bar{\Pi}} \\ &\quad \cdot \exp\left(-\frac{1}{2}\left[-\frac{\left(\frac{(x - \xi_t)\chi_t}{\sigma_T^2} + \frac{\mu_{s,t}}{\sigma_{s,t}^2}\right)^2}{\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}} + \frac{(x - \xi_t)^2}{\sigma_T^2} + \frac{\mu_{s,t}^2}{\sigma_{s,t}^2}\right]\right) \\ &\propto \bar{\Pi} \cdot \exp\left(\frac{1}{2} \frac{\left(\frac{(x - \xi_t)\chi_t}{\sigma_T^2}\right)^2 + 2\frac{(x - \xi_t)\chi_t}{\sigma_T^2} \frac{\mu_{s,t}}{\sigma_{s,t}^2} + \left(\frac{\mu_{s,t}}{\sigma_{s,t}^2}\right)^2 - \frac{(x - \xi_t)^2 \chi_t^2}{\sigma_T^2 \sigma_{s,t}^2} - \frac{\mu_{s,t}^2 \chi_t^2}{\sigma_{s,t}^2 \sigma_T^2} - \frac{(x - \xi_t)^2 \frac{1}{\sigma_{s,t}^2}}{\sigma_T^2} - \frac{\mu_{s,t}^2 \frac{1}{\sigma_{s,t}^2}}{\sigma_{s,t}^2 \sigma_T^2}}{\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}}\right) \\ &\propto \bar{\Pi} \cdot \exp\left(-\frac{1}{2\sigma_T^2 \sigma_{s,t}^2} \frac{(x - \xi_t)^2 - 2(x - \xi_t)\chi_t \mu_{s,t} + \mu_{s,t}^2 \chi_t^2}{\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}}\right) \\ &\propto \bar{\Pi} \cdot \exp\left(-\frac{1}{2} \frac{(x - \xi_t - \chi_t \mu_{s,t})^2}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2}\right). \end{aligned}$$

¹³This simplified updating equation only using the latest prior and the latest observation is a consequence of our convenient choice of the conjugate prior.

The following predictive distribution P_{t+1} governs the temperature realization in period $t + 1$ incorporating stochasticity and parameter uncertainty

$$P_{t+1}(x) = \int_{-\infty}^{\infty} l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, \dots, \hat{T}_t)ds \propto \exp\left(-\frac{1}{2}\frac{(x - \xi_t - \chi_t\mu_{s,t})^2}{\chi_t^2\sigma_{s,t}^2 + \sigma_T^2}\right).$$

It is the normal distribution $\mathcal{N}(\chi_t\mu_{s,t}, \chi_t^2\sigma_{s,t}^2 + \sigma_T^2)$. We find the posterior

$$\begin{aligned} \Pi(s|\hat{T}_1, \dots, \hat{T}_{t+1}) &= \frac{l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, \dots, \hat{T}_t)}{\int_{-\infty}^{\infty} l_t(x_{t+1}|s)\Pi(s|\hat{T}_1, \dots, \hat{T}_t)ds} \\ &\propto \exp\left(-\frac{1}{2}\left(\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}\right)\left(s - \frac{(\hat{T}_{t+1}-\xi_t)\chi_t + \frac{\mu_{s,t}}{\sigma_{s,t}^2}}{\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}}\right)^2\right). \end{aligned}$$

Thus, if $\Pi(s|\hat{T}_1, \dots, \hat{T}_t)$ is distributed normally with expected value $\mu_{s,t}$ and variance $\sigma_{s,t}$, then the posterior in the subsequent period $\Pi(s|\hat{T}_1, \dots, \hat{T}_{t+1})$ is also distributed normally with expected value

$$\mu_{s,t+1} = \frac{\frac{\chi_t^2}{\sigma_T^2} \frac{\hat{T}_{t+1}-\xi_t}{\chi_t} + \frac{1}{\sigma_{s,t}^2} \mu_{s,t}}{\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}} = \frac{\chi_t^2 \sigma_{s,t}^2 \frac{\hat{T}_{t+1}-\xi_t}{\chi_t} + \sigma_T^2 \mu_{s,t}}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2}$$

and variance

$$\sigma_{s,t+1} = \left(\frac{\chi_t^2}{\sigma_T^2} + \frac{1}{\sigma_{s,t}^2}\right)^{-1} = \frac{\sigma_T^2 \sigma_{s,t}^2}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2}.$$

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Table 1: Parameters of the model

Economic Parameters		
η	2	intertemporal consumption smoothing and risk aversion preference
RAA	10, 100	coefficient of relative ambiguity aversion
b_1	0.00284	damage coefficient; for uncertain scenario normally distributed with standard deviation 0.0013 (low) and 0.0025 (high)
b_2	2	damage exponent; for uncertain scenario normally distributed with standard deviation 0.35 (low) and 0.5 (high)
δ_u	1.5%	pure rate of time preference
L_0	6514	in millions, population in 2005
L_∞	8600	in millions, asymptotic population
g_L^*	0.035	rate of convergence to asymptotic population
K_0	137	in trillion 2005-USD, initial global capital stock
δ_K	10%	depreciation rate of capital
κ	0.3	capital elasticity in production
A_0	0.0058	initial labor productivity; corresponds to total factor productivity of 0.02722 used in DICE
$g_{A,0}$	1.31%	initial growth rate of labor productivity; corresponds to total factor productivity of 0.9% used in DICE
δ_A	0.1%	rate of decline of productivity growth rate
σ_0	0.1342	CO ₂ emission per unit of GDP in 2005
$g_{\sigma,0}$	-0.73%	initial rate of decarbonization
δ_σ	0.3%	rate of decline of the rate of decarbonization
a_0	1.17	cost of backstop 2005
a_1	2	ratio of initial over final backstop cost
a_2	2.8	cost exponent
g_Ψ	-0.5%	rate of convergence from initial to final backstop cost
Climatic Parameters		
T_0	0.76	in °C, temperature increase of preindustrial in 2005
M_{preind}	596	in GtC, preindustrial stock of CO ₂ in the atmosphere
M_0	808.9	in GtC, stock of atmospheric CO ₂ in 2005
$\delta_{M,0}$	1.7%	initial rate of decay of CO ₂ in atmosphere
$\delta_{M,\infty}$	0.25%	asymptotic rate of decay of CO ₂ in atmosphere
δ_M^*	3%	rate of convergence to asymptotic decay rate of CO ₂
B_0	1.1	in GtC, initial CO ₂ emissions from LUCF
g_B	-1%	growth rate of CO ₂ emission from LUCF
s	3.08	climate sensitivity, i.e. equilibrium temperature response to doubling of atmospheric CO ₂ concentration with respect to preindustrial concentrations
EF_0	-0.06	external forcing in year 2000
EF_{100}	.3	external forcing in year 2100 and beyond
σ_{forc}	3.2%	warming delay, heat capacity atmosphere
σ_{ocean}	0.7%	warming delay, ocean related